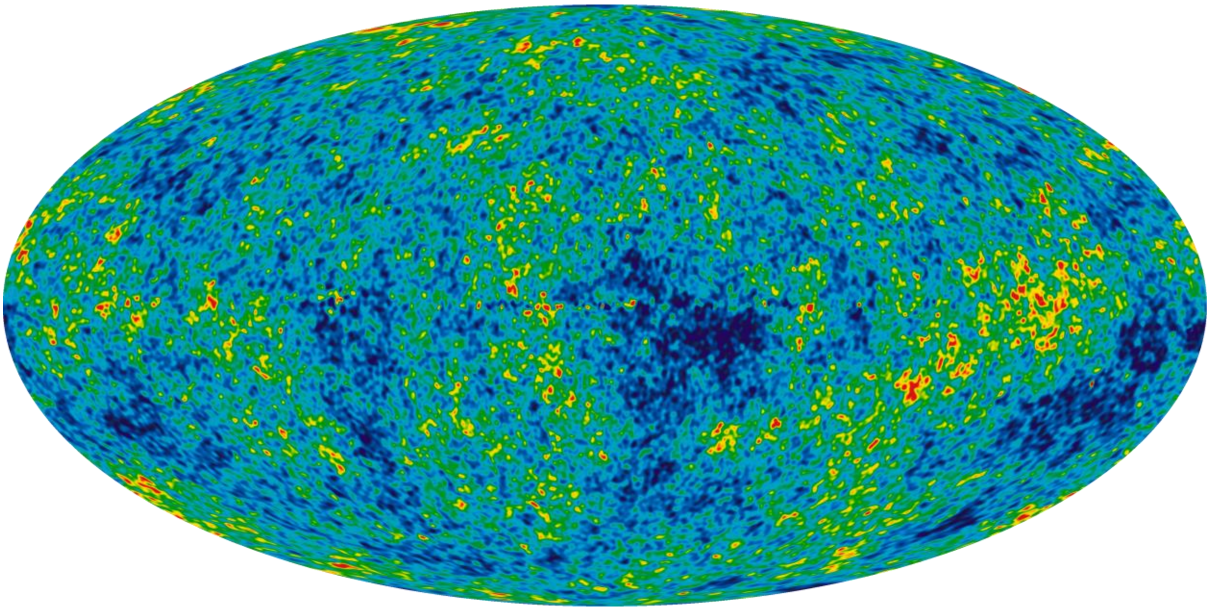
CURIOSITY IN UNIVERSAL PHYSICAL CONSTANTS
BOOK ONE



Roya Arefyan

Edited by: Ali Tabrizi nejad

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Introduction

Physics is an exact science and therefore depends on measurement, while all measurements themselves require sensory understanding. As a result, all the ideas used in physics come from the world of sensory perception. It follows that the laws of physics ultimately refer to the events of the senses. These sensing can be done by human and human-made sensing apparatus. Given this fact, many scientists and philosophers tend to believe that in the end, physics deals exclusively with this particular world. What they have in mind, of course, is the world of the human senses. This is called, for example, when from a physics point of view in terms of the "object" of view, what is taken in ordinary terms is simply a combination of different sensory data placed in one place. It is worth noting that logic cannot refute this view, because logic itself cannot take us out of the realm of the senses.

Accurate and reliable measurements are the foundation of international trade, high-tech manufacturing, human health and safety, environmental protection, global climate studies, and many other aspects of the world today. Accurate measurements make possible the basic experiments of scientific theory on which all these depend. This introduction examines the performance of accurate measurements, the current state of our knowledge of fundamental constants, and their increasing use as the basis for the International System of Units (SI). This preface will also address the broad role of metrology. The fundamental constants of physics play an increasing role in the science of measurement. This was predicted by Maxwell, who made this famous statement in a speech to the Society for the Advancement of Science in Britain in 1870.

"If we want to achieve standards of length, time, and mass that are completely permanent, we must meet them not in the dimensions, motion, or mass of our planet, but in the wavelengths, vibrations, and absolute masses of these incorruptible immutable and quite similar molecules".

In all the formulations of the basic theories of physics and their application in the real world, certain fundamental fixed quantities appear again and again. These quantities are called fundamental physical constants. They have special and widely used symbols. These constants are so important that they must be known as accurately as possible. They include the velocity of light in vacuum ***c***, the charge of an electron whose absolute value is the principal unit of electric charge ***e***, the mass of the electron ***me***, the Planck constant ***h***, and the microstructure constant ***α***.

Of course, there are other important values that can be measured with high accuracy, such as the density of a particular piece of silver, or the lattice distance (distance between the plates of atoms) of a particular crystal of silicon, or the distance from the earth to the sun. However, these quantities are not usually considered as fundamental constants. First, they are not universal variables because they are very specific, very close to the specific properties of the substance or system on which the measurements are based. Such quantities lack generality because they do not consistently appear in the basic theoretical equations of physics on which all science is based, nor they are the properties of the fundamental particles of physics of which all matter is composed.

Knowing the numerical values of fundamental constants with high accuracy is important for at least two reasons. First, the quantitative predictions of the basic theories of physics depend on the fixed numerical values that appear in the theories. Therefore, if one hopes to achieve a slightly accurate description of the physical world, it is necessary to know their exact values. Second, and more importantly, the careful study of the numerical values of these constants, as determined by various experiments in various fields of physics, can in turn test the general consistency and accuracy of the basic theories of physics.

The five constants above are listed because they represent different sources of fundamental constants. The speed of light **c** and the Planck constant **h** are examples of quantities that occur naturally in the mathematical formulation of some fundamental physical theories, the first in James Clerk Maxwell's theory of electric and magnetic fields and Albert Einstein's theory of relativity, and the second in particle theory i.e. atomic or quantum theory. For example, in Einstein's theories of relativity, mass and energy are equivalent, energy **E** is directly proportional to mass **m**, with a proportionality constant, the square of the speed of light **c^2** - the famous equation **$E = m c^2$**. In this equation **E** and **m** are variables and **c** are the constant. In quantum theory, energy and frequency symbolized as **E** and **v** respectively. A photon i.e. a quantum unit of electromagnetic energy such as light or heat radiation, is associated with $E = h v$, where, **h** is the proportionality constant.

In mathematics, two sequences of numbers (data) are proportional or direct if their corresponding elements have a fixed ratio. This ratio is called the proportionality constant or the proportionality constant. If the corresponding elements have a fixed product, also called the coefficient of proportion, the two sequences are inversely proportional. This definition is usually extended to different related committees, commonly called variables.

A natural unit system, in which the values of fundamental constants such as c and $\hbar$ are set to one and all quantities are expressed in units, is usually referred to as a computational convenience. However, we show that this system of natural units also has a physical justification. We discuss natural units, including the definitions of each of the seven base units in the International System of Units (**SI**), in terms of one unit. We also consider fundamental constants that can be classified as unit dependent or unit independent. Unit-independent constants, the values of which are not determined by the human contracts of the units, may be interpreted as inherent constants of nature.

In the fascinating drama of scientific ideas, the physical constants are definitely the main characters. We will see that the correct interpretation of constants and their role often involves discussing the basic problems of physics, with the participation of the best minds who have advanced science: Newton, Einstein, Boltzmann, Planck, and others. These debates created new ideas and sometimes completely changed the human worldview. To measure constants, as we shall see. The most delicate and accurate experimental methods have been used.

The term "dimensions" emphasizes that, along with the problems of unit systems, much attention has been paid to dimensional systems. It should be emphasized that it should only be related to the word "units" and not to the word "quantity".

Measurements made in a scientific experiment give a definitive answer to a major question posed by science, allow for a choice between two hypotheses, and sometimes even lead to the emergence of a new theory or even a new branch of science. Thus, measuring the speed of light in different media facilitated the establishment of the light wave theory, an attempt to measure the absolute speed of the earth's motion led to the emergence of the theory of relativity, measuring the distribution of energy on the planet. The black body spectrum gave rise to quantum theory.

No branch of engineering, from structural mechanics to complex chemical companies, from communications engineering to nuclear power plants, can exist without a measurement system developed to determine the dimensions and properties of the product, and to provide adequate control conditions for machinery and Production processes exist. One of the remarkable things about the behavior of the universe is that it seems to be based on mathematics with a remarkable degree of precision. The more we understand about the physical world, and the deeper we delve into the laws of nature, the more it seems that the physical world is almost evaporating and we are stuck in mathematics. The deeper we understand the laws of physics, the more we are drawn to this world of mathematics and mathematical concepts. Let's look at the scales we have to deal with in the world, as well as the role of our place in the world. We can summarize all these scales in one diagram (Figure 1). On the left side of the chart are the time scales and on the right side are the corresponding distance scales. At the bottom of the chart, on the left, is the shortest time scale that is physically significant.

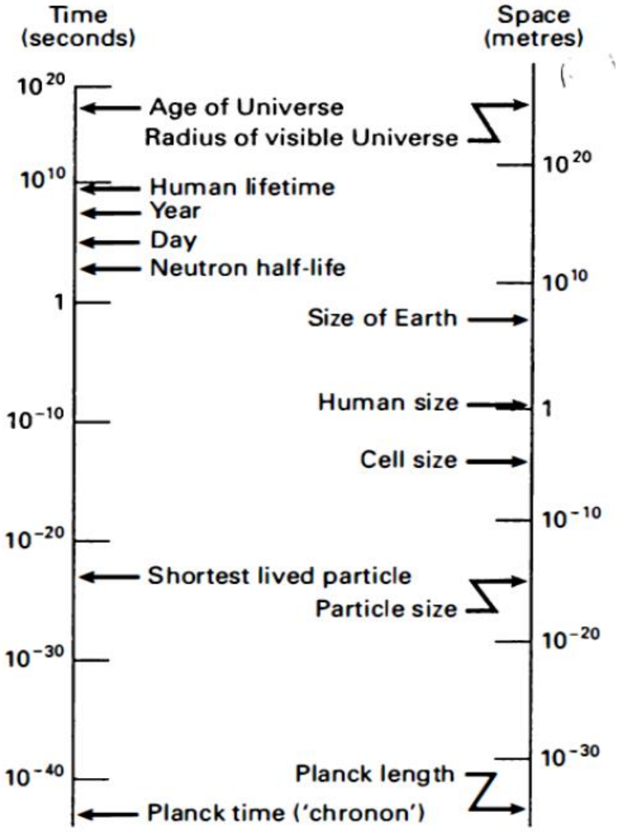


Figure above shows Spatial and temporal measurements in the world.

The key point is that, in physics, we use two very different types of methods. To describe small-scale behavior, we use quantities at the level of quantum mechanics, and to describe large-scale behaviors, we use measurements at the level of classical mechanics in Figure 2. One of the things people say about quantum mechanics is that it is fuzzy and uncertain, but that is not true. As long as you stay at this level, quantum theory is definitive and accurate. In its most familiar form, quantum mechanics involves the use of the equation known as the Schrيندinger equation, which governs the behavior of the physical state of a quantum system - called its quantum state - and this is a definite equation. I used the letter U to describe this quantum surface activity. Uncertainty in quantum mechanics only arises when you do something called magnifying an event from the quantum level to the classical "measurement" level.

On a large scale, we use classical physics, which is quite definite - these classical laws include Newton's laws of motion, Maxwell's laws for the electromagnetic field, which include electricity, magnetism, and light, and Einstein's theories of relativity, the theory of special relativity which It deals with large velocities and the theory of general relativity dealing with large gravitational fields. These very, very strict rules apply on a large scale.

Metrology is defined as the science of measurement and, if interpreted broadly, encompasses much of experimental physics. The term usually refers in a narrower sense to a part of the science of measurement for the preparation, maintenance, and propagation of fixed sets of units. It can also be used to provide support in trade with weights and measurement rules and data for quality control in production. In this limited sense, metrology has taken on the nature of an art or skill instead of science, and has attracted little academic attention.

Measurement is a set of manipulations of objects or physical systems based on a defined protocol that results in a number. The number uniquely indicates the magnitude or intensity of some of the quantities present in the experimental object. This number is used to form the basis of a decision that affects a human goal or meets human needs, the satisfaction of which depends on the characteristics of the experimental body.

The study of the laws governing physical phenomena and the application of these laws in other fields such as engineering involves the measurement of physical quantities. Measuring a quantity means comparing it with a standard quantity of the same nature. This standard quantity is known as the unit of measurement. The unit is selected for a given physical quantity in such a way that it is convenient to measure those quantitative values that are often encountered in practice.

When humans began to qualify their description of natural phenomena, metrology, the science of measurement alongside geometry, and mathematics developed. However, from the flames of antiquity to modern times, the role of metrology was more limited to the need for commercial, social, or scientific transactions at the local or national maximum. With the onset of the Renaissance, and especially in Western Europe over the last century, metrology quickly gained international character as a result of the growing need for more accurate measurements and common standards in the emerging industrial society. Although metrological concerns are deeply rooted in the basic sciences, until recently they were largely considered a caring aspect by many in the scientific community.

With the advent of the twentieth century - which was scientifically demonstrated by Planck's constant discovery of relativity and quantum theory of matter - the role and scope of metrology changed dramatically. The science of measurement was now firmly and closely associated with an increasing number of emerging fundamental atomic constants. As profound as this change was in the first half of this century, in the last two or three decades there have been even more dramatic advances in metrology. The discovery of lasers and the Josephson effect have made revolutionary advances in accurate measurements of length, time, electricity and other derived quantities. We only have to mention the sudden improvement in the speed of light by two large times in the early 1970s. Another example is the amazing advancement in measuring electromagnetic constants using ultra-sensitive Josephson superconductors.

With these and other stunning results in preclinical measurements and the definition of "supervision" standards and fundamental constants, metrology has dramatically shifted its image from its reputation to a discipline that now pays attention to the borderline advances of the physical sciences. Eventually, metrology is becoming more and more involved in the needs of high-precision technologies, so that today it is a truly universal discipline that encompasses a wide range of interests, from quantum electrodynamics and particle physics to global communications and interplanetary navigation.

The SI system consists of seven main units: meters, seconds, kilograms, amps, kelvins, moles and candelas. In the time since its adoption, certain basic units have been corrected as science, and our ideas about the nature of some physical phenomena related to the corresponding values have been developed. In this book, we discuss metrology and its application in physics, which manifests itself in constants and units. We hope this is a worthwhile endeavor.

gravitational constant

The first constant in physics is the gravitational constant. So, it's wise to start with this constant. Because the story of the fundamental constants of physics begins with the arrival of the gravitational constant. It was in the late seventeenth century that Isaac Newton introduced it to quantify the gravitational pull of physical objects. Over the next two centuries, abundant observational evidence seemed to dispel any possible doubts about the validity of Newton's theory. Horizon - and the mechanism of gravity remained unknown until, in the early years of the twentieth century, Albert Einstein's genius gave us a whole new perspective on the world. Today, with his theory of general relativity and discoveries in astrophysics, the theory of gravity and the gravitational constant are back in the spotlight.

By the way, the word gravity is derived from the Latin gravis meaning heavy. The force that is felt and observed. An invisible force that keeps the Earth's atmosphere close to the planet so that life on the planet can remain as it is today. This force is applied to low mass objects by very massive objects. Apart from such a gravitational effect, we usually do not see any other effect of gravitational attraction. For example, when we talk to each other, we never feel that we are being brought together by a force of about 3×10^{-7} N. Here N stands for Newton, the force that accelerates 1 m/s^2 to 1 kg of mass. Thus, the two ships that sail at sea do not collide. Of course, the reason is obvious: gravitational forces are very weak in these cases. In fact, gravity is the weakest force known in nature. Electric force with a coefficient of 10^{42} is stronger than gravitational force. This force is much weaker than the nuclear forces that hold the particles that make up the nucleus together.

However, in contrast, the role of nuclear and electric forces in interaction among celestial bodies is insignificant. This is due to the fact that, nuclear forces, decrease very rapidly with distance therefore, have no effect outside nuclei. While electrical forces act at large distances among charges. However, since large bodies are electrically neutral, they have no electrical effect on each other. Finally, we say that, the strength of the gravitational force depends on the mass of the body.

Two 50 kg balls at a distance of 10 cm from each other with a force of 1.6×10^{-5} N absorb each other. This is a small force. But the earth with mass $M_E = 6 \times 10^{24}$ kg attracts moon with mass $M_m = 7 \times 10^{22}$ with a force of 10^{20} N and itself is attracted with a force of 10^{22} N by the sun with mass $M_S = 2 \times 10^{30}$.

In fact, these forces operating our universe in large distances among planets, stars, galaxies and supercluster of galaxies. They determine and operate the evolution of the universe in such distances, as well as our tiny planet.

As mentioned above, gravitational forces or interactions, in fact *fields*, are very weak among small terrestrial objects, while they are very large and distant among large celestial bodies. Consequently, we have many natural limitations for the experimental study of this force. But man, always has a trick up his sleeve. Thus, despite these problems, progress has been made in the theory.

A physical theory must be expressed " in terms of mathematical expressions and equations, to present a quantitative description of observed phenomena". Thus, a theory in physics should "explain experimental facts already established" i.e. the theory must be testable and it should "predict new facts" i.e. be predictable. If these two conditions of testability and predictability be fulfilled then the theory is considered to be true. This is also the story happened to Newton’s law of universal gravitation and its constant. With this saying, it is natural to ask how was the law discovered.

The question is, "what actually makes two bodies attract each other?". Aristotle (384-322 B.C.) classified all bodies as light and heavy. Light bodies go up like smoke, heavy bodies such as, stones fall down. He declared that, falling freely with no force to act on a particular body is so natural such that there is no need to explain. He also believed that heavy bodies fall down faster that light bodies. In contrast with mathematician like Pythagoras who believed in a heliocentric universe, Aristotle claimed that our earth is the center of universe. This is in fact because of the philosophical idea that human being is the most supreme existence that creator created it thus its home is the most supreme place in universe. This idea also was extended to Islamic golden age.

after the fall of the Roman and Persian empires about 600-700 AD the Muslim scientist come to the scene about 800-1300 AD. Therefore, it wasn't the dark age. However about 200 years mostly speeded to translate texts from Greek, Roman, Persian and Indian.

In gravity, Persian polymath pur Sina (980–1037) or as it is known Avicenna (pur"پور" is a Persian word for the "the son of" as it said in Arabic Ibn "ابن") agreed with John Philoponus of Alexandria (490 - 570) i.e. "the moved object acquires an inclination from the mover" as in projectile motion. This is known as impetus theory. Pur Sina in "the book of healing" stated that there exist a *mayl* or acquired power i.e. inclination that cause any motion. Thus, in his idea there is a difference between force and inclination. He "argued that an object gained *mayl* when the object is in opposition to its natural motion. Consequently, the continuous inclination the continuous motion such that an "object will be in motion until the *mayl* is spent". However the difference between pur Sina and Philoponus idea on inclination is that Philoponus believed it as a declining temporary virtue even in a vacuum but Pur Sina thought it as a "persistent, requiring external forces such as air resistance to dissipate it".

It was another Persian polymath Biruni proposed that all celestial bodies have mass, weight and gravity just like planet earth. He also studied the theory of center of gravity for 3-dimential objects. In fact, he and Khazini established the theory of gravity for the first time. Biruni was naturalist than being a philosopher so he believed that every theory must be proved with observational and experimental methods. Biruni is the only Islamic scientist who questioned the geocentric model. However, he was on Ptolemaic system. he showed that astronomical facts can also be explained by assuming the Earth revolves around the Sun.

although there was always a dispute from ancient time that it is earth round the sun or it is all heaven round the earth. It was very natural for most ancient scientist to believe in geocentric model since they were seeing everything in heaven apparently turning around the earth. But the first recorded scientist that proposed a heliocentric model was Pythagoras and his school. Their idea was based on the fact that the earth's position in universe was so gross to be the center. Thus, they placed the sun on the center and all other planets was rounding in uniform circles. In later years Aristotle proposed geocentric model with philosophical dispute. Thus, philosophy was right or mathematics?

Biruni proposed that there is no reasonable cause to reject the heliocentric model although for practical calculation i.e. for calendars or so, a geocentric model is easier. Now we know that all things from atom to galaxies are rounding around each other and nothing in universe is stationary.

A more serious criticism on the issue was made by Leonardo da Vinci (1452-1519). He proposed that the "Earth resided not in the center of the Universe but in the center of its own elements". Gravity was showed experimentally by Galileo Galilei (1564-1642), by dropping a pair of balls of wood and iron from the top of the tower of Pisa at the same time. They hit the ground simultaneously. With this experiment the value of experiment against mere philosophical reasoning was proved. This rejected Aristotle simple idea that heavier objects hit the ground faster than lighter ones. Instead it proved that, "in the absence of air resistance all objects fall to the earth at the same acceleration independent of their mass". Same acceleration means a not changing and uniform or constant speed. He roughly measured the magnitude of this free fall acceleration. Now we corrected it as $g = 9.8$ m/s^2. This is in fact the milestone of Newton's idea of gravitation. However, some other evidence was also needed for Newton to complete his theory.

Nicolaus Copernicus (1473-1543) was the first astronomer and mathematician who proposed the first heliocentric view of our universe. It was a uniform circular model. But what was the reason of this circular motion of the planets round the sun? he couldn't answer because of the lack mathematical tools on that time. However, Johannes Kepler (1571-1630) was able to discover the mathematical description of planetary motion round the sun.

With the help of, planetary angular position data exactly observed and recorded by Tycho Brahe (1546-1601), and adopted the heliocentric approach of Copernicus, he proposed that, "all bodies attract each other with a force proportional to their masses and inversely proportional to the square of the distance between them". Thus, he showed the planets round the sun in elliptical orbits.

Kepler was not able to show "any link between the force of gravitation and the behavior of the planets" due to the lack of mathematical tools. Here the great mind of Newton was needed to gather all efforts from ancient time to 17th century in a law of universal gravitation in a more rigorous mathematical formulation. The word "universal" means the most general. It was a voyage of motion, from the world of kinematics toward the world of dynamics i.e. from how to why!

Sir Isaac Newton (1642-1727) proposed the laws of dynamics on earth and heaven a mathematical method called calculus. But here we consider it in a simpler way. His question was, how the motion of the planets around the sun could happen? What is the reason of this motion? He found this motion as an interaction or because of an interaction among different celestial bodies which was expressed as an attraction among the bodies before him. Thus, he should find the main features of this interaction to establish the law of gravitation. Thus, he corrected Kepler's laws and sated them as:

"motion of the planets was governed by a radial attractive force directed to the Sun and inversely proportional to the square of the distance between the Sun and a planet".

Newton found attraction force between an apple and earth is the same as with the moon. They are exactly from the same origin i.e. difference in their masses. Firstly, he considered the moon orbit around the earth, approximately circular. Then find the equation for centripetal acceleration a as:

$$a = r\omega^2$$

Here r is the radius of the orbit thus its value is 60 times of the Earth's radius $r = 60\ \times\ 6370\ km$. The quantity ω is the Moon's angular acceleration. In fact, it is the period of the moon revolution around the earth, i.e. 3.7 days. Thus;

$$\omega = \frac{2\pi}{3.7}\quad \text{rad/day}$$

Now he simply found the value of acceleration a as:

$$a = r\omega^2 = 60 \times 6370 \times 10^5 \left(\frac{2\times 3.14}{27.3\times 86400}\right)^2 \text{cm/ s}^2 = 0.27\ \text{cm}/s^2$$

Now if Kepler's inverse square law is hold then:

$$g_M = \frac{g_0}{60^2} = \left(\frac{980}{3600}\right) = 0.27\ \text{cm}/s^2$$

Thus $g_M = a$ is a mutual attraction force F between the mass of earth (M_E) and the mass of moon (M_m). Now newton could write the law of gravitation as:

$$F = G\frac{M_E M_m}{r^2}$$

The combination of this law and the law of dynamics i.e. $F = ma$ enabled scientists such as Hamilton and Lagrange to calculate the orbits of the planets.

In spite of the diverse uncountable phenomena in universe from atom to galaxies, some constants are existed i.e. things that repeat themselves. According to three Kepler's law of placatory motion around the sun, assuming circular orbits, universal gravitational law of Newton for planet A can be expressed as;

$$F_A = G\frac{M_{sun} M_A}{r_A^2}$$

The centripetal acceleration a for the planet A would be:

$$a_A = r_A {\omega_A}^2 = r_A \left(\frac{2\pi}{T_A}\right)^2$$

Now based on newton law of dynamic i.e. his second law ($a = F/m$) and mutual attraction force i. e. $g_M = a$ we have;

$$a_A = \frac{F_A}{M_A} = G\frac{M_{sun}}{r_A^2}$$

$$r_A\left(\frac{2\pi}{T_A}\right)^2 = G\frac{M_{sun}}{r_A^2}$$
$$\frac{4\pi^2}{T_A^2} = G\frac{M_{sun}}{r_A^3}$$

This last result for any planet such as planet B is true.

$$\frac{4\pi^2}{T_B^2} = G\frac{M_{sun}}{r_B^3}$$

Now if we divide the last result for planet A with this for planet B we get;

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

This is Kepler third law which was formulated about 50 years before Newton's Universal law of gravitation. This is in fact a proof for correctness of Newton's Universal gravitational law. However, some other proofs such as Edmund Halley's prediction of return time of Halley's comet and discovering a new planet Neptune by Johann Galle based on the irregularities in the orbit of planet Uranus are existed as well.
But what is *G* and its value? We know that it is a constant. A gravitational constant for law of universal gravitation. Universal! Thus, it must be true for earth-bound objects as it is true for objects in the sky. Therefore, there was a need to prove the law and determine the value of *G* on earth between two objects. To this aim, an earth-bound experiment must be conducted.
The universal gravitational constant G, determined by Henry Cavendish in 1798 for the first time. Remember, to measure G, he had to measure the two masses m_1 and m_2, the force *F*, and the distance *r* between them. He also knew that, the determination of G for massive celestial bodies is impossible and for non-massive terrestrial bodies is elusive.
We know that the universal law of gravitation in words would be;

" the attraction between two particles of matter is directly proportional to the product of their masses, inversely proportional to the square of the distance between them, and independent of the kind of matter and of the intervening medium." Thus, Cavendish must find the amount attraction force between "two masses of known amount at a known distance apart". The masses such as a tenth of an inch of metal sphere to a huge mountain mass, or of 1250 feet of the earth's crust.

The planet earth is a sphere and "the value of G is connected with the value of the mean specific gravity of the earth Δ with earth's gravity acceleration g and radius R by the equation

$$G\Delta = \frac{3g}{4\pi R}$$

Therefore, one can say that the aim of experiments is to find the *mean density* of the earth.

However, beside Cavendish experiment, there was other remarkable endeavors to determine the value of G. French scientists Bouguer and de la Condamine sent to Peru to consider the attraction force of mountain masses near the equator. "The Figure of the Earth" published by them in 1749. It was about experiments and observations on gravitation with remarks on the causes of the figure of the earth.

M. Eicher was the first stated that the force of gravity does not have the same value in different longitude i.e. "it is greater towards the poles, and less towards the equator". This is in agreement with the ellipsoid figure of the earth. Thus, we have a difference between two diameters of our planet. Now the question was;

"Does the effect correspond exactly to the cause upon which we desire it to depend? Is the difference in attraction so great that we can attribute to it all the inequality which exists, as we have seen, between the two diameters of our globe?"

To answer this question, they must determine the distribution of force of gravity in different longitudes of the earth. They used two methods two observe alteration in the attraction force in several regions. They examined "how much more quickly or more slowly a pendulum of given length oscillates" and "find the length of the pendulum whose time of vibration is exactly a second". The differences which they found in the length of the pendulum determined the changes of the attraction in different regions. But they found that the length of the pendulum is also connected with other properties such as temperature and centrifugal force. Finally, they stated that, "the distribution of density in the earth may be such that the maximum attraction of the earth is not at its surface but at some distance beneath it".

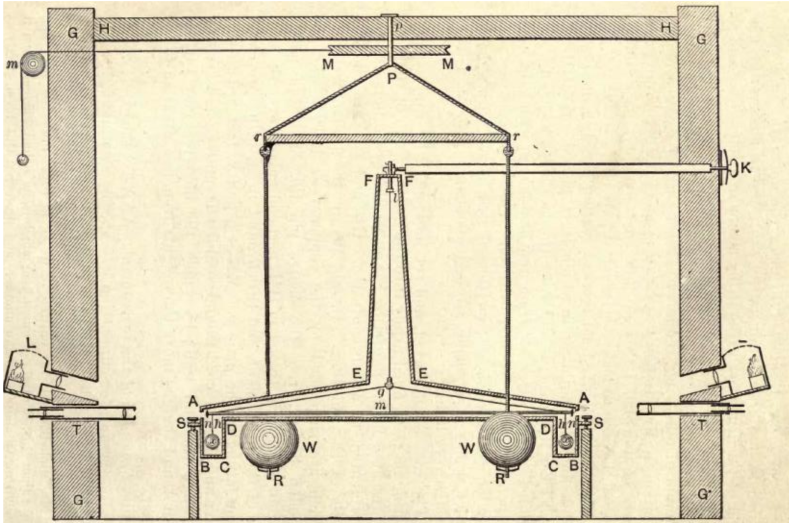
Another experiment called "The Schehallien Experiment" conducted by the British. Bouguer himself had suggested that his experiment on the attraction of a mountain should be repeated in Britain. In IN 1772, Maskelyne, the English Astronomer did this trial in two convenient hills. Then it repeated for other suitable hills. Maskelyne found that the attraction should be twice that found by observation such that the mean density of the earth is twice that of the hill. According to his experiments, on the attraction of the hill Schehallien, the density of the earth determined as 4.5 times that of water.

For more about such experiments the reader is referred to Stanley Mackenzie's "The laws of gravitation" 1900.

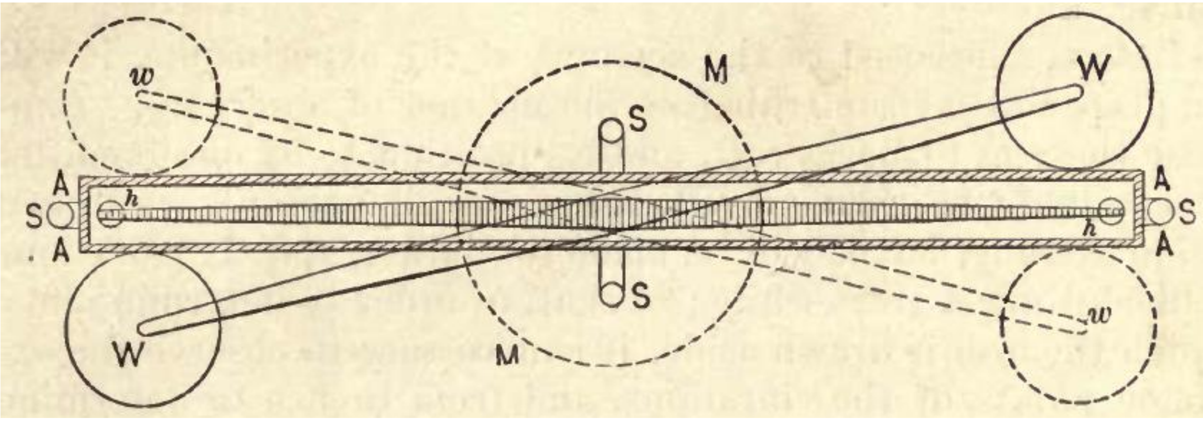
In "Experiments to determine density of the earth" published in 1798, Henry Cavendish complete all endeavors to determine the value of *G* with the help of an apparatus called "torsion balance" made by Rev. John Michell. It was an apparatus contrived a method of determining the density of the earth, by "rendering sensible the attraction of small quantities of matter". He did not live to make any experiments with it. But Cavendish complete his work. However, such device is also invented and used by in 1784, Charles Coulomb. By this device he discovered the law of interaction of electric charges i.e. Coulomb law.

Michell apparatus consisted of a 6 feet (long) wooden arm suspending horizontally, by a slender 40 inches (long) wire. A 2 inches (diameter) leaden ball was hung to each edge. All in a narrow wooden case.

In order to determine the density of the earth by this apparatus, it is necessary to ascertain what force is required to draw the arm aside (torque) through a given space. John Michell tried to do this by putting the arm in motion, and observing the time of its vibrations. But there was a familiar problem similar to previous experiment i.e. the effect of heat on the apparatus sensitivity. Thus, Mr. Cavendish thought that it is better to put the apparatus in room with constant temperature and observe it indirectly. He did this by making some changes in the Michell's apparatus as below (a vertical view).



A more convenient plan (a horizontal view) of this instrument is as below.



He moved the weights from their rest positions which caused the arm be drawn aside and thereby it will be made to vibrate, and its vibrations will continue a great while. Determining how much the arm is drawn aside, it was necessary to observe the extreme points of the vibrations and also the points it would be at rest. He recorded successive extreme points of a vibration and rest, and take the mean of them.

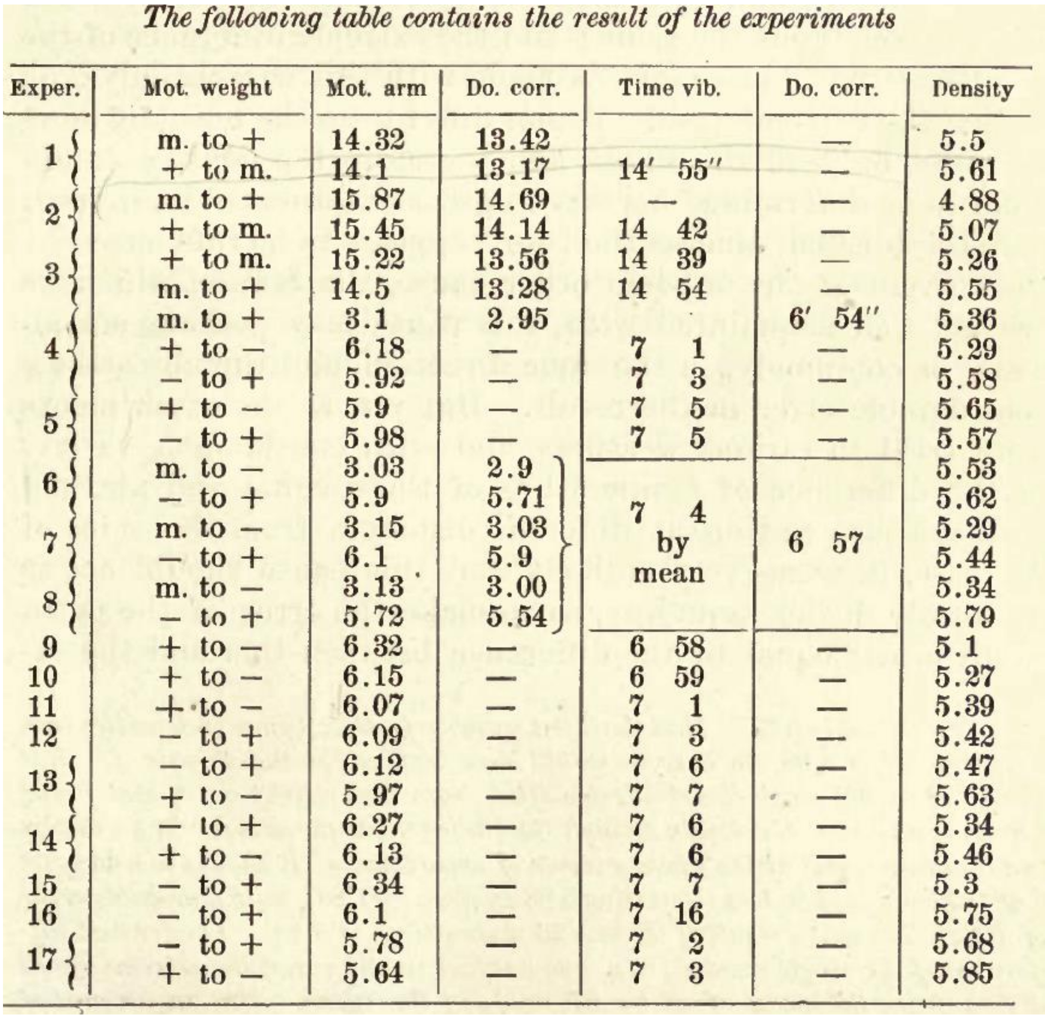
The time of vibration was the next item to be determined i.e. the times at which the arm arrives at given divisions between these extreme points of a vibration. Then he computed the middle point of the vibration and, by proportion, found the time at which the arm comes to this middle point. After repeating this process, he " divide the interval of time, between the coming of the arm to these two middle points, by the number of vibrations, which gives the time of one vibration ". his consideration showed that;

"The effect of motion in the point of rest is, that when the arm is moving in the same direction as the point of rest, the time of moving from one extreme point of vibration to the other is increased, and it is diminished when they are moving in contrary directions ; but, if the point of rest moves uniformly, the time of moving from one extreme to the middle point of the vibration, will be equal to that of moving from the middle point to the other extreme, and, moreover, the time of two successive vibrations will be very little altered ; and, therefore, the time of moving from the middle point of one vibration to the middle point of the next, will also be very little altered".

However due to the air resistance some inaccuracy appeared in the time at which the arm comes to the middle point of the vibration and the mean between the times of its coming to the extreme points. But those errors were not very considerable.

Seventeen successive experiment performed by placing the weights in different (positive and negative) positions along with changing the wires. The results of these experiments by Mr.

Cavendish is showed in below. The last column from right is representing the values for earth's density with respect to each experiment.



The following table contains the result of the experiments

Exper.	Mot. weight	Mot. arm	Do. corr.	Time vib.	Do. corr.	Density
1	m. to +	14.32	13.42		—	5.5
	+ to m.	14.1	13.17	14′ 55″	—	5.61
2	m. to +	15.87	14.69	—	—	4.88
	+ to m.	15.45	14.14	14 42	—	5.07
3	+ to m.	15.22	13.56	14 39	—	5.26
	m. to +	14.5	13.28	14 54	—	5.55
4	m. to +	3.1	2.95		6′ 54″	5.36
	+ to −	6.18	—	7 1	—	5.29
	− to +	5.92	—	7 3	—	5.58
5	+ to −	5.9	—	7 5	—	5.65
	− to +	5.98	—	7 5	—	5.57
6	m. to −	3.03	2.9	7 4 by mean	6 57	5.53
	− to +	5.9	5.71			5.62
7	m. to −	3.15	3.03			5.29
	− to +	6.1	5.9			5.44
8	m. to −	3.13	3.00			5.34
	− to +	5.72	5.54			5.79
9	+ to −	6.32	—	6 58	—	5.1
10	+ to −	6.15	—	6 59	—	5.27
11	+ to −	6.07	—	7 1	—	5.39
12	− to +	6.09	—	7 3	—	5.42
13	− to +	6.12	—	7 6	—	5.47
	+ to −	5.97	—	7 7	—	5.63
14	− to +	6.27	—	7 6	—	5.34
	+ to −	6.13	—	7 6	—	5.46
15	− to +	6.34	—	7 7	—	5.3
16	− to +	6.1	—	7 16	—	5.75
17	− to +	5.78	—	7 2	—	5.68
	+ to −	5.64	—	7 3	—	5.85

Finally, he stated that "by a mean of the experiments made with the wire first used, the mean density of the earth comes out 5.48 times greater than that of water with deviation of .38 to .75 with respect to different wires. This was 20 % greater than Maskelyne and Hutton’s value. In fact, he used Mitchell’s torsion balance to determine the value *G,* "which in turn allowed him to calculate the mass and density of the earth" i.e. “weighed the world.”

The mathematical method for computing these values of density is briefly as below:

According to Kepler's inverse square law of proportionality interred in Newton's formula;

$$F = G\frac{m_1 m_2}{r^2}$$

The planet earth was one of the masses and should be measured to determine the value of G or vice versa. Here instead of "the force is required to draw the arm aside" we use the familiar term "torque" which is generated by the torsion of the wire and gravitational pull of the masses in opposite direction as well. This differs the method we represent with Cavendish's original method.

We start with Hooke's law. According to this law and definition of torque i.e. the moment of the force, "the torque (τ) on the torsion wire is proportional to the deflection angle of the balance (φ)" is equal to a product of the attractive forces between the balls (F) and the distance to the suspending wire d. thus;

$$\tau\varphi = dF$$

As we said, we have two sources of torque here; torsion of the wire and gravitational pull of the two masses (balls) in opposite direction. Thus, the equilibrium point happens when these two cancel out each other and the amount of torque is equal to zero.

Now if we solve the equation of Newton's law of universal gravitation for F we have;

$$F = \frac{GmM}{r^2}$$

Thus, we can write;

$$\tau\varphi = d\frac{GmM}{r^2}$$

This is equivalent with considering of extreme points of vibration by Cavendish we mentioned above.

Now he considered, the times at which the arm arrives at given divisions between these extreme points of a vibration i.e. the period of the vibration. We call it, resonant oscillation period T. this helped him to find the torsion coefficient (τ) as;

$$T = 2\pi\sqrt{I/\tau}$$

Now according to first constraints made by cavendish on his experiment, the moment of inertia (I) of the balance is just due to the small balls. Therefore;

$$I = \frac{md^2}{2}$$

This change the above relation of period (T) as;

$$T = 2\pi\sqrt{\frac{md^2}{2\tau}}$$

Now solve for τ give us the required relation to find G relation as:

$$G = \frac{2\pi^2 dr^2 \varphi}{MT^2}$$

Another way to calculate G is the following. We know that free-fall acceleration is about g_o=9.8 m/s^2. For a ball falling toward earth with a mass(m) due to the attractive force produce by the earth (F) would be;

$$F = G\frac{mM_E}{R_E^2}$$

Now second law of dynamics says, this force apply to the ball an acceleration equal to that of g_o i.e.

$$g_0 = \frac{F}{m} = G\frac{M_E}{R_E^2}$$

$$G = g_0\frac{R_E^3}{M_E}$$

With $R = 6370 \times 10^3$ and $M_E = 6 \times 10^{24}$ we can write the value of $G = 6.6 \times 10^{-11} \frac{m^3}{S^2 kg}$.

Avogadro number

These are chemical elements that made up all substances. An atom is the smallest part i.e. particle of each element with high small mass proportional to its atomic weight.

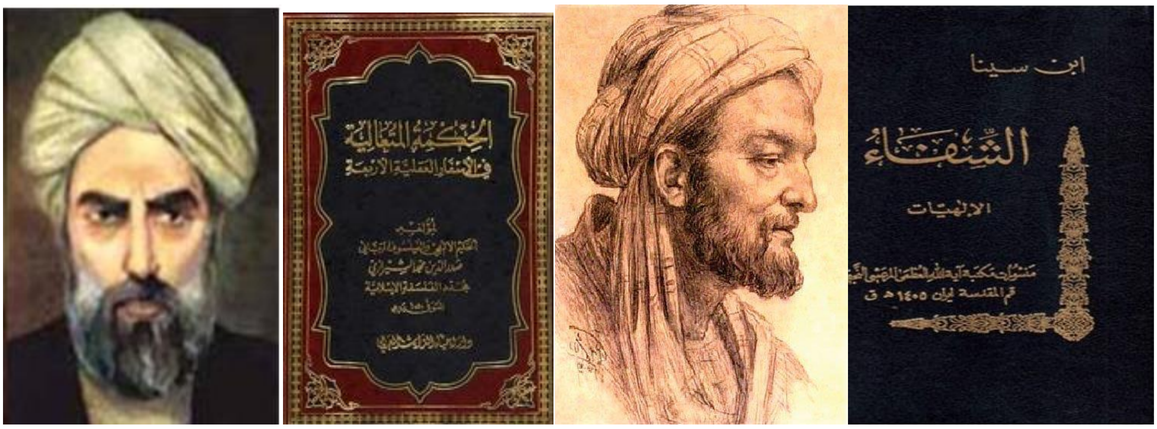
The number of molecules in a mole of any substance is a constant called Avogadro number. We passed along way in understanding the structure of substances in universe to reach this constant. The name for this number proposed in 1908. This is 50 years after Avogadro's death.

From ancient time there was two major approaches to the problem of primary elements that other matters are made from them. The "substance" and the "atom". The substance never appeared alone but, in a combination, with a pair of qualities (warmness, dryness, dampness and coldness) to produce the conventional elements of fire, water, air and earth. For example, a combination of substance + warmness+ dryness gives fire.

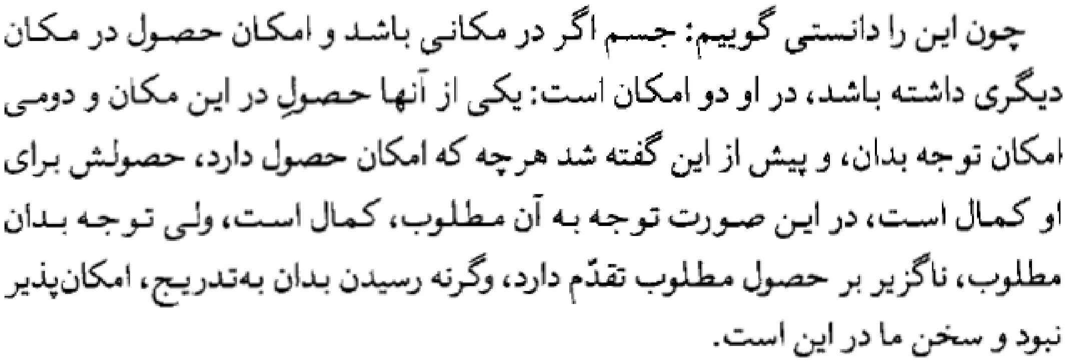
This was Aristotle view to the universe while Plato believed that "one primary element could change into another". Substance is viewed as "a continuous solid and uniform mass".

The second approach was proposed by Leucippus, Epicurus and especially Democritus. In their view the substance was composed of "tiny indivisible and impenetrable particles of various sizes and forms". They called them *atoms* which is a Greek word with same meaning. These atoms had distinguished " shape, size, position and arrangement". They could collide and bound to each other in a random motion. Some may "interlock because of the symmetry of their shapes, sizes, positions and arrangements, and remain together". Thus, unit bodies with different compositions would begin to exist.

Pur Sina's views on the natural sciences, was in the tradition of Aristotle. However, his theory of substance is the distinction between existence and actual existence. In his view natural bodies as the subject of physics are changeable and have different manners of movement and rest based on their existence. In fact, the word existence and its intensities are the driver of universe for him and other Islamic (more Persian scholars) such as Farabi, Mola sadra and even the more recent Hasanzadeh Amoli. Although scholars such as Biruni or Razi in their efforts to convert things into gold have more observational and experimental attitude in their researches but in the case of fundamental substance or atoms all of their contributions have a philosophical point of view in consistence with their monotheistic theology. Endeavors of these scholars remained vague and philosophical due to the lack of quantitative methods in their researches. However, one can find concepts of quantum mechanics such as quantum entanglement in mola sadra's famous book Asfar Arbae "four voyages" or as it known as "The Transcendent Philosophy of the Four Journeys of the Intellect" in a metaphysical way.


الحكمة المتعالية في الأسفار العقلية الأربعة
ابن سينا
الشفاء
الالهيات

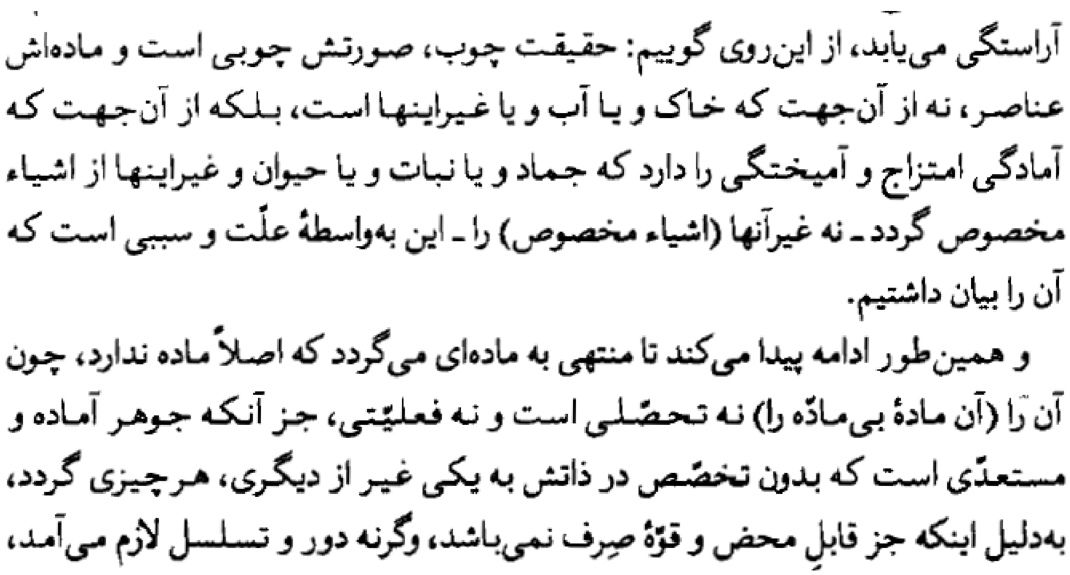
Mola sadra in describing "rest and motion" stated that:



چون این را دانستی گوییم: جسم اگر در مکانی باشد و امکان حصول در مکان دیگری داشته باشد، در او دو امکان است: یکی از آنها حصولِ در این مکان و دومی امکان توجه بدان، و پیش از این گفته شد هرچه که امکان حصول دارد، حصولش برای او کمال است، در این صورت توجه به آن مطلوب، کمال است، ولی توجه بدان مطلوب، ناگزیر بر حصول مطلوب تقدّم دارد، وگرنه رسیدن بدان به‌تدریج، امکان‌پذیر نبود و سخن ما در این است.

when you know this, we say: If an object is in a place and you have the possibility of obtaining it in another place, there are two possibilities in it: one of them is obtaining it in this place and the second is the possibility of "paying attention to it", and it was said before that everything that is possible to obtain its obtaining is a perfection for it, in this case "paying attention to it" is the desired of perfection, but paying attention to the desired, inevitably takes precedence over achieving the desired, otherwise it would not be possible to reach the body gradually, and this is what we are talking about.

In describing the "form and substance" he proposed that:



آراستگی می‌یابد، از این‌روی گوییم: حقیقت چوب، صورتش چوبی است و ماده‌اش عناصر، نه از آن‌جهت که خاک و یا آب و یا غیراینها است، بلکه از آن‌جهت که آمادگی امتزاج و آمیختگی را دارد که جماد و یا نبات و یا حیوان و غیراینها از اشیاء مخصوص گردد ـ نه غیرآنها (اشیاء مخصوص) را ـ این به‌واسطهٔ علّت و سببی است که آن را بیان داشتیم.

و همین‌طور ادامه پیدا می‌کند تا منتهی به ماده‌ای می‌گردد که اصلاً ماده ندارد، چون آن را (آن مادهٔ بی‌مادّه را) نه تحصّلی است و نه فعلیّتی، جز آنکه جوهر آماده و مستعدّی است که بدون تخصّص در ذاتش به یکی غیر از دیگری، هرچیزی گردد، به‌دلیل اینکه جز قابلِ محض و قوّهٔ صِرف نمی‌باشد، وگرنه دور و تسلسل لازم می‌آمد،

..:The truth of wood is that its form is wood, and its substance is elements, not because it is soil or water or any of these, but because it is ready to mix and mingle, to become, inanimate objects, or plants, or animals, etc. i.e. any particular objects, and no other than them (particular objects). This is due to the cause and effect that we have stated before. And it (cause and effect) continues in the same way until it leads to a substance that has no substance at all, because it (the matter-less substance) is neither a creation nor an actuality, except that it is a ready and susceptible substance (johar) that is not allocated in its essence to one of the other become anything, due to the fact that, it is nothing but absolute power and mere potential, and otherwise, it would be necessary to go around and sequence...

Sadri Hassani in "From Atom to Galaxies" states that, "The atomic theory of Democritus, Epicurus, and Lucretius lay dormant for over sixteen centuries until post-Renaissance natural philosophers adorned it with experimental testing and mathematical precision". This will be the basis to go on in our discussion.

From the second half of the seventeenth century quantitative methods introduced to the word of science. Scottish polymath Robert Boyle (1627-1691) with "The Origin of forms and Qualities" in 1666 and Russian polymath Mikhail Lomonosov (1711-1765) with "Elements of Mathematical Chemistry" (a draft) proposed the first modern view of structure of matter.

J. Kerseboom pinxit
B. Baron Sculp.
The Honourable ROBERT BOYLE.
NATIONAL GALLERIES SCOTLAND

The Honourable Robert Boyle, 1627 - 1691. Philosopher, 1744, multiple artists

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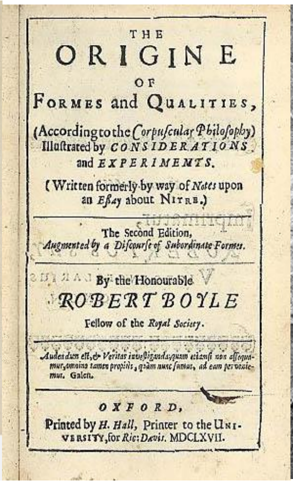
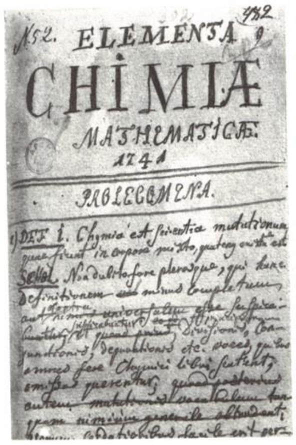

THE
ORIGINE
OF
FORMES and QUALITIES,
(According to the *Corpuſcular Philoſophy*)
Illuſtrated by *CONSIDERATIONS*
and *EXPERIMENTS*.
(Written formerly by way of *Notes* upon
an *Eſſay* about NITRE.)

The Second Edition,
Augmented by a Diſcourſe of Subordinate Formes.

By the Honourable
ROBERT BOYLE
Fellow of the *Royal Society*.

Audendum eſt, & Veritas inveſtiganda, quam etiamſi non aſſequamur, omnino tamen propius, quàm nunc ſumus, ad eam perveniemus. Galen.

OXFORD,
Printed by *H. Hall*, Printer to the UNIVERSITY, for *Ric: Davis*. MDCLXVII.

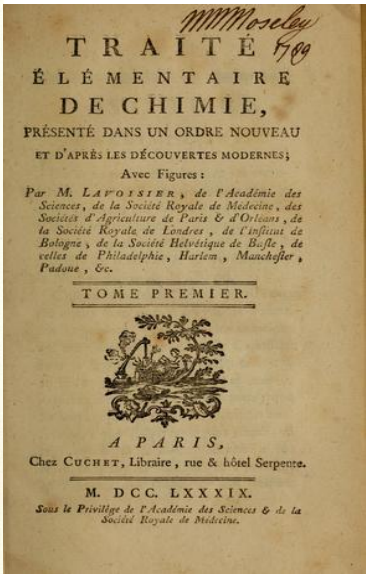

№ 52. ELEMENTA
CHIMIÆ
MATHEMATICÆ.
1741
PROLEGOMENA.
DEF. 1. Chymia est scientia mutationum quae fiunt in corpore mixto, quatenus mixtum est.

Boyle believed that there exist "one universal matter common to all bodies" i.e. an extended, divisible, dynamic and impenetrable substance. Being dynamic and in motion this universal matter cause variety of qualities and forms in natural bodies. He stated that, "the grand cause of forms is motion, which by variously dividing, grouping, transposing, and so connecting the portions of matter, produces in them the accidents and qualities that qualify the body in question to belong to this or that determinate species of natural bodies".

Lomonosov had more mathematical contributions than Boyle maybe because he was a friend of Leonard Euler the famous swiss mathematician. He called corpuscles (atoms/molecules) the "insensible particles" and similar to most of his contemporaries used the theory of kinetic motion for them. He stated that, "chemistry is the science of changes occurring in mixed bodies since they are mixed" and… "with an understanding of mixed bodies we can explain all their possible changes, including separations, combinations, etc.".

Thus, they used a corpuscular theory to explain it from atomistic point of view. According to Boyle "the corpuscle (molecule) is a combination of elements (atoms); the element is a part of a body, not composed of any other, smaller and different bodies".

The opening classification of chemical elements suggested by Antoine Lavoisier (1743-1794) in his "treaties in elements of chemistry" in 1793 in two volumes. Although his classification had some fundamental errors but the atomism was toddling on those years.



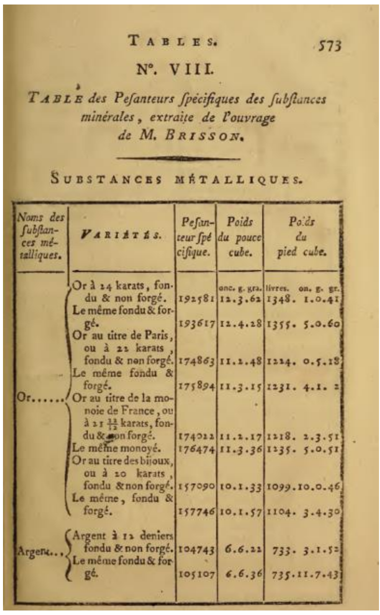
WMMoseley 1789

TRAITÉ
ÉLÉMENTAIRE
DE CHIMIE,
PRÉSENTÉ DANS UN ORDRE NOUVEAU
ET D'APRÈS LES DÉCOUVERTES MODERNES;
Avec Figures:
Par M. LAVOISIER, de l'Académie des Sciences, de la Société Royale de Médecine, des Sociétés d'Agriculture de Paris & d'Orléans, de la Société Royale de Londres, de l'Inſtitut de Bologne, de la Société Helvétique de Baſle, de celles de Philadelphie, Harlem, Mancheſter, Padoue, &c.

TOME PREMIER.

A PARIS,
Chez CUCHET, Libraire, rue & hôtel Serpente.

M. DCC. LXXXIX.
Sous le Privilège de l'Académie des Sciences & de la Société Royale de Médecine.



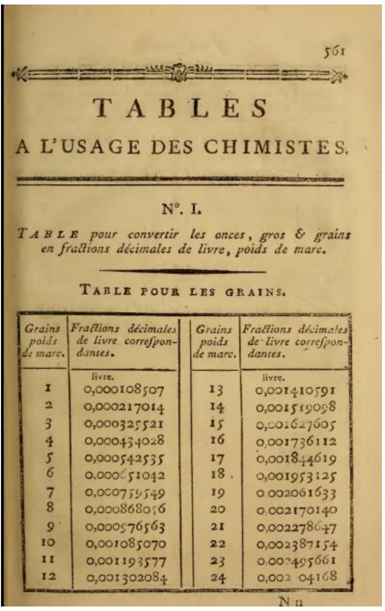
TABLES. 573

N°. VIII.

TABLE des Peſanteurs ſpécifiques des ſubſtances minérales, extraite de l'ouvrage de M. BRISSON.

SUBSTANCES MÉTALLIQUES.

Noms des ſubſtances métalliques.	*VARIÉTÉS.*	*Peſanteur ſpécifique.*	*Poids du pouce cube.*	*Poids du pied cube.*
			onc. g. gra.	livres. on. g. gr.
Or......	Or à 24 karats, fondu & non forgé.	192581	12.3.62	1348. 1.0.41
	Le même fondu & forgé.	193617	12.4.28	1355. 5.0.60
	Or au titre de Paris, ou à 22 karats, fondu & non forgé.	174863	11.2.48	1224. 0.5.18
	Le même fondu & forgé.	175894	11.3.15	1231. 4.1. 2
	Or au titre de la monoie de France, ou à 21 $\frac{22}{32}$ karats, fondu & non forgé.	174022	11.2.17	1218. 2.3.51
	Le même monoyé.	176474	11.3.36	1235. 5.0.51
	Or au titre des bijoux, ou à 20 karats, fondu & non forgé.	157090	10.1.33	1099.10.0.46
	Le même, fondu & forgé.	157746	10.1.57	1104. 3.4.30
Argent...	Argent à 12 deniers fondu & non forgé.	104743	6.6.22	733. 3.1.52
	Le même fondu & forgé.	105107	6.6.36	735.11.7.43



561

TABLES
A L'USAGE DES CHIMISTES.

N°. I.

TABLE pour convertir les onces, gros & grains en fractions décimales de livre, poids de marc.

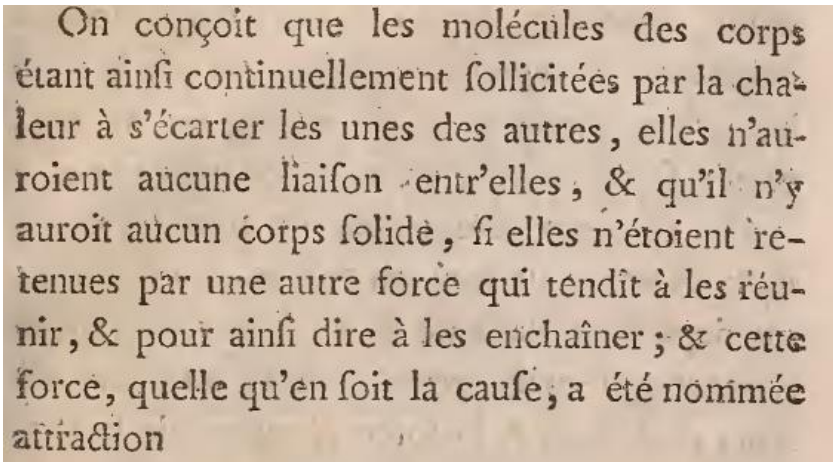
TABLE POUR LES GRAINS.

Grains poids de marc.	*Fractions décimales de livre correſpondantes.*	*Grains poids de marc.*	*Fractions décimales de livre correſpondantes.*
	livre.		livre.
1	0,000108507	13	0,001410591
2	0,000217014	14	0,001519098
3	0,000325521	15	0,001627605
4	0,000434028	16	0,001736112
5	0,000542535	17	0,001844619
6	0.000651042	18	0,001953125
7	0,000759549	19	0 002061633
8	0,000868056	20	0.002170140
9	0,000976563	21	0,002278647
10	0,001085070	22	0,002387154
11	0,001193577	23	0.002495661
12	0,001302084	24	0,002 04168

Nn

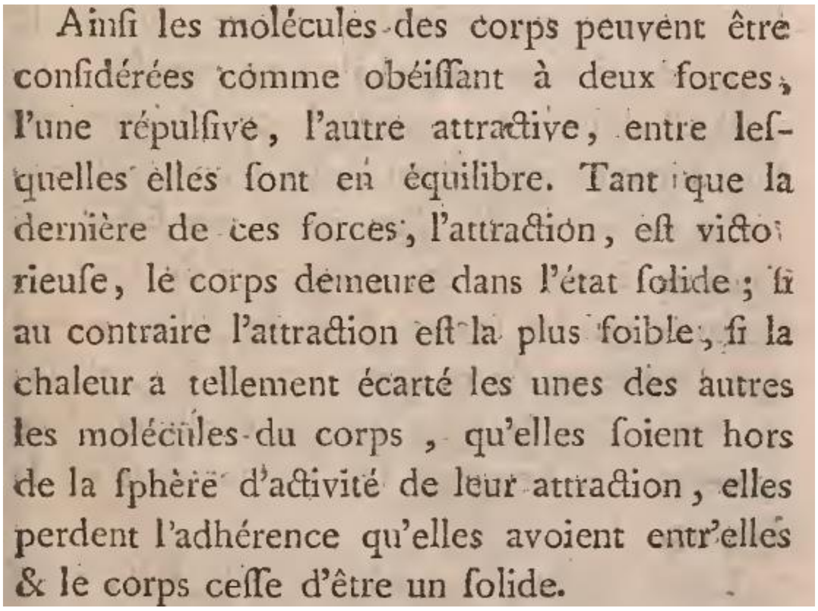
Russian chemist D.I. Mendeleev used the works of him and John Dalton (1766-1844) develop his theory of elements in his "On the Relation of the Properties to the Atomic Weights of the Elements" in 1869. He recognized that the apparent randomness of elements is fitted into a system.

In the first volume of his book the father of modern chemistry, Lavoisier stated that:



On conçoit que les molécules des corps étant ainſi continuellement ſollicitées par la chaleur à s'écarter les unes des autres, elles n'auroient aucune liaiſon entr'elles, & qu'il n'y auroit aucun corps ſolide, ſi elles n'étoient retenues par une autre force qui tendît à les réunir, & pour ainſi dire à les enchaîner; & cette force, quelle qu'en ſoit la cauſe, a été nommée attraction

We conceive that the molecules of bodies being thus continually favored by the heat away from each other, they did not no connection between them & that there would have no solid body, if they were not retained by another force which tended to unite them, & so to speak chain them; & this force, whatever the cause has been appointed attraction.



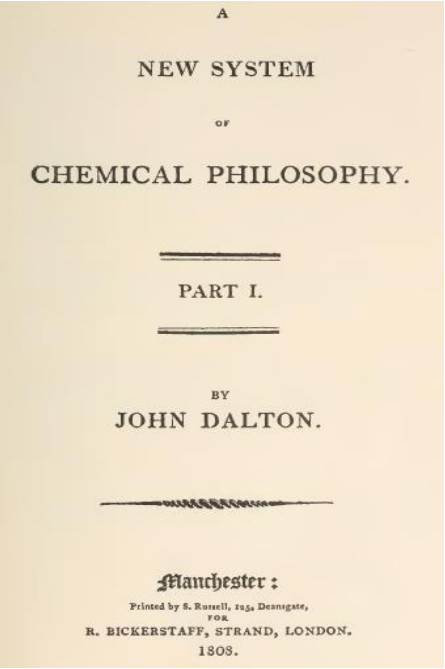
Ainſi les molécules des corps peuvent être conſidérées comme obéiſſant à deux forces, l'une répulſive, l'autre attractive, entre leſquelles elles ſont en équilibre. Tant que la dernière de ces forces, l'attraction, eſt victorieuſe, le corps demeure dans l'état ſolide; ſi au contraire l'attraction eſt la plus foible, ſi la chaleur a tellement écarté les unes des autres les molécules du corps, qu'elles ſoient hors de la ſphère d'activité de leur attraction, elles perdent l'adhérence qu'elles avoient entr'elles & le corps ceſſe d'être un ſolide.

Thus, the molecules of bodies can be considered as obeying two forces, one repellent, the other attractive, between which they are in balance. As long as the last of these forces, attraction, is victorious" causing, the body remains in the solid state; on the contrary, attraction is the most weakest, if the heat has spread so far from each other body molecules, whether they are out of the sphere of activity of their attractions they lose the bond they had with each other & the body ceases to be a solid.

However, John Dalton proposed that, "all bodies at all times, and in every situation, attract one another; yet in certain circumstances, they are likewise actuated by a repulsive power;

the only efficient motive force is then the difference of these two powers".

The first modern definition of the atomic weight of an element was suggested by Dalton in 1808 in his "A New System of Chemical Philosophy" as; "a ratio of the mass of one atom of an element to the mass of one atom of hydrogen". He also proposed that, "elements may only combine with one another in proportions 1: 1, 1: 2, 1:3…". thus, he considered all gases to be monatomic. He built a table of "relative atomic masses of elements". According to him "relative molecular (atomic) mass of a substance is the ratio of the mass of its molecule (atom) to 1/12 of the mass of a carbon atom".



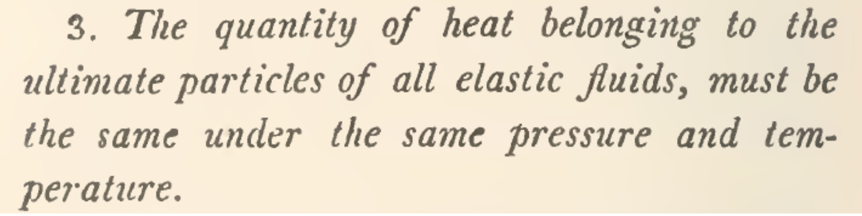
A
NEW SYSTEM
OF
CHEMICAL PHILOSOPHY.

PART I.

BY
JOHN DALTON.

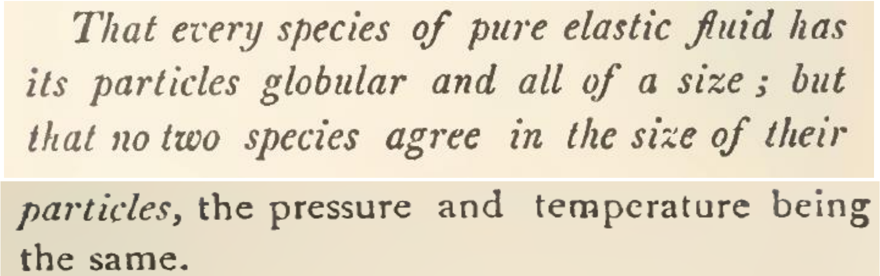
Manchester:
Printed by S. Russell, 125, Deansgate,
FOR
R. BICKERSTAFF, STRAND, LONDON.
1808.

In the first part of his book he considered "Heat or Caloric "and in the second part "constitution of Bodies". In states that:



3. The quantity of heat belonging to the ultimate particles of all elastic fluids, must be the same under the same pressure and temperature.

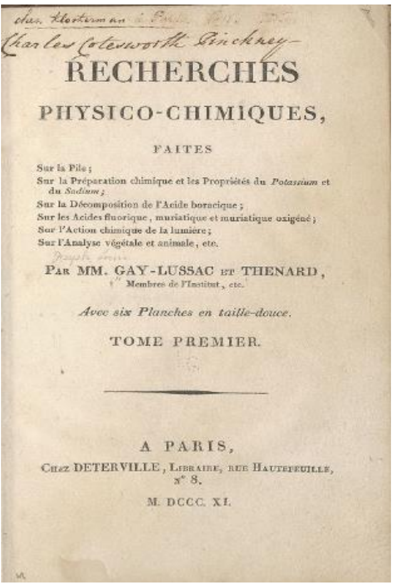
In high temperature "the particles will condense their respective atmospheres of heat, by which their mutual repulsion will be diminished, and the external pressure will therefore effect a proportionate condensation in the volume of air : neither an increase nor diminution in the quantity of heat around each molecule, or around the whole, will take place". Having been involved with meteorology for some time, he chose air as his point of departure, and later generalized his investigation to all gases.



That every species of pure elastic fluid has its particles globular and all of a size ; but that no two species agree in the size of their particles, the pressure and temperature being the same.

In contrast with Dalton atomistic view, J.L. Gay-Lussac proposed a *volume* view of substances in his paper "on the combination of gaseous substances with each other" and his book" Recherches physico-chimiques" with T. L Jacques (1777-1857) as the joint author.

He suggested a law of Combining Volumes in 1808. This law states that "if gases interact and form a gaseous product, the volumes of the reacting gases and the volumes of the products are in simple proportion".



chas. Klosterman

Charles Cotesworth Pinckney

RECHERCHES
PHYSICO-CHIMIQUES,
FAITES
Sur la Pile;
Sur la Préparation chimique et les Propriétés du *Potassium* et du *Sodium*;
Sur la Décomposition de l'Acide boracique;
Sur les Acides fluorique, muriatique et muriatique oxigéné;
Sur l'Action chimique de la lumière;
Sur l'Analyse végétale et animale, etc.

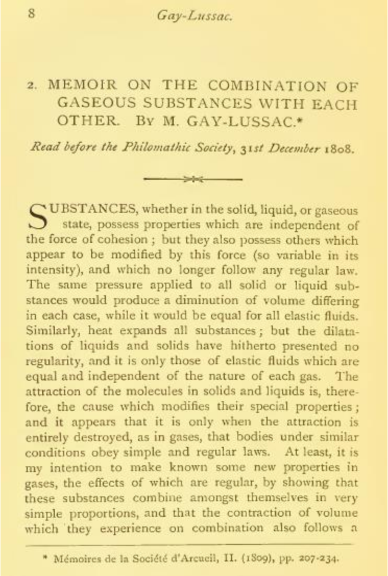
Joseph Louis

PAR MM. GAY-LUSSAC ET THENARD,
Membres de l'Institut, etc.

Avec six Planches en taille-douce.

TOME PREMIER.

A PARIS,
CHEZ DETERVILLE, LIBRAIRE, RUE HAUTEFEUILLE, n° 8.
M. DCCC. XI.



8 Gay-Lussac.

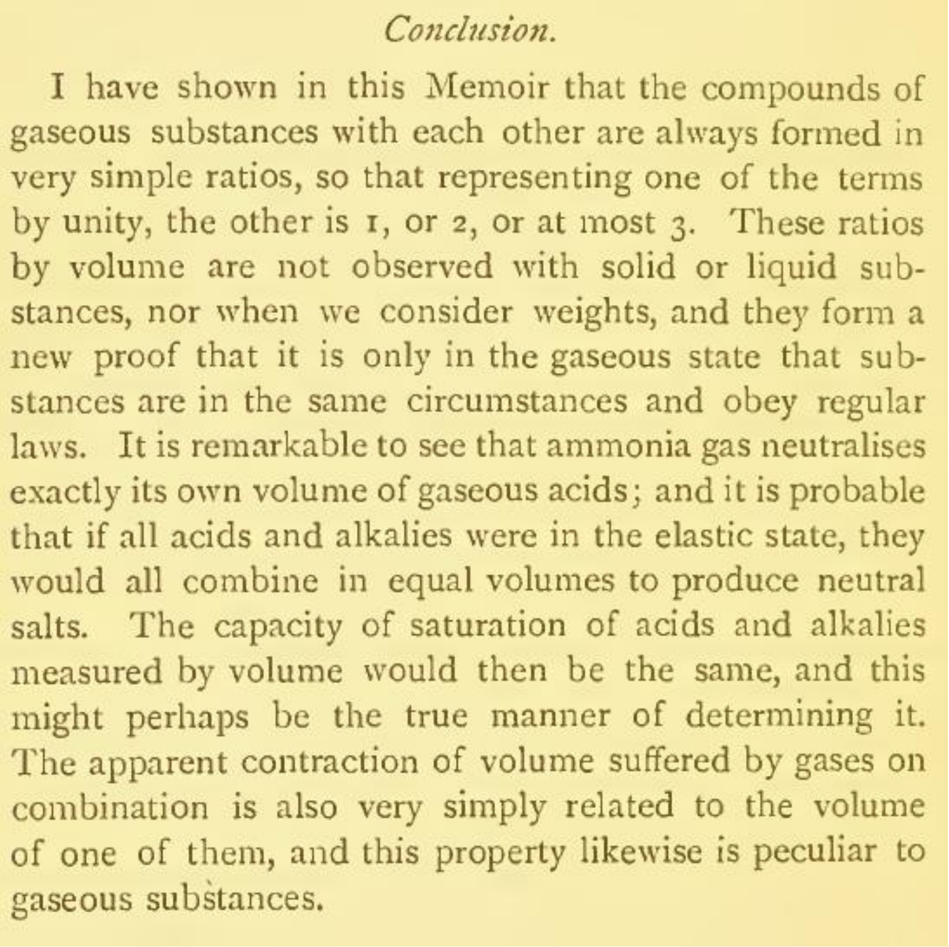
2. MEMOIR ON THE COMBINATION OF GASEOUS SUBSTANCES WITH EACH OTHER. BY M. GAY-LUSSAC.*

Read before the Philomathic Society, 31st December 1808.

SUBSTANCES, whether in the solid, liquid, or gaseous state, possess properties which are independent of the force of cohesion ; but they also possess others which appear to be modified by this force (so variable in its intensity), and which no longer follow any regular law. The same pressure applied to all solid or liquid substances would produce a diminution of volume differing in each case, while it would be equal for all elastic fluids. Similarly, heat expands all substances ; but the dilatations of liquids and solids have hitherto presented no regularity, and it is only those of elastic fluids which are equal and independent of the nature of each gas. The attraction of the molecules in solids and liquids is, therefore, the cause which modifies their special properties ; and it appears that it is only when the attraction is entirely destroyed, as in gases, that bodies under similar conditions obey simple and regular laws. At least, it is my intention to make known some new properties in gases, the effects of which are regular, by showing that these substances combine amongst themselves in very simple proportions, and that the contraction of volume which they experience on combination also follows a

* Mémoires de la Société d'Arcueil, II. (1809), pp. 207-234.

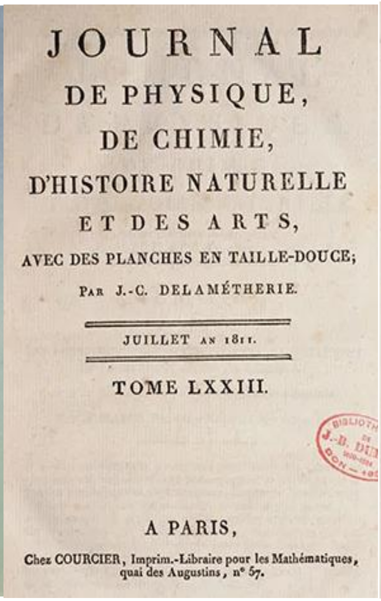
According to his view instead of having a compound of hydrogen and oxygen atoms for water we have "two volumes of hydrogen combine with one volume of oxygen to produce two volumes of water vapor". In conclusion of his paper he stated that:



Conclusion.

I have shown in this Memoir that the compounds of gaseous substances with each other are always formed in very simple ratios, so that representing one of the terms by unity, the other is 1, or 2, or at most 3. These ratios by volume are not observed with solid or liquid substances, nor when we consider weights, and they form a new proof that it is only in the gaseous state that substances are in the same circumstances and obey regular laws. It is remarkable to see that ammonia gas neutralises exactly its own volume of gaseous acids; and it is probable that if all acids and alkalies were in the elastic state, they would all combine in equal volumes to produce neutral salts. The capacity of saturation of acids and alkalies measured by volume would then be the same, and this might perhaps be the true manner of determining it. The apparent contraction of volume suffered by gases on combination is also very simply related to the volume of one of them, and this property likewise is peculiar to gaseous substances.

In fact, the subject of this paper is what is known as Gay-Lussac's law:" Not only do gases combine in very simple proportions " by volume, " but the apparent contraction of volume which they experience on combination has also a simple relation to the volume of the gases, or at least to that of one of them."
Italian Amedeo Avogadro, combined these efforts together by considering every molecule as a compound of some atoms. In 1811, he published "Essay on a manner of Determining the Relative Masses of the Elementary Molecules of Bodies and the Proportions by Which They Enter These Combinations" in Journal de Physique de chimie.

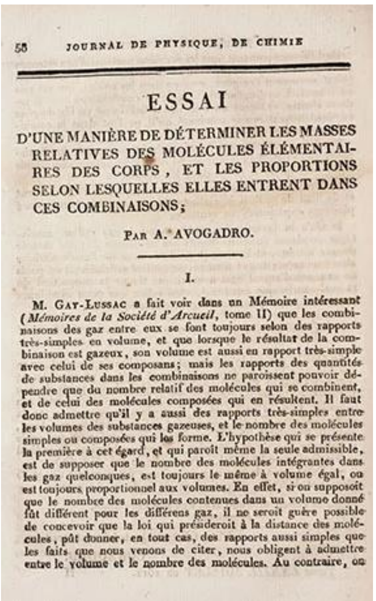


JOURNAL
DE PHYSIQUE,
DE CHIMIE,
D'HISTOIRE NATURELLE
ET DES ARTS,
AVEC DES PLANCHES EN TAILLE-DOUCE;
PAR J.-C. DELAMÉTHERIE.

JUILLET AN 1811.

TOME LXXIII.

A PARIS,
Chez COURCIER, Imprim.-Libraire pour les Mathématiques,
quai des Augustins, n° 57.



58 JOURNAL DE PHYSIQUE, DE CHIMIE

ESSAI
D'UNE MANIÈRE DE DÉTERMINER LES MASSES RELATIVES DES MOLÉCULES ÉLÉMENTAIRES DES CORPS, ET LES PROPORTIONS SELON LESQUELLES ELLES ENTRENT DANS CES COMBINAISONS;

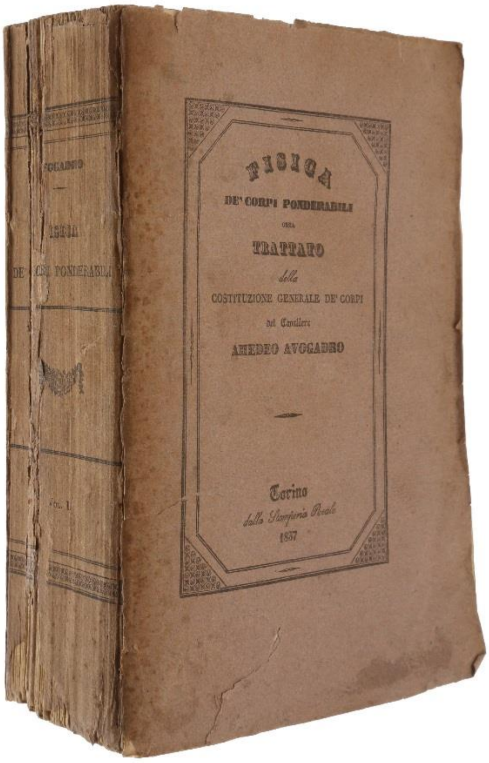
PAR A. AVOGADRO.

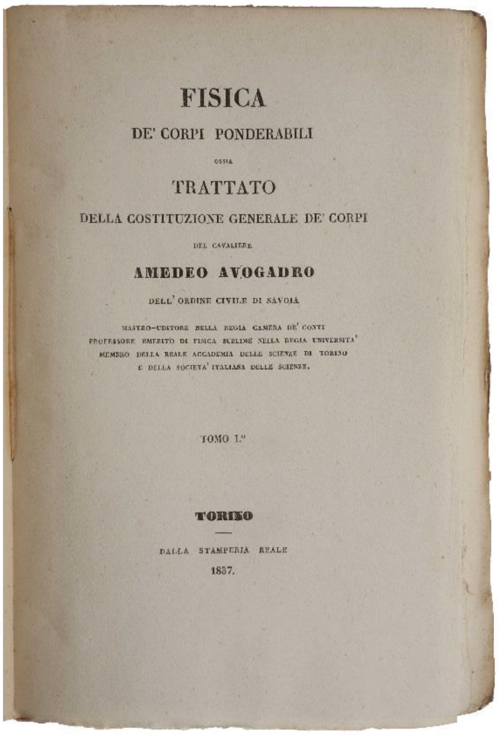
I.

M. GAY-LUSSAC a fait voir dans un Mémoire intéressant (*Mémoires de la Société d'Arcueil*, tome II) que les combinaisons des gaz entre eux se font toujours selon des rapports très-simples en volume, et que lorsque le résultat de la combinaison est gazeux, son volume est aussi en rapport très-simple avec celui de ses composans; mais les rapports des quantités de substances dans les combinaisons ne paroissent pouvoir dépendre que du nombre relatif des molécules qui se combinent, et de celui des molécules composées qui en résultent. Il faut donc admettre qu'il y a aussi des rapports très-simples entre les volumes des substances gazeuses, et le nombre des molécules simples ou composées qui les forme. L'hypothèse qui se présente la première à cet égard, et qui paroît même la seule admissible, est de supposer que le nombre des molécules intégrantes dans les gaz quelconques, est toujours le même à volume égal, ou est toujours proportionnel aux volumes. En effet, si on supposoit que le nombre des molécules contenues dans un volume donné fût différent pour les différens gaz, il ne seroit guère possible de concevoir que la loi qui présideroit à la distance des molécules, pût donner, en tout cas, des rapports aussi simples que les faits que nous venons de citer, nous obligent à admettre entre le volume et le nombre des molécules. Au contraire, on

In fact, on consideration of Gay-Lussac law, Avogadro proposed his hypothesis, namely, " the number of (integral) molecules in any gases is always the same for equal volumes, or always proportional to the volumes". This means that "quantitative ratios of substances in compounds only depend on the relative number of atoms comprising the compound and on the number of molecules they form" and "The number of molecules is always the same in equal volumes of all gases or it is always proportional to the volumes".

He was a prolific scientist and published several important papers such as: "Note on the Relative Masses of Elementary Molecules, or Suggested Densities of Their Gases, and on the Constituents of Some of Their Compounds", in 1811 and "New Considerations on the Theory of Proportions Determined in Combinations, and on Determination of the Masses of Atoms" in 1821. However in 1841, he published his work in *Fisica dei corpi ponderabili, ossia Trattato della costituzione materiale de' corpi*, in 4 volumes.



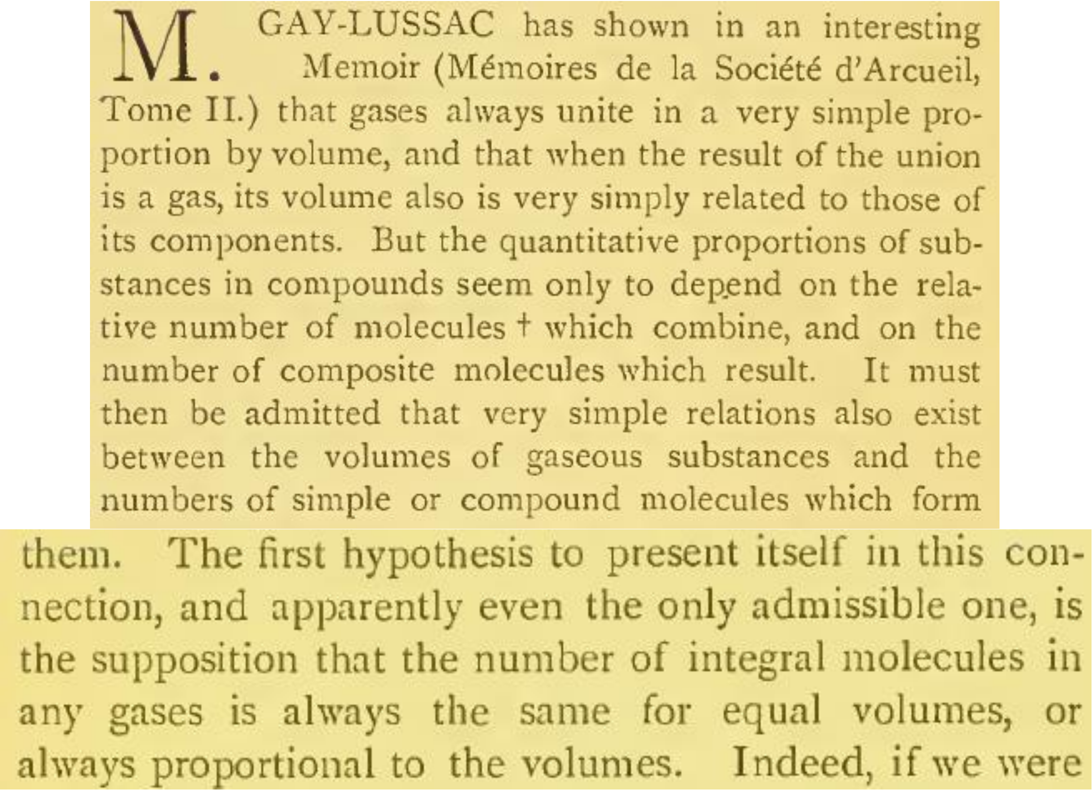


FISICA
DE' CORPI PONDERABILI
OSSIA
TRATTATO
DELLA COSTITUZIONE GENERALE DE' CORPI
DEL CAVALIERE
AMEDEO AVOGADRO
DELL' ORDINE CIVILE DI SAVOIA
MASTRO-UDITORE NELLA REGIA CAMERA DE' CONTI
PROFESSORE EMERITO DI FISICA SUBLIME NELLA REGIA UNIVERSITA'
MEMBRO DELLA REALE ACCADEMIA DELLE SCIENZE DI TORINO
E DELLA SOCIETA' ITALIANA DELLE SCIENZE.

TOMO I.°

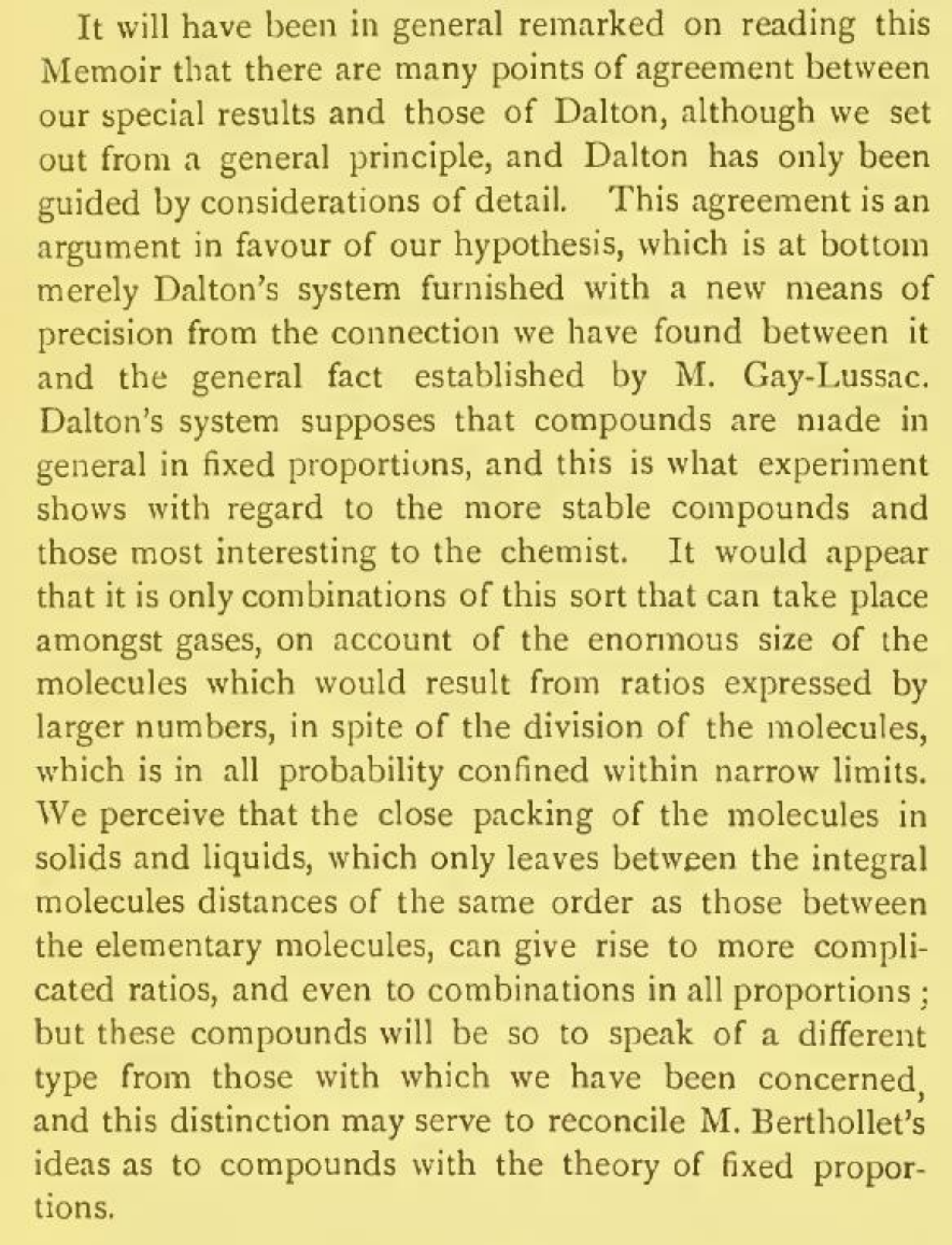
TORINO
—
DALLA STAMPERIA REALE
1837.

In "Essay on a manner of Determining the Relative Masses of the Elementary Molecules of Bodies and the Proportions by Which They Enter These Combinations" he starts as below:



M. GAY-LUSSAC has shown in an interesting Memoir (Mémoires de la Société d'Arcueil, Tome II.) that gases always unite in a very simple proportion by volume, and that when the result of the union is a gas, its volume also is very simply related to those of its components. But the quantitative proportions of substances in compounds seem only to depend on the relative number of molecules † which combine, and on the number of composite molecules which result. It must then be admitted that very simple relations also exist between the volumes of gaseous substances and the numbers of simple or compound molecules which form them. The first hypothesis to present itself in this connection, and apparently even the only admissible one, is the supposition that the number of integral molecules in any gases is always the same for equal volumes, or always proportional to the volumes. Indeed, if we were

… then finally he concludes that:



It will have been in general remarked on reading this Memoir that there are many points of agreement between our special results and those of Dalton, although we set out from a general principle, and Dalton has only been guided by considerations of detail. This agreement is an argument in favour of our hypothesis, which is at bottom merely Dalton's system furnished with a new means of precision from the connection we have found between it and the general fact established by M. Gay-Lussac. Dalton's system supposes that compounds are made in general in fixed proportions, and this is what experiment shows with regard to the more stable compounds and those most interesting to the chemist. It would appear that it is only combinations of this sort that can take place amongst gases, on account of the enormous size of the molecules which would result from ratios expressed by larger numbers, in spite of the division of the molecules, which is in all probability confined within narrow limits. We perceive that the close packing of the molecules in solids and liquids, which only leaves between the integral molecules distances of the same order as those between the elementary molecules, can give rise to more complicated ratios, and even to combinations in all proportions; but these compounds will be so to speak of a different type from those with which we have been concerned, and this distinction may serve to reconcile M. Berthollet's ideas as to compounds with the theory of fixed proportions.

In Principia, Newton proposed that "the smallest indivisible bodies followed the laws of motion; that certain hitherto unknown forces were responsible for the adherence of these particles, or their recession from one another". Dalton and others were working on these indivisible particles.

Now let's go on in a more sensible way and expressed, Gay-Lussac law as: "two volumes of molecules of hydrogen in combination with one volume of molecules of oxygen produce two volumes of molecules of water vapor".

$$2H_2 + O_2 = 2H_2O$$

Consequently,; "the ratio of the volumes of reacting gases could be used to determine the composition and relative molecular masses of chemical compounds".
As we said before, a mole is the quantity equal to the molecular weight in mass units (as grams). Avogadro stated that, a mole of any substance M (a molar mass) contains a constant number of molecules N_A. Then, if m be a relative molecular mass we have;

$$M = N_A m$$

Therefore, $N_A = \frac{M}{m} = 6.022 \times 10^{23}$ is the constant number of molecules per mole. Consequently, "the same number of gas molecules always occupies the same volume".
In normal conditions i.e. in 0° C and one atmosphere of pressure; 1.013×10^5 Pascal the volume(V_o) of a mole of any gas is a constant quantity as well measured as:

$$V_0 = 22.41 \times 10^{-3} m^3$$

Thus N_A and V_0 showing us the most important properties of atoms and molecules. For instance, the mass of one mole of hydrogen H_2 is 2.016 g and that is the mole contains N_A molecules as well. Thus, the mass of one molecule and atom of hydrogen are $3.35 \times 10^{-24} g$ and $1.675 \times 10^{-24} g$.
Now we go on with dimensions and distances among molecules and atoms. This is an estimation process. For example, one cubic centimeter (cm^3) of water is equivalent with 1/18 of the mole of water, so that it contains 3.34×10^{22} molecules of water. The approximate volume of this molecule is; $1/3.34 \times 10^{22}$ which is

equivalent with $3 \times 10^{-23}\ cm^3$. Thus, the linear dimension (r) of a water molecule is;

$$r = \sqrt[3]{3 \times 10^{-23}}\ cm$$

What about the distance among molecules of a gas? The occupied volume of gas molecules (V_M) in one mole is expressed as;

$$V_M = \frac{4}{3}\pi r^3 N_A \approx 70\ cm^3$$

Now we can say that,

$$\frac{V_M}{V_0} = \frac{70\ cm^3}{22.41\ \times\ 10^{-3} m^3} \approx 3.2 \times 10^{-3}$$

Thus, the volume per the molecules is a tiny fraction of V_0 expressed as:

$$V_{perM} = \frac{V_0}{N_A} \approx 3.7 \times 10^{-20} cm^3$$

With respect to this, the average distance among gas molecules would be:

$$r_{ave} = \sqrt[3]{V_{perM}} \approx 3.5 \times 10^{-7} cm$$

We can easily calculate such infinitesimal values using Avogadro incredible number! However, his hypothesis also proposed the *discrete* structure of matter. This is very important in the development of atomic theory.

For instance, at normal conditions, one cubic centimeter of gas contains $2.687\ \times 10^{19}$ molecules. This is the ratio of ${}^{N_A}/{}_{V_0}$ which is called Loschmidt's number. This infinitesimal world with a very large number of tiny bodies (particles) couldn't be described by Newtonian mechanics, because this is impossible to solve equation of motion for a large number of particles with random motions simultaneously. The particles about 10^{-8} in diameter and 10^{-24} in mass. Even large distances among the particles (in a gas), linear motions with uniform velocities in the case of equilibrium condition does not help. If collision, happens then energy will exchange and the direction would change. Also, in the case of non-equilibrium condition there would exist a flux of gas. These would make the calculation more complicated. This is the birth of quantum mechanics.

Boltzmann constant

There was a strong philosophical view in 19th century about atomic world insists that "all attempts to reveal inner mechanisms of phenomena were denounced as nonscientific".

But there were men who believe in atoms. Austrian theoretical physicist and philosopher Ludwig Boltzmann (1844-1906) was the most persistent and stubborn one. He proposed that, atoms do really exist. They have interactions which leads the bodies be seen and sensed.

His greatest achievements were the development of statistical mechanics, and the statistical explanation of the second law of thermodynamics. He defined entropy (S) in 1877 as:

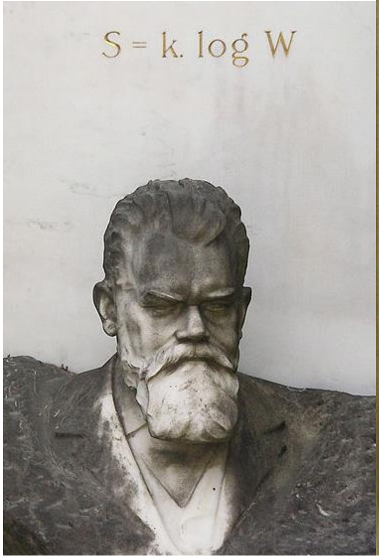
$$S = K_B ln\Omega$$

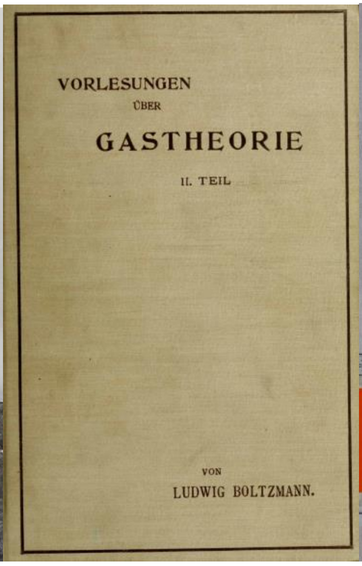
The Greek letter Ω is the number of microstates whose energy equals the system's energy and K_B is a constant of proportionality between temperature and energy. Its value differs based on unit choice for temperature and energy. In Joules per Kelvin its value is;

$$1.380\,649\ \times\ 10^{-23}\, J \cdot K^{-1}$$

However, this famous formula is engraved in his tombstone as: $s = k\ log\ w$. The K_B is proposed by Max Planck later to be called Boltzmann constant. The letter w or Ω is probability (statistical disorder) measure of the system.

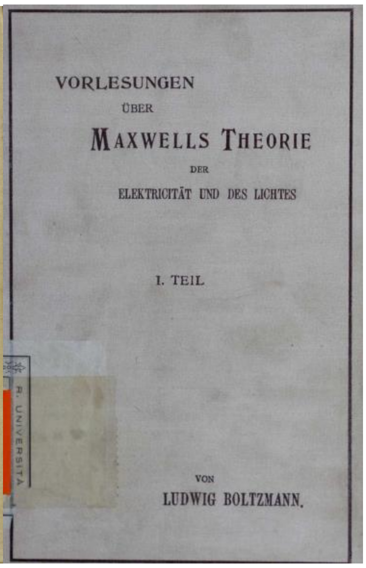
He wrote "Gastheorie" and "Maxwellis theorie" in two volumes. The first one is translated to English by Stephen G. Brush as; "Lectures on Gas Theory" in 1964. This book "contains a comprehensive exposition of the kinetic theory of gases by a scientist who devoted a large part of his own career to it, and brought it very nearly to completion as a fundamental part of modern physics."





VORLESUNGEN
ÜBER
GASTHEORIE
II. TEIL

VON
LUDWIG BOLTZMANN.

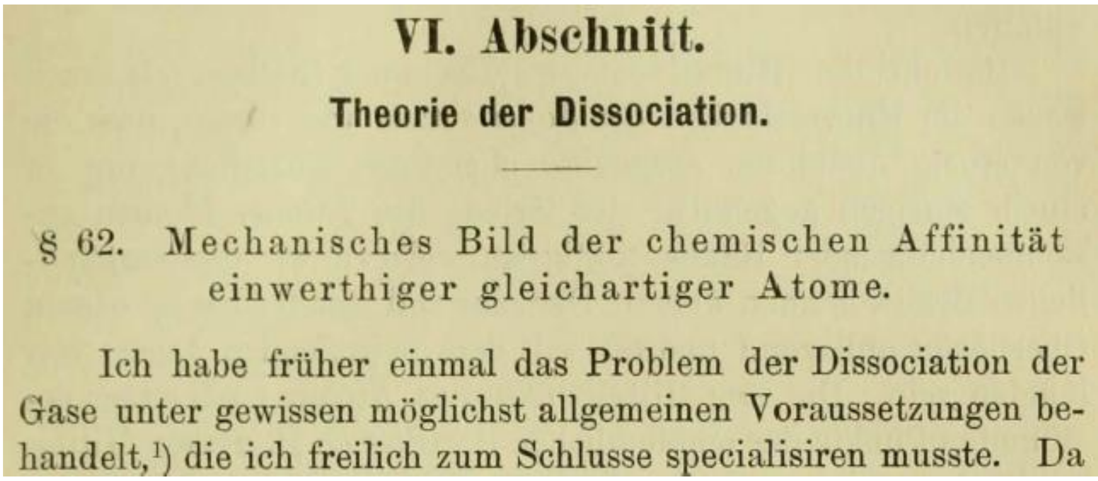


VORLESUNGEN
ÜBER
MAXWELLS THEORIE
DER
ELEKTRICITÄT UND DES LICHTES

I. TEIL

VON
LUDWIG BOLTZMANN.

His kinetic theory of gases, presupposed the existing of atoms. In "Gastheorie" in chapter VI begins as;



VI. Abschnitt.

Theorie der Dissociation.

§ 62. Mechanisches Bild der chemischen Affinität einwerthiger gleichartiger Atome.

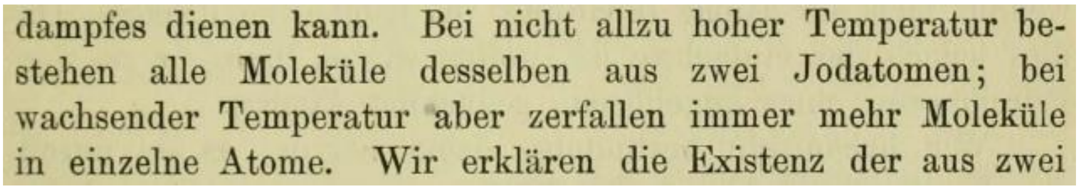
Ich habe früher einmal das Problem der Dissociation der Gase unter gewissen möglichst allgemeinen Voraussetzungen behandelt,[1]) die ich freilich zum Schlusse specialisiren musste. Da

"Theory of dissociation.

§ 62. Mechanical picture of the chemical affinity of monovalent similar atoms.

I have once previously treated the problem of the dissociation of gases, on the basis of the most general possible assumptions, which of course I had to specialize at the end...."

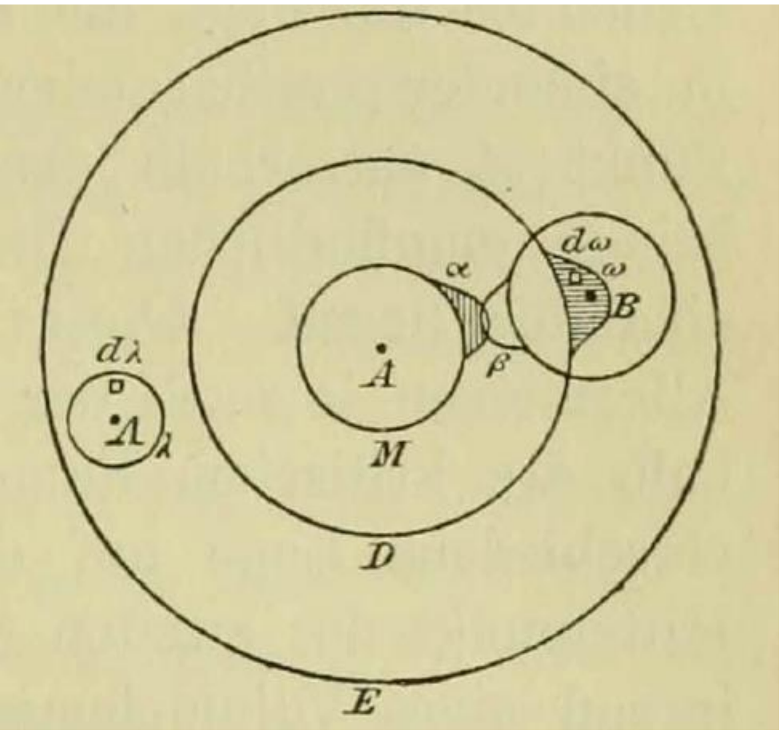
He goes on with the simplest case of dissociation, of iodine vapor as a prototype. He proposed that;



dampfes dienen kann. Bei nicht allzu hoher Temperatur bestehen alle Moleküle desselben aus zwei Jodatomen; bei wachsender Temperatur aber zerfallen immer mehr Moleküle in einzelne Atome. Wir erklären die Existenz der aus zwei

"At not too high temperatures, all molecules consist of two iodine atoms; as the temperature increases, however, more and more molecules decompose into single atoms".

He suggests the "existence of molecules composed of two atoms by an attractive force acting between the atoms". He calls them, "chemical attraction". This force assumed to be effective in a small space compared to the size of the atom, i.e. "the sensitive region" if "two atoms are situated so that their sensitive regions are in contact, or partly overlap".... "The sensitive region shall be such a small part of the entire surface of the atom that the possibility that the sensitive regions of three atoms can be in contact (chemically bounded), or partly overlapping, is completely excluded". He assumes a spherical atom with diameter σ. Then;



Here A is the center and M, D and E are particular atoms. The shaded region alpha and omega are the sensitive regions. they are drawn outside the atom since the atom imagined to be impenetrable. For more information you can refer to Gastheorie" in chapter VI or its English translation "Lectures on Gas Theory" by Stephen G. Brush, (1964).

Boltzmann published "Further Studies on the Thermal Equilibrium of Gas Molecules" in 1872 and "On the Relation of a General Mechanical Theorem to the Second Law of Thermodynamics" In 1877. He also responded to zermelo's philosophical dispute with, "Reply to Zermelo's Remarks on the Theory of Heat" in 1896 and "On the Mechanical Explanation of Irreversible Processes" in 1897.

We know that, gases tend to uniformly spread in a given volume. Consequently, under equilibrium condition, their molecules (particles) properties are independent of initial positions. Thus, knowing the path or trajectory of molecules is useless. In large number of particles what matter is their behavior on average.

The kinetic theory of gases is better to begines with pressure. But what is pressure exactly? The answer is, molecules of gas have random continues motions thus they collide with each other and with the walls of its container, and this will reverse their directions of their velocities. Finally, this causes a force acting on the inside surface of the container and this is pressure. This force is an average (vector) quantity. Thus, this quantity is not exact and always along with some errors (noise) due to the natural fluctuations among the particles of a gas.

The value of these fluctuations is ${N_A}^{-1/2}$ which is commonly is equal to 10^{-12} percent. So ignorable!

Now we can go on with the simple laws of gases. These laws are correct for all gases with different chemical compositions. Why? Since they are just "systems made up of large numbers of weakly interacting particles and the average characteristics of such systems are insensitive to the nature of the particles".

At constant temperature, $pV = constant$. The Boyle's law. The Gay-Lussac law relates ΔV to ΔT as $V = V_0 \alpha T$. Here α is factor of expansion 1/273 K^{-1}. With These two equations, we can describe the state of an ideal gas i.e. the state equation.

$$\frac{pV}{T} = R = constant$$

But what is R? it is called the **universal gas constant**. At normal condition namely when $p = 1.013 \times 10^5 \text{Pa}$ and $T = 273\ K$, one mole of a gas fills the volume $V_0 = 22.41\ \times 10^{-3}\ m^{-3}$. Thus;

$$R = \frac{pV_0}{T} = 8.31\, J/mole\ .K$$

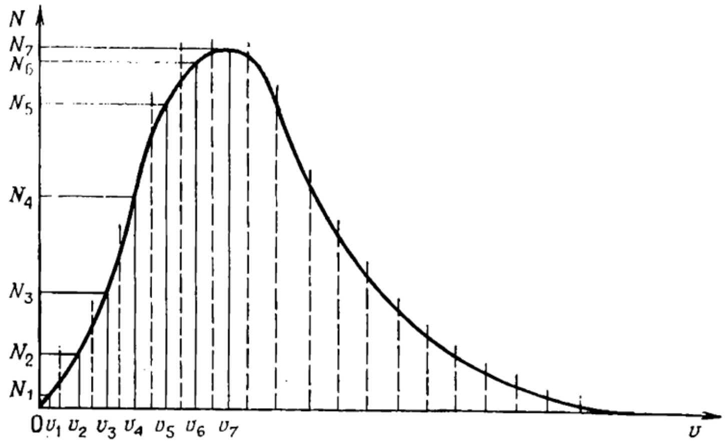
"The universal gas constant is in fact the amount of energy acquired (or lost) by the molecules of a mole of gas if its temperature is varied by one degree."

From recent description it is clear that, in this averaging process of calculation we are dealing with a velocity distribution of gas particles. This is due to the fact that, collision of gas particles in a given volume exchange (kinetic) energy among them an also leads to dissipation and gradient (difference) of energy. thus;

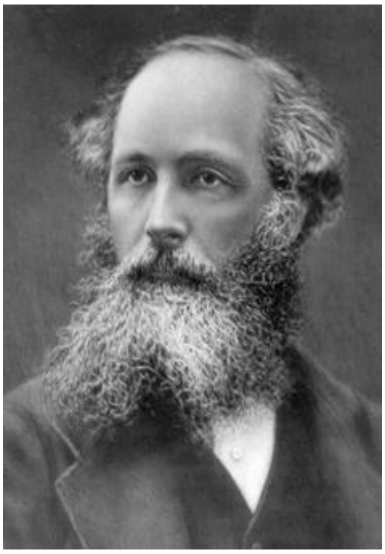
$$\bar{v} = \frac{N_1 v_1 + N_2 v_2 + \cdots N_n v_m}{N_1 + N_2 + N_3 + \cdots N_k} = \sum_1^{n,m,k} \frac{N_n v_m}{N_k}$$

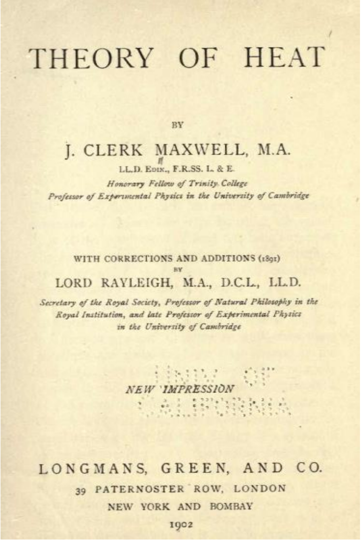
Here, $\bar{v}$ is the average velocity, $N_1, N_2, \ldots N_n$ the number of particles, $v_1, v_2, v_3 \ldots v_m$ and $N_1 + N_2 + N_3 + \cdots N_k$ the total number of particles. Thus, the *velocity distribution of gas particles* is a discrete function of *N(v)*.

For this function we can have a diagram such as below.



In the middle of the nineteenth century, James Clerk Maxwell (1831-1879) derived first function of this Kind. On his book, "Theory of Heat" published in four editions from 1871 to 1902, on the last chapter, he proposed such functions.





THEORY OF HEAT

BY

J. CLERK MAXWELL, M.A.
LL.D. Edin., F.R.SS. L. & E.
Honorary Fellow of Trinity College
Professor of Experimental Physics in the University of Cambridge

WITH CORRECTIONS AND ADDITIONS (1891)
BY
LORD RAYLEIGH, M.A., D.C.L., LL.D.
Secretary of the Royal Society, Professor of Natural Philosophy in the Royal Institution, and late Professor of Experimental Physics in the University of Cambridge

NEW IMPRESSION

UNIV. OF CALIFORNIA

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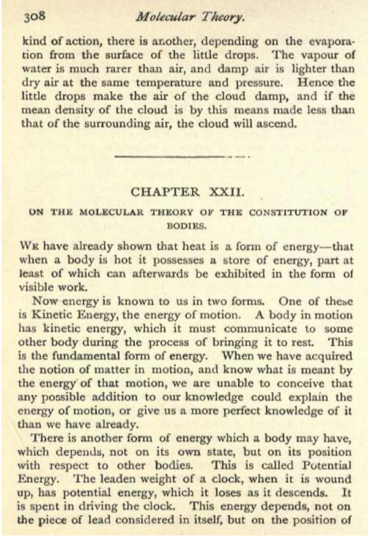


308 *Molecular Theory.*

kind of action, there is another, depending on the evaporation from the surface of the little drops. The vapour of water is much rarer than air, and damp air is lighter than dry air at the same temperature and pressure. Hence the little drops make the air of the cloud damp, and if the mean density of the cloud is by this means made less than that of the surrounding air, the cloud will ascend.

CHAPTER XXII.

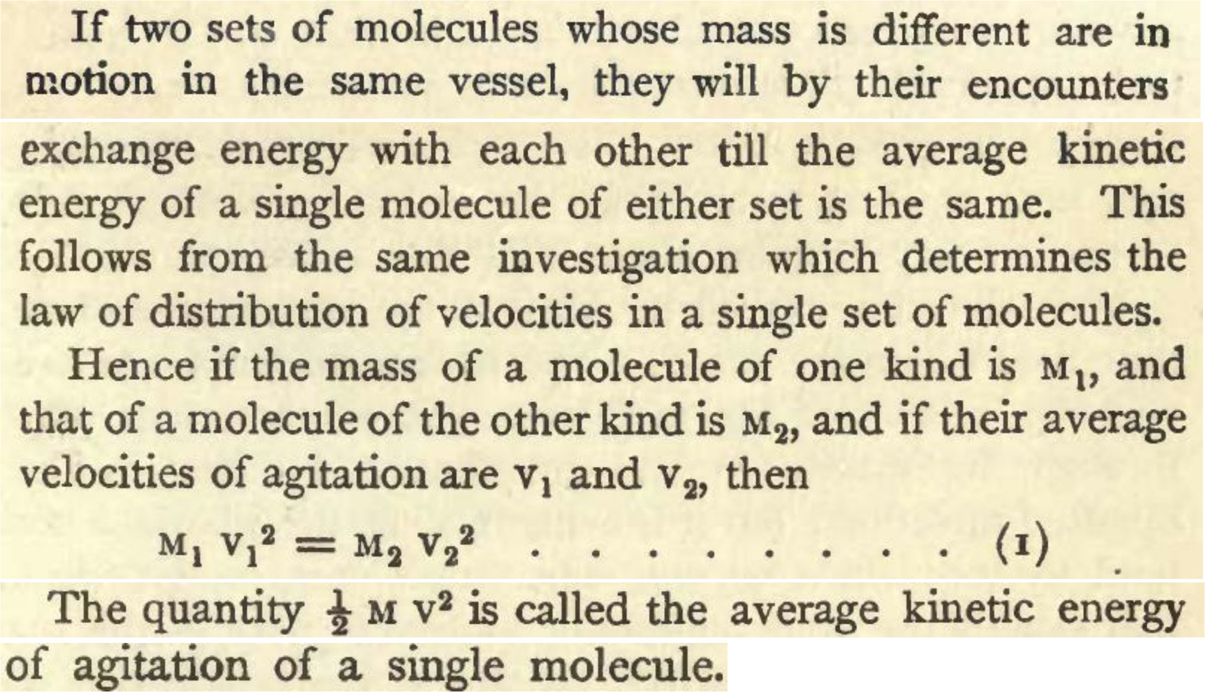
ON THE MOLECULAR THEORY OF THE CONSTITUTION OF BODIES.

We have already shown that heat is a form of energy—that when a body is hot it possesses a store of energy, part at least of which can afterwards be exhibited in the form of visible work.

Now energy is known to us in two forms. One of these is Kinetic Energy, the energy of motion. A body in motion has kinetic energy, which it must communicate to some other body during the process of bringing it to rest. This is the fundamental form of energy. When we have acquired the notion of matter in motion, and know what is meant by the energy of that motion, we are unable to conceive that any possible addition to our knowledge could explain the energy of motion, or give us a more perfect knowledge of it than we have already.

There is another form of energy which a body may have, which depends, not on its own state, but on its position with respect to other bodies. This is called Potential Energy. The leaden weight of a clock, when it is wound up, has potential energy, which it loses as it descends. It is spent in driving the clock. This energy depends, not on the piece of lead considered in itself, but on the position of

He considers distribution of kinetic energy between two sets of molecules and states that;



If two sets of molecules whose mass is different are in motion in the same vessel, they will by their encounters exchange energy with each other till the average kinetic energy of a single molecule of either set is the same. This follows from the same investigation which determines the law of distribution of velocities in a single set of molecules.

Hence if the mass of a molecule of one kind is M_1, and that of a molecule of the other kind is M_2, and if their average velocities of agitation are V_1 and V_2, then

$$M_1 V_1^2 = M_2 V_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The quantity $\frac{1}{2} M V^2$ is called the average kinetic energy of agitation of a single molecule.

But it was Ludwig Boltzmann who consider this more deeply and also went further to solve a number of fundamental problems of the new kinetic theory. He extended Maxwell's results and proved that even if rotations and vibrations of molecules are taken into consideration, "the same velocity (or energy) distribution of molecules holds for all gases". Then he went on further and deduce the energy (E) distribution for (n) molecules affected by external forces as;

$$n(E) = Ae^{-BE}$$

Here B and A are coefficients. Using this result, he concluded that, molecules concentration is a function of height (altitude) as;

$$n_h = n_0 e^{-ch}$$

Here c is a coefficient, n_0 molecules concentration at zero (sea) level and n_h this concentration at an arbitrary height.

He also found that "in large systems the energy is equally divided among all the degrees of freedom". Consequently;

$$\bar{E} \propto T$$

Here E is the average kinetic energy and T the absolute temperature in kelvin (K). The significance of this relation is that, T is a macroscopic parameter of a gas and in the same time is an energy measure of microscopic particles. This fact, paves the way for further development in this fabulous microworld.

Now we go on with, fundamental equation of kinetic theory and equation of state for an ideal gas. The first is expressed as;

$$P = 2/3\, \rho \bar{E}$$

Here ρ stands for concentration (density) of gas molecules. Multiplying both sides by molar volume V_0 we get;

$$pV_0 = \frac{2}{3} N_A \bar{E}$$

Here $N_A = \rho V_0$

The second is expressed as;

$$pV_0 = RT$$

With substitution we get;

$$\frac{2}{3}\,N_A\bar{E} = RT$$

Dividing both side by $2/3$ and then N_A we get;

$$\bar{E} = \frac{3}{2}\,\boldsymbol{R/N_A}T$$

This ratio R/N_A, as we know is called Boltzmann constant K. Now substituting the known values, we get;

$$k = \frac{R}{N_A} = \frac{8.31\,J.m}{6.023 \times 10^{23}K.m} = 1.38 \times 10^{-23}J/K$$

Here *J, m and K* stand for Joules, mole and Kelvin respectively.

Now one can write average kinetic energy equation of the particles as; $\bar{E} = \frac{3}{2}\,\boldsymbol{k}\,T$ and fundamental equation of the kinetic theory as $P = \rho kT$.

Consequently, Boltzmann's constant connects changes of $\bar{E}$ of a microscopic (infinitesimal) world to the changes of T and p of a macroscopic one.

The usefulness of this constant is so wide in physics. For instance, if we express average kinetic energy as;

$$\bar{E} = \frac{1}{2}\,\boldsymbol{m\bar{v}^2}$$

Here $\boldsymbol{\bar{v}^2}$ is the average squared velocity of a molecule which is expressed as;

$$\boldsymbol{\bar{v}^2} = \frac{2\bar{E}}{m} = \frac{3kT}{m}$$

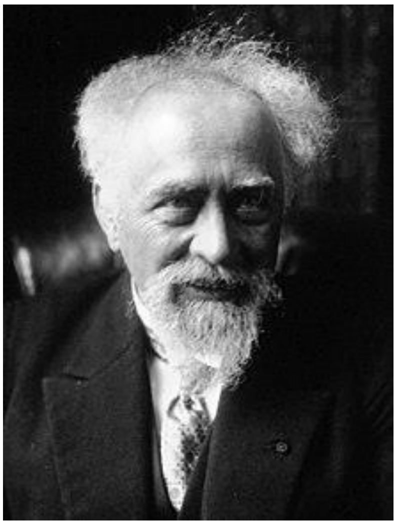
For hydrogen H_2 molecule with the mass of $3.35 \times 10^{-24}g$ at 273 K, the amount of this quantity would be;

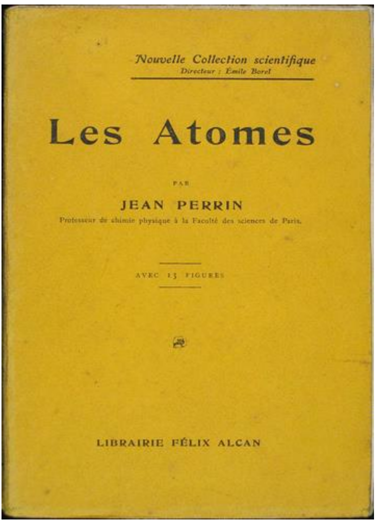
$$\bar{v}_{H_2}^2 = \frac{2\bar{E}}{3.35 \times 10^{-24} g} = \frac{3(1.38\ \times 10^{-23}\, J/K)(273)}{3.35 \times 10^{-24} g}$$

$$v = \ 1800\, m/s$$

Thus, as *"Avogadro's number has provided us with the dimensions and masses of (micro) particles, Boltzmann's constant enabled us to get an idea of their energies and speeds"*.

However, Boltzmann revolutionary ideas was not recognized in his life time but in the last few years before his death it was begun to be considered more seriously. The French physicist Jean Baptiste Perrin (1870-1942) in his book "Les Atomes" proved the existence of atoms based on an experiment for Avogadro's number under Brownian motion. He won the Noble prize in physic 1926 for his work on discontinuous structure of matter and sedimentation equilibrium.





Nouvelle Collection scientifique
Directeur : Émile Borel

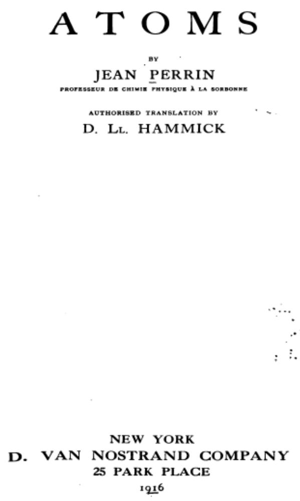
Les Atomes

PAR

JEAN PERRIN
Professeur de chimie physique à la Faculté des sciences de Paris.

AVEC 13 FIGURES

LIBRAIRIE FÉLIX ALCAN



ATOMS

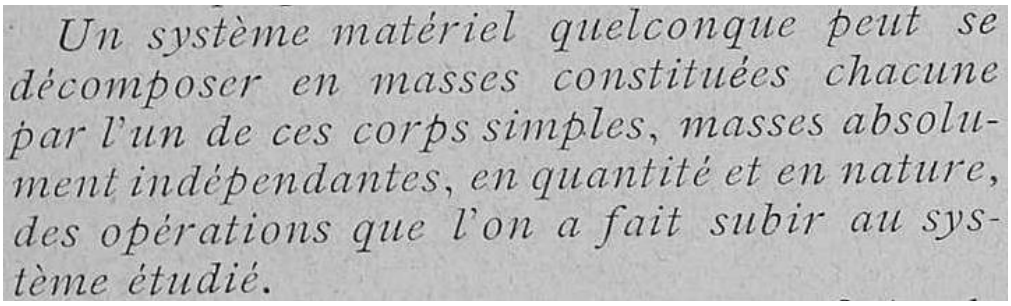
BY

JEAN PERRIN
PROFESSEUR DE CHIMIE PHYSIQUE À LA SORBONNE

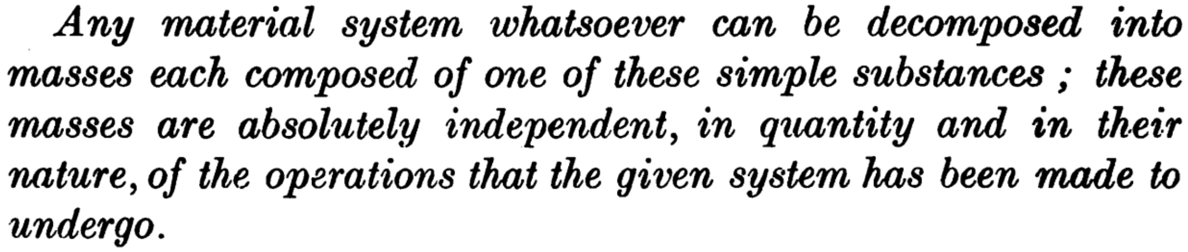
AUTHORISED TRANSLATION BY
D. LL. HAMMICK

NEW YORK
D. VAN NOSTRAND COMPANY
25 PARK PLACE
1916

Nature in 10 July of 1913 wrote about his book and discovery that, Perrin has "virtually made atoms visible and stablished their reality". He expressed that;

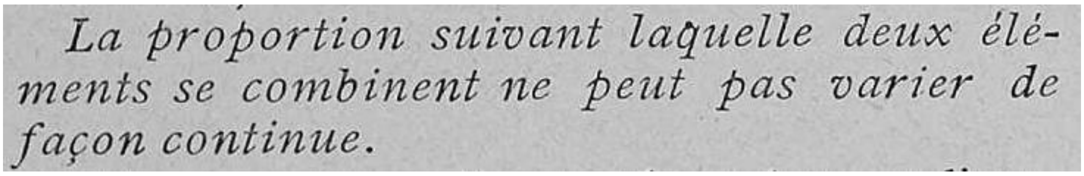


Un système matériel quelconque peut se décomposer en masses constituées chacune par l'un de ces corps simples, masses absolument indépendantes, en quantité et en nature, des opérations que l'on a fait subir au système étudié.



Any material system whatsoever can be decomposed into masses each composed of one of these simple substances ; these masses are absolutely independent, in quantity and in their nature, of the operations that the given system has been made to undergo.

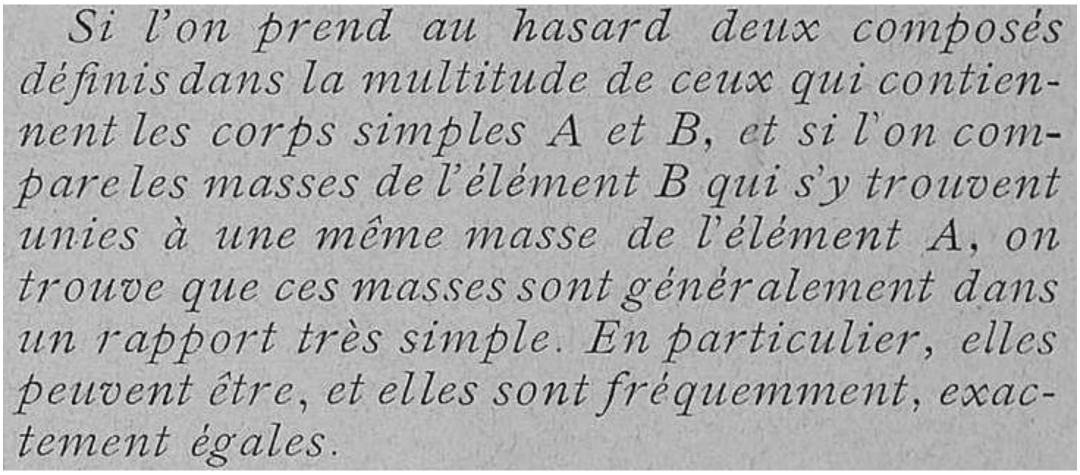
Then he defines the definite proportions as;



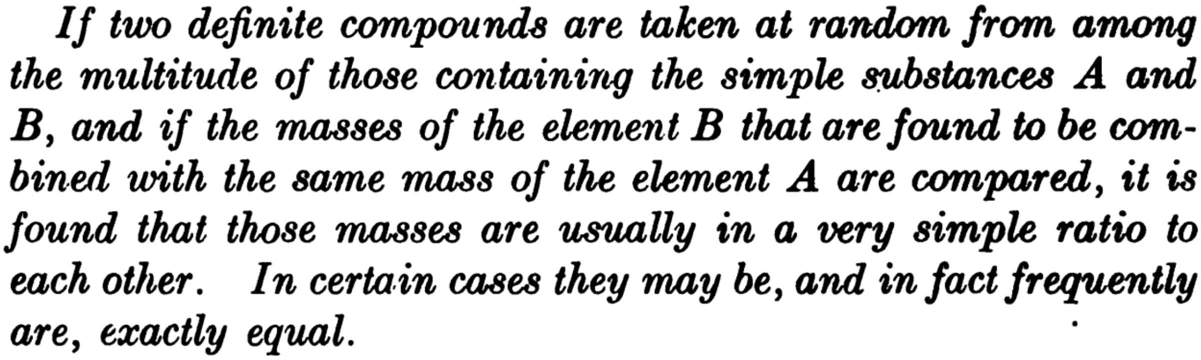
La proportion suivant laquelle deux éléments se combinent ne peut pas varier de façon continue.

The proportions in a which two elements combine cannot vary continuously.

And multiple proportions as;

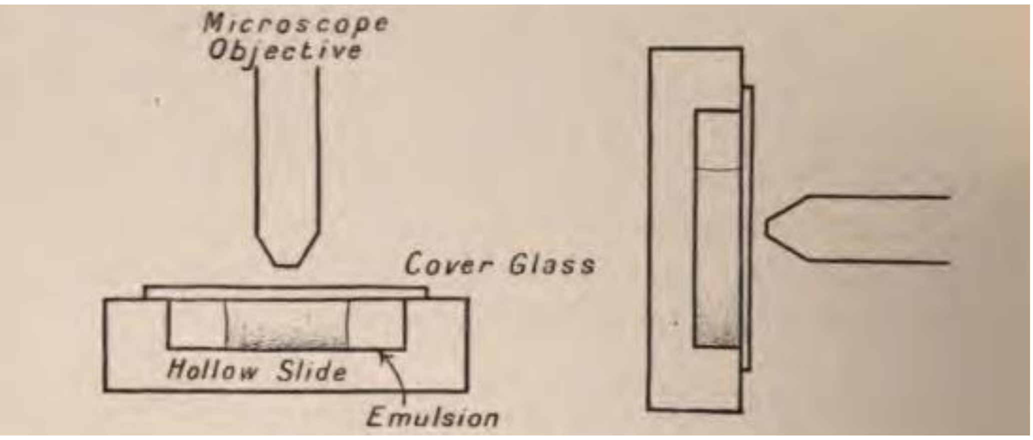


Si l'on prend au hasard deux composés définis dans la multitude de ceux qui contiennent les corps simples A et B, et si l'on compare les masses de l'élément B qui s'y trouvent unies à une même masse de l'élément A, on trouve que ces masses sont généralement dans un rapport très simple. En particulier, elles peuvent être, et elles sont fréquemment, exactement égales.

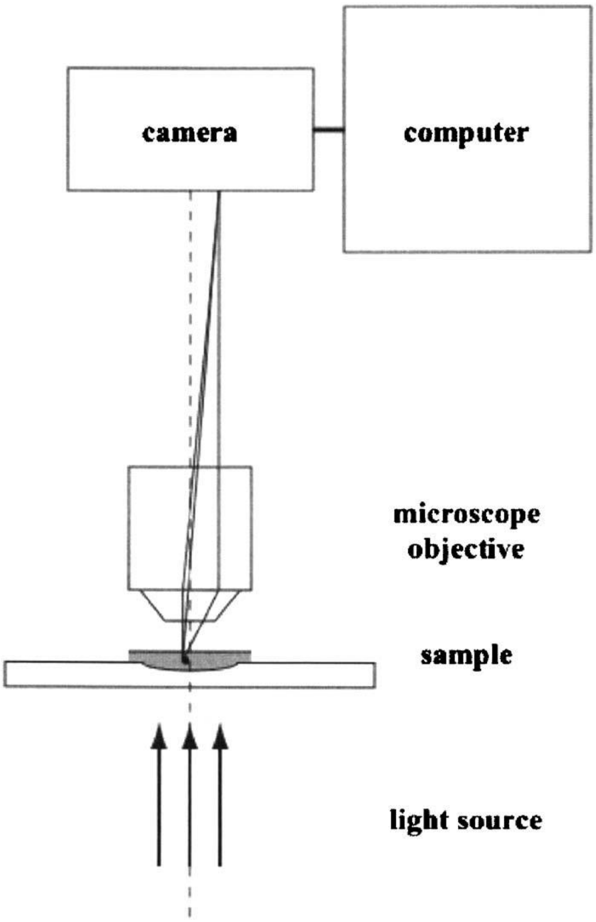


If two definite compounds are taken at random from among the multitude of those containing the simple substances A and B, and if the masses of the element B that are found to be combined with the same mass of the element A are compared, it is found that those masses are usually in a very simple ratio to each other. In certain cases they may be, and in fact frequently are, exactly equal.

He performed his experiment with a microscope and camera (to photograph), then investigate, a drop of emulsion in horizontal and vertical positions.

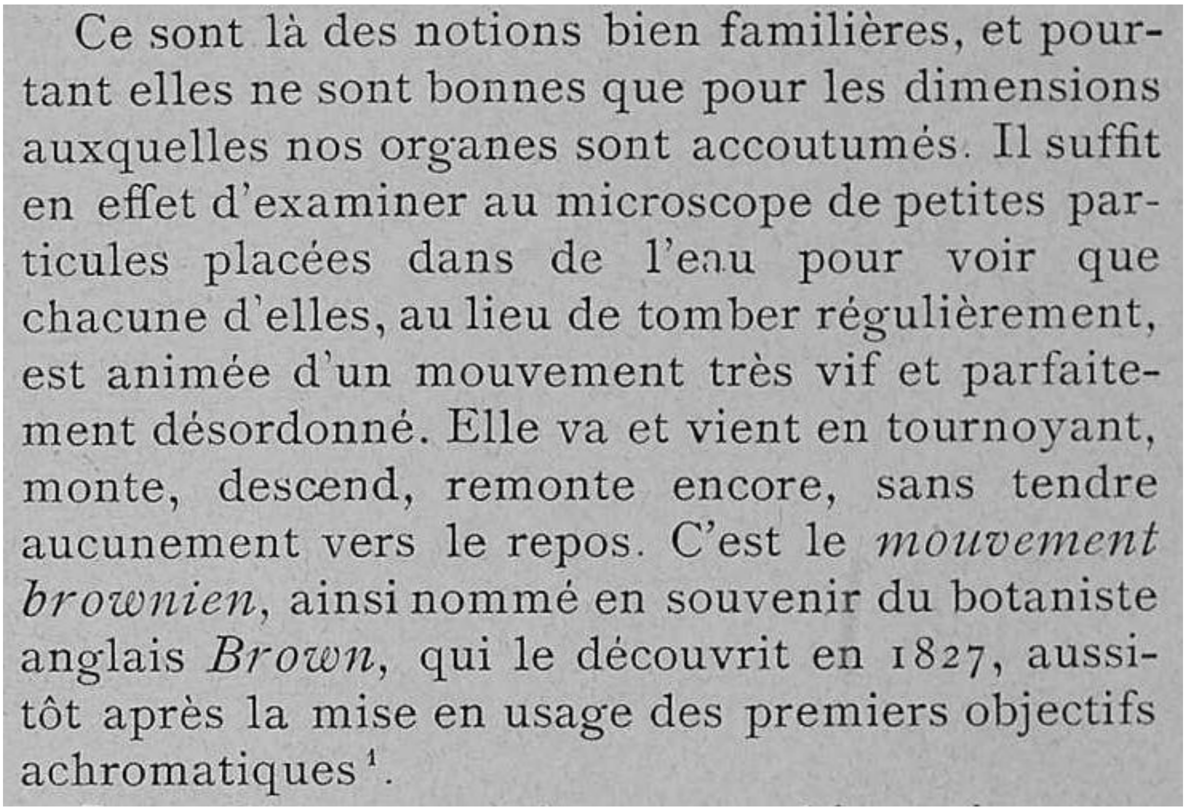


However, Ronald Newburgh et al (2006) repeated Perrin's 1908 experiment for the determination of Avogadro's number by determining the mean square displacement of small particles undergoing Brownian motion. They used a CCD camera in their apparatus as shown below.

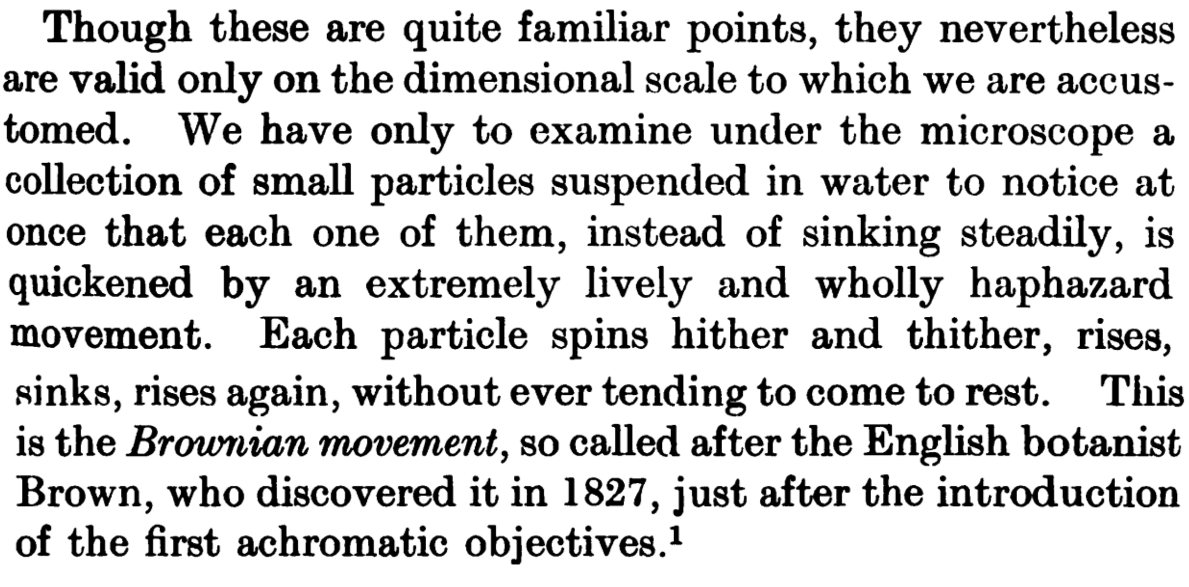


They also "review Einstein's 1905 analysis of Brownian motion and Langevin's alternative derivation of the Einstein equation for the mean square displacement and show how Einstein's thinking was a reflection of his belief in the validity of molecular-kinetic theory".

To define Brownian motion Perrin states that, it is not possible to see agitation of molecules or microscopic particles suspended in a fluid directly except with a microscope. These particles will remain at rest in the bottom of the container (sedimentation equilibrium). Then he continue as;



Ce sont là des notions bien familières, et pourtant elles ne sont bonnes que pour les dimensions auxquelles nos organes sont accoutumés. Il suffit en effet d'examiner au microscope de petites particules placées dans de l'eau pour voir que chacune d'elles, au lieu de tomber régulièrement, est animée d'un mouvement très vif et parfaitement désordonné. Elle va et vient en tournoyant, monte, descend, remonte encore, sans tendre aucunement vers le repos. C'est le *mouvement brownien*, ainsi nommé en souvenir du botaniste anglais *Brown*, qui le découvrit en 1827, aussitôt après la mise en usage des premiers objectifs achromatiques[1].



Though these are quite familiar points, they nevertheless are valid only on the dimensional scale to which we are accustomed. We have only to examine under the microscope a collection of small particles suspended in water to notice at once that each one of them, instead of sinking steadily, is quickened by an extremely lively and wholly haphazard movement. Each particle spins hither and thither, rises, sinks, rises again, without ever tending to come to rest. This is the *Brownian movement*, so called after the English botanist Brown, who discovered it in 1827, just after the introduction of the first achromatic objectives.[1]

In fact, he did a series of experiments. But Perrin's experimentalists on suspended particles, proved molecular reality. His results proved Einstein's mean square displacement relation for suspended particles and therefore the molecular-kinetic theory.

Albert Einstein was also one of the physicists who worked on the existing of atoms. " He accepted Maxwell-Boltzmann statistics and its relation to the molecular-kinetic theory of heat". He did this on his dissertation "*On a new determination of molecular dimensions*" (University of Zürich, 1905) and his paper *"Investigations on the Theory of Brownian Movement"*(1956).

In this paper he showed that, "particles suspended in a liquid behave much like solute molecules dissolved in a liquid". However, those suspended particles display an osmotic pressure. An osmosis is the process by which a liquid pass through a thin piece of solid substance such as the roots of a plant. The only difference between these two was in their dimensions. He proved this "calculating the entropy and free energy of the entire system of particles and liquid".

His calculation explained that, kinetic theory of molecules which gave us, the ideal gas law, it also "explain the osmotic pressure of a suspension of particles". In fact, in his investigation, he started from solution to suspension and stablished his equation of mean square displacement $\langle x^2 \rangle$ for suspended particles as;

$$\langle x^2 \rangle = t(\frac{RT}{3\pi\eta r N_A})$$

This leads to an expression for Avogadro's number as;

$$N_A = \left(\frac{1}{\langle x^2 \rangle}\right)(RT/3\,\pi\eta r)t$$

Here, η is the viscosity coefficient, r the radius of the particle and t stands for inverted time between particle positions. Thus, he described the Brownian motion as a diffusion process.

Using his equation of state for gases and liquid, Dutch theoretical physicist Johannes Van der Waals, showed in 1873 that, $N_A > 10^{23}$ per mole namely $6.2 \times 10^{23}\, mole^{-1}$ approximately. Although his results were denied in scientific communities but other scientists went on.

English physicist Lord Rayleigh investigated the effect of dispersion (scattering) of light by atmosphere in 1871. The different wavelength (color) the different scattering intensity. Thus, he derived a ratio (formula) of incident intensity to scattering intensity of light in inverse relation with the air density.

However, his formula was become confirmed by William Thomson (Lord Kelvin) on the mountain Monte Rosa, in Italy due to the cleaner air to affect the measurements. Actually, the intensity (brightness) of the Sun light was measured "at the zenith and at 40 degrees above the horizon". These measurements leaded Thomson to find "the limits for Avogadro’s number" as:

$$3 \times 10^{23}\ \text{mole}^{-1} \leqslant N_{\mathbf{A}} \leqslant 15 \times 10^{23}\ \text{mole}^{-1}.$$

However, as we said before it was discovering of a continuous random motion of very small particles suspended in a liquid by Robert Brown in 1828. This paved the way for displaying the reality of microparticle (atoms and molecules) by determining more accurate Avogadro's number. This is showing us the significance of constants in physics.

Now the question was what was the cause of this continues random motion. It was discovered that, the smaller particles and more temperature the faster they move. This was another credit for kinetic theory.

It was Jean Perrin in 1908 who confirmed Einstein's equation of mean square displacement with his great experiment. Using nearly equal size of suspending particles in a drop of liquid and observing the traveling distances among particles under Brownian motion he found $N_A \approx 6.85 \times 10^{23}$per mole. After this discovery the existence of atoms and molecules could no longer be doubted.

There exist other procedures to determine Avogadro's number such as the use of electrolysis data, the study of the radioactive decay of radium and the measurement of the blackbody radiation. However, one of the most precise one is using X-ray scattering on crystalline solids. In this method, if the X-rays have wavelength λ and the interatomic separation in the crystal is α, then;

$$N_A = nM/V_0\rho_c$$

Here n stands for the number of particles, M for their relative mass ρ_c the density of crystal and V_0 stands for initial cell volume. This formula and method give us;

$$N_A = (6.02205 \mp 0.00016) \times 10^{23}\ per\ mole$$

We finish this discussion with a statements of Stephen G. Brush Introduction in "Lectures on Gas Theory" in 1964 that; "Boltzmann's model for dissociation, employing molecules with small "sensitive regions" on parts of their surfaces, is an interesting historical curiosity, but the need for such ad hoc assumptions to explain the directional character of chemical bonds has been eliminated by quantum mechanics".

The story of electron

It was investigation of electron in 1897 that permitted us to explain the common nature of electricity and magnetism, light and electromagnetic waves despite their different manifestations. It also exposed the structure of atom and though elementary particles; it was our travel from classical physics to modern physics and beyond.

Basically, it is an elementary particle carrying a unit of negative charge. The laws of electric currents will explain the behavior of a large collections of these elementary particles. In fact, from now on, electron, will play us the role of a lantern. Thus, it is fair to title this part with its name.

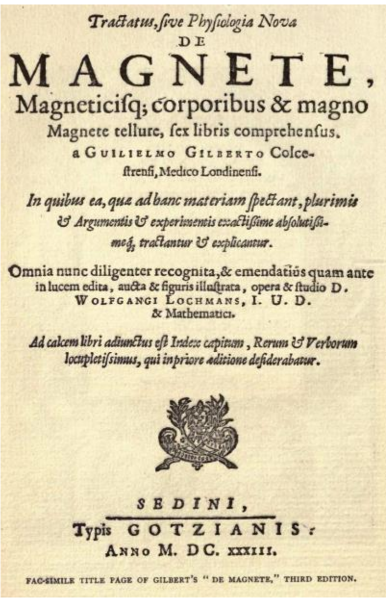
The idea of an impenetrable smallest substance called "atom" turns back to ancient Greek but the existence of smallest particles with electrical effects namely "electrons" is belonging to the late 18'th century. Although, for a long-time people knew that, with rubbing a piece of amber on object such as wool or silk makes it capable of attracting small light objects like a feather. One of them was Thales of Miletus about 600 BC who is also known for his theorem in geometry. He believed that, "it is the soul which somewhat producing motion, and that the loadstone (magnet) has a soul, since it moves iron". Another Greek philosopher, Plutarch of Delphi (46-119) AD believed that "there is in amber something flame-like, or having the nature of the breath, and that, when the paths are cleared by friction of the surface, is emitted and attracts bodies".

Such a *motive soul* of Thales, is also could be found in the works of Islamic scholars such as Avicenna, Tusi, Mola sadra to the recently died polymath Hasanzadeh Amoli. However, one can extend and interpret this "soul" as the word "energy" we use today, because we do not know what actually energy is as we do not know what exactly soul is. The only way we recognize both is their manifestations and appearance in different phenomena of nature.

Thus, a long time after Greeks (about 2000 years!) all things remained almost untouched. Even in the hands of brilliant Islamic scholars, due to the weak observational and experimental methods they use. Another reason is that, they were dealing with a phenomenon of nature which they couldn't see it directly but observe its effects. Thus, they just started to classify them.

Until, British scientist and physician William Gilbert (1544-1603) published his famous book "De Magnet" in 1600. He defined and coined the term "electrica" (electrical bodies) as, those substances which have properties of an amber namely they could attract small objects after being rubbed. It is fossilized tree resin using as a gemstone in decoration and jewelry and Greeks call it electron (ήλεκτρον).

William Gilbert showed that, this attracting phenomenon is existed in peace of magnet, a glass rod rubbed against silk and the planet earth as well. He is mostly known for his book "De Magnete, et de Magno Magnete Tellure" (on the magnet (loadstone) and magnetic bodies and on the great magnet the earth) wrote in Latin. The word "Tellure" is the name of his model of planet earth which he built to do his experiments.



Tractatus, ſive Phyſiologia Nova
DE
MAGNETE,
Magneticiſq; corporibus & magno
Magnete tellure, ſex libris comprehenſus,
a GUILIELMO GILBERTO Colce-
ſtrenſi, Medico Londinenſi.

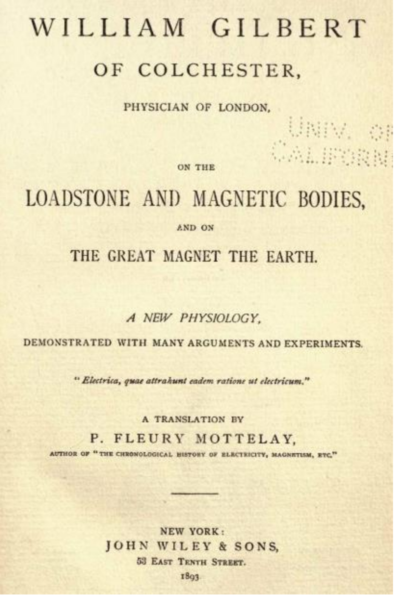
In quibus ea, quæ ad hanc materiam ſpectant, plurimis
& Argumentis & experimentis exactiſſime abſolutiſſi-
meq; tractantur & explicantur.

Omnia nunc diligenter recognita, & emendatiùs quam ante
in lucem edita, aucta & figuris illuſtrata, opera & ſtudio D.
WOLFGANGI LOCHMANS, I. U. D.
& Mathematici.

Ad calcem libri adiunctus eſt Index capitum, Rerum & Verborum
locupletiſsimus, qui in priore æditione deſiderabatur.

SEDINI,
Typis GOTZIANIS:
ANNO M. DC. XXXIII.

FAC-SIMILE TITLE PAGE OF GILBERT'S "DE MAGNETE," THIRD EDITION.



WILLIAM GILBERT
OF COLCHESTER,
PHYSICIAN OF LONDON,

ON THE
LOADSTONE AND MAGNETIC BODIES,
AND ON
THE GREAT MAGNET THE EARTH.

A NEW PHYSIOLOGY,
DEMONSTRATED WITH MANY ARGUMENTS AND EXPERIMENTS.

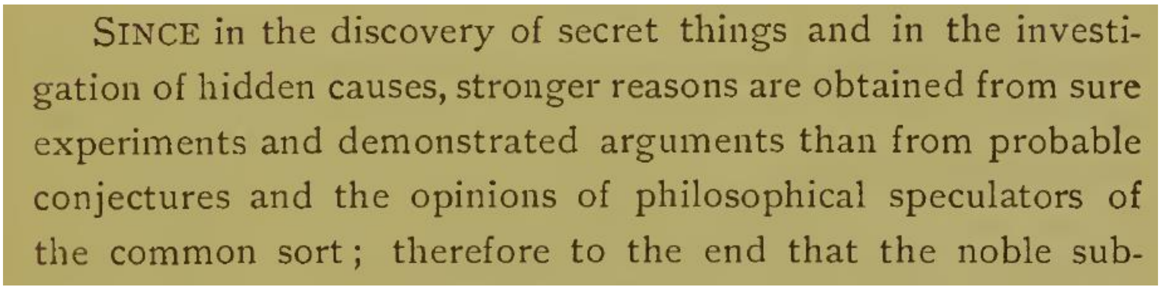
"*Electrica, quae attrahunt eadem ratione ut electricum.*"

A TRANSLATION BY
P. FLEURY MOTTELAY,
AUTHOR OF "THE CHRONOLOGICAL HISTORY OF ELECTRICITY, MAGNETISM, ETC."

NEW YORK:
JOHN WILEY & SONS,
53 EAST TENTH STREET.
1893

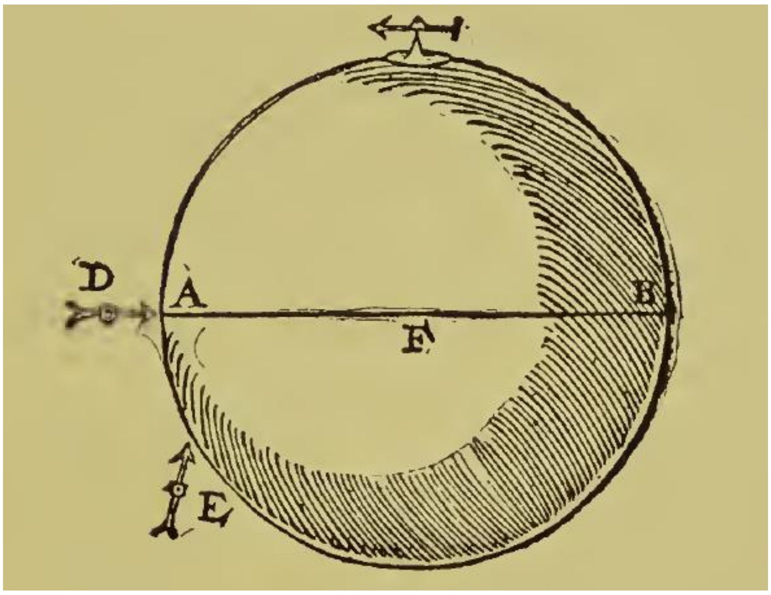
UNIV. OF
CALIFORN

In the preface of his book, he begins with emphasis on experiments in natural sciences. We use here the translated book by P.Fleury Mottelay published in 1893.



SINCE in the discovery of secret things and in the investigation of hidden causes, stronger reasons are obtained from sure experiments and demonstrated arguments than from probable conjectures and the opinions of philosophical speculators of the common sort; therefore to the end that the noble sub-

In the first book he reviews ancient to modern (Gilbert's time) writing concerning loadstone. In the second he investigates magnetic movements for the earth and also for the loadstone. In book three and four he considers the orientations and variations of magnetic force on planet earth and loadstone respectively. Book five investigates the relations between dip (sudden turning of the compass needle) and magnetic force in planet earth and loadstone. In the book six he considers "the globe of earth as a great loadstone". He did his experiments mostly using his model of planet earth "Tellure" namely, little earth. It was a homogeneous "globe-shaped loadstone".

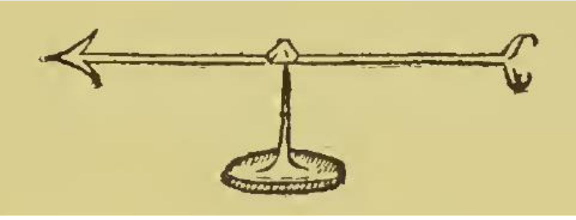


"In the heavens, astronomers give to each moving sphere two poles ; thus do we find two natural poles of excelling importance even in our terrestrial globe, constant points related to the movement of its daily revolution, to wit, one pole pointing to Arctos (Ursa) and the north; the other looking toward the opposite part of the heavens. In like manner the loadstone has from nature its two poles, a northern and a southern; fixed, definite points in the stone, which are the primary termini (limits) of the movements and effects, and the limits and regulators of the several actions and properties".

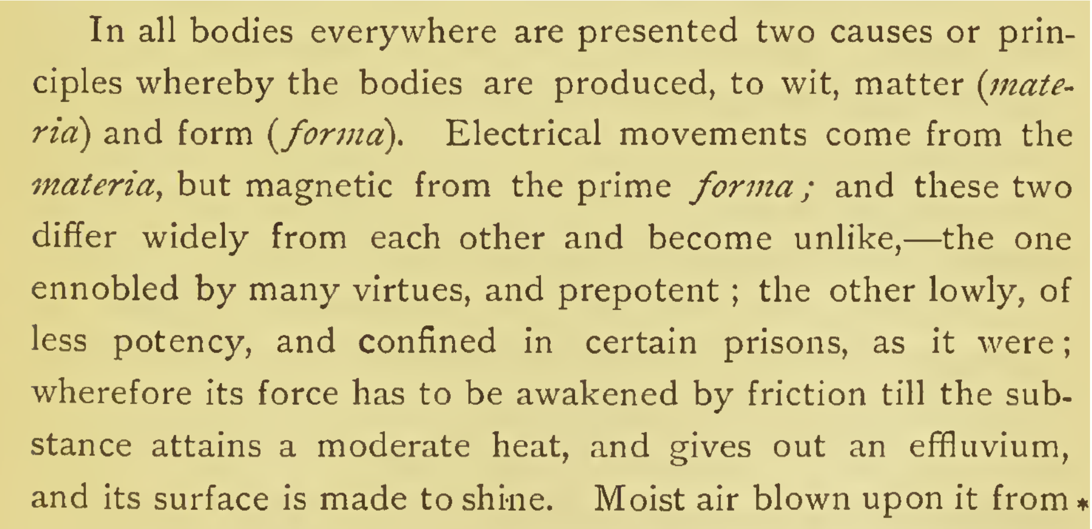
He follows magnet footprints across history and languages and finally states that; Greeks called the magnet as *Σιδηρι'της* because of its property of attracting iron; and the diamond is called *Σιδηρι'της* from the glistening of polished iron".

Gilbert proposed that, the source of magnetic force (attraction) in a loadstone and other "homogeneous magnetic matter" is the same with "potency of the earth's core". He expresses that, ancient observe that (rubbed) objects such as amber, jet and glass attract straws and chaff. But no one tried to explained the cause of this phenomenon.

Gilbert stated that, lots of other material such as crystals, sealing-wax and resin also possess this attracting force. Theses "electric bodies" are able to attract not only straws and chaff, but also, "whatever things appeal to our senses or are solid". He showed this fact by using a simple device which was a rotating needle with a polished amber or gem on one end. "at once the instrument revolves, several objects are seen to attract"



He expresses that, scholars such as Avicenna and others just numerate and list three kind of such attractions in nature. First those with rubbing, second those with "in—rush into a vacuum" and third those in their entire mass. These dividing do not tell us what is the causes of this attracting force. He believed that, "there is no attraction outside of the class of magnetic and electric bodies". He proposed that electric bodies are those which are polished and rubbed. If not, they are "anelectrics" or non-electrics. Then in searching for the cause of this attracting force (electrical movements) he writes;



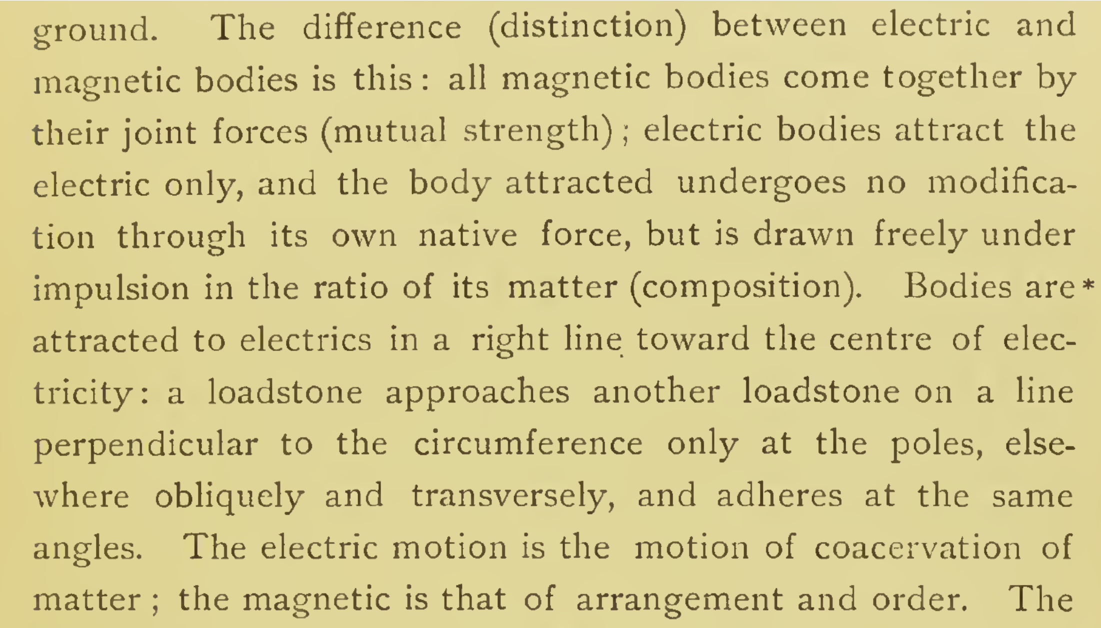
> In all bodies everywhere are presented two causes or principles whereby the bodies are produced, to wit, matter (*materia*) and form (*forma*). Electrical movements come from the *materia*, but magnetic from the prime *forma;* and these two differ widely from each other and become unlike,—the one ennobled by many virtues, and prepotent ; the other lowly, of less potency, and confined in certain prisons, as it were ; wherefore its force has to be awakened by friction till the substance attains a moderate heat, and gives out an effluvium, and its surface is made to shine. Moist air blown upon it from

Then he proposed that, a loadstone attracts and lifts only magnetic (heavy) bodies; electrics attract everything with light weights. He asks, "what state is brought about by rubbing; also, what causes are evoked that seize all sorts of substances?".

His crude answer is; rubbing will agitate the inner heat ("their own heat") of the amber or any substances and this "heat produced by friction dissolves bodies into **effluvia**" (effluxes, currents) and this makes electrics ready for action.

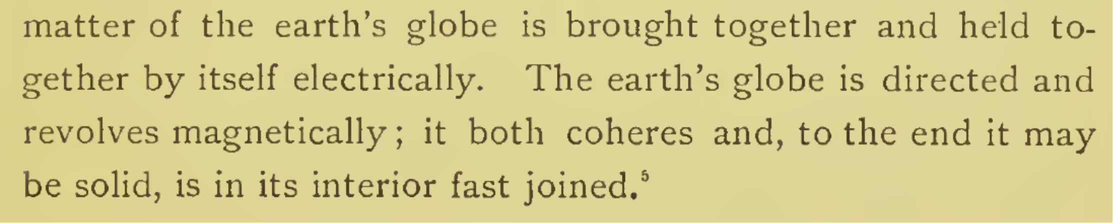
But finally, with some experiments and more considerations on the effects of heat, moisture, air, earth etc. on the effluvium of electrical bodies, he concludes that; *any electrical bodies have their own natural effluvium*. Thus, "corpuscles are carried to the electrical bodies themselves". Then he states, the effluvia (currents) which are generated from rubbing rod attracts straws and other light bodies until this attracting force is vanished. Then the earth attracts them toward itself (by its magnetic force) and they fall down.

He uses the traditional medical term "humor" i.e. the dominant health characteristic of a person e.g. dry, cold etc. to show that this attracting force is an *intrinsic characteristic* in electric and magnetic bodies. Then he states that, the effluvia generated from this humor and spread in different directions. According to him the distinction between electric and magnetic bodies is;



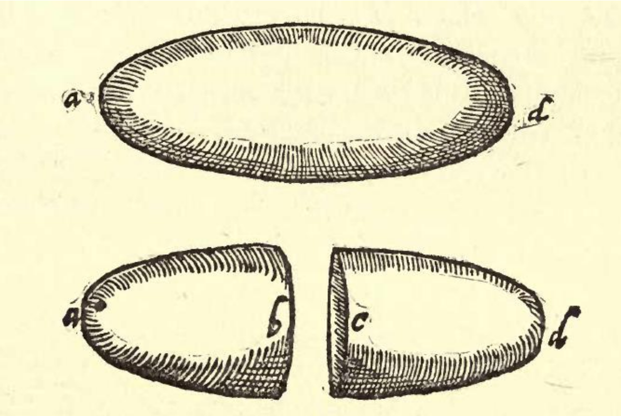
ground. The difference (distinction) between electric and magnetic bodies is this: all magnetic bodies come together by their joint forces (mutual strength); electric bodies attract the electric only, and the body attracted undergoes no modification through its own native force, but is drawn freely under impulsion in the ratio of its matter (composition). Bodies are* attracted to electrics in a right line toward the centre of electricity: a loadstone approaches another loadstone on a line perpendicular to the circumference only at the poles, elsewhere obliquely and transversely, and adheres at the same angles. The electric motion is the motion of coacervation of matter; the magnetic is that of arrangement and order. The

Even he proposed that;

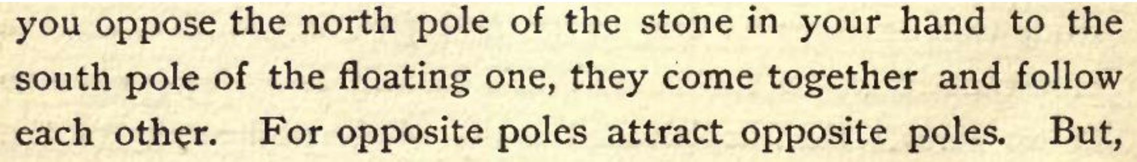


matter of the earth's globe is brought together and held together by itself electrically. The earth's globe is directed and revolves magnetically; it both coheres and, to the end it may be solid, is in its interior fast joined.[5]

In chapter V of the book one he considers the attraction and repulsion of a loadstone in terms of its poles (N and S). He does this by a simple experiment using two pieces one loadstone as below;

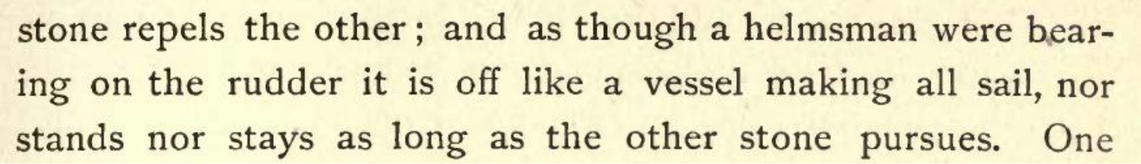


He float one of the pieces in a say, "vessel" then shows that if;



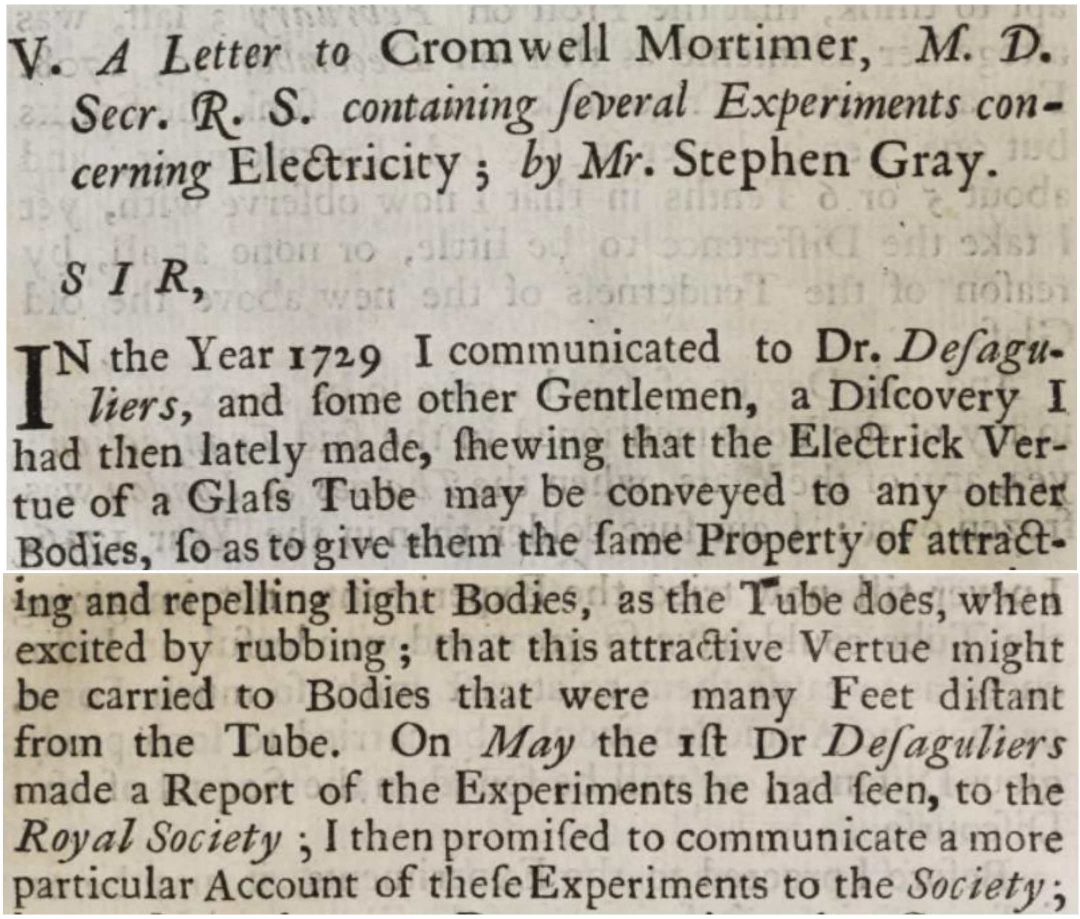
you oppose the north pole of the stone in your hand to the south pole of the floating one, they come together and follow each other. For opposite poles attract opposite poles. But,

But if N approaches N or S to S then one;



stone repels the other; and as though a helmsman were bearing on the rudder it is off like a vessel making all sail, nor stands nor stays as long as the other stone pursues. One

In 1729, Stephen Gray (1666-1736) performed the first systematic experiment on the phenomena of electrical conduction. He classified all bodies into two large categories, namely, the conductors of electricity and insulators as Gilbert classified as, electric bodies and anelectrics. He is known for proposing "electrical conductivity" in his letter to Cromwell Mortimer. (f stans for s where it is required)

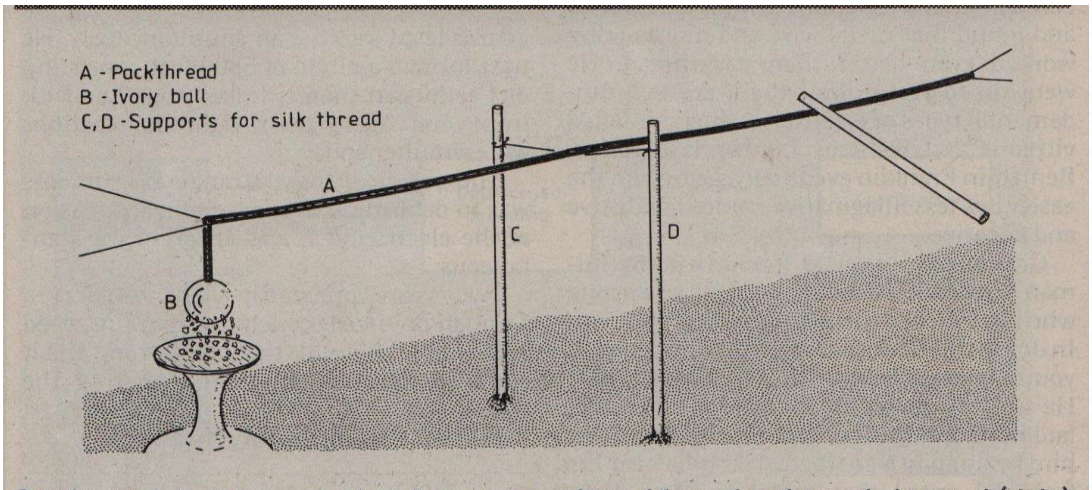


V. *A Letter to* Cromwell Mortimer, *M. D. Secr. R. S. containing ſeveral Experiments concerning* Electricity; *by Mr.* Stephen Gray.

S I R,

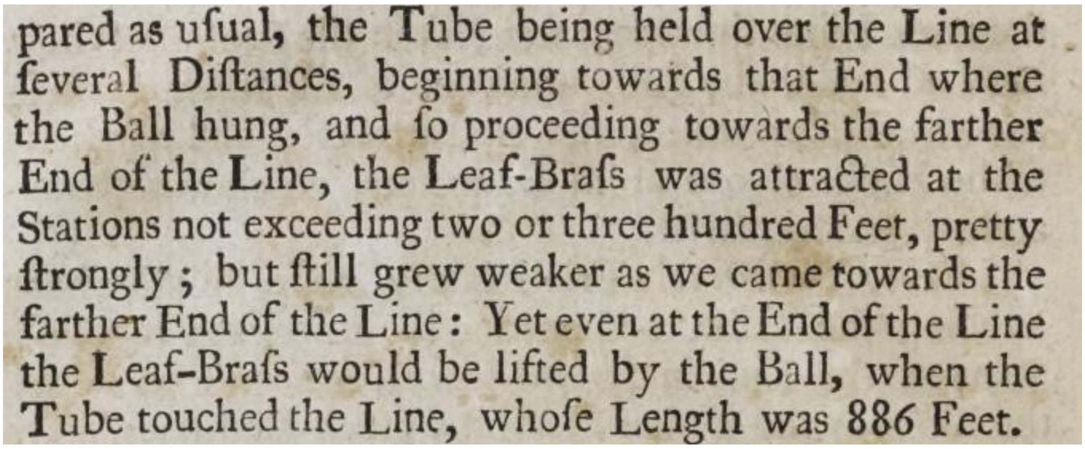
IN the Year 1729 I communicated to Dr. *Deſaguliers*, and ſome other Gentlemen, a Diſcovery I had then lately made, ſhewing that the Electrick Vertue of a Glaſs Tube may be conveyed to any other Bodies, ſo as to give them the ſame Property of attracting and repelling light Bodies, as the Tube does, when excited by rubbing; that this attractive Vertue might be carried to Bodies that were many Feet diſtant from the Tube. On *May* the 1ſt Dr *Deſaguliers* made a Report of the Experiments he had ſeen, to the *Royal Society*; I then promiſed to communicate a more particular Account of theſe Experiments to the *Society*;

He knew that, all insulators such as amber, resin and glass could be electrically excited i.e. charged by rubbing. He observed a charged glass tube is able to pass its charges to a piece of metal. He called this "electric virtue". He also observed this phenomenon for the glass tube and cork, fir stick, Ivory ball, brass wire, iron wire, silk etc. with an arrangement as below.

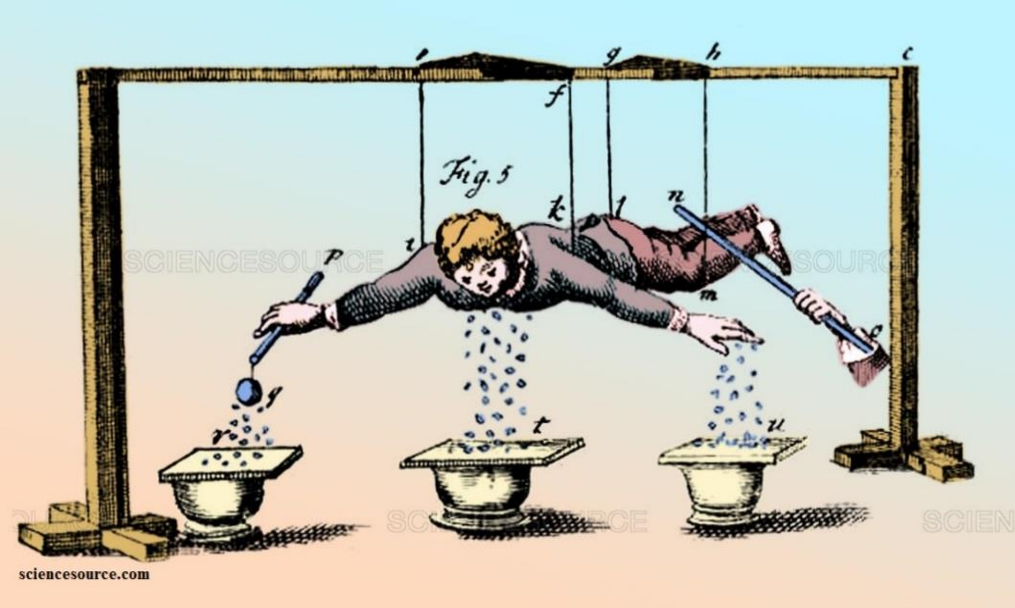


Gray's crucial experiment: electricity was conducted from the charged glass rod (right) to the ivory ball. Though neglected in our own time, Gray and his achievement were recognized by some, at least of his contemporaries: in 1732 he was elected a fellow of the Royal Society.

He could conduct charges from a few feet to 35 to 147 feet, but not more, since the charges escaped from the brass wires namely, they dissipated. By these observations, he and his colleagues Weller found that silk prevent scaping of charges (electricity) while brass wire does not. This was the first manifestation of electrical conduction and insulation. After some corrections in their experiment they could extend the maximum distance to 765 and then 886 feet long.

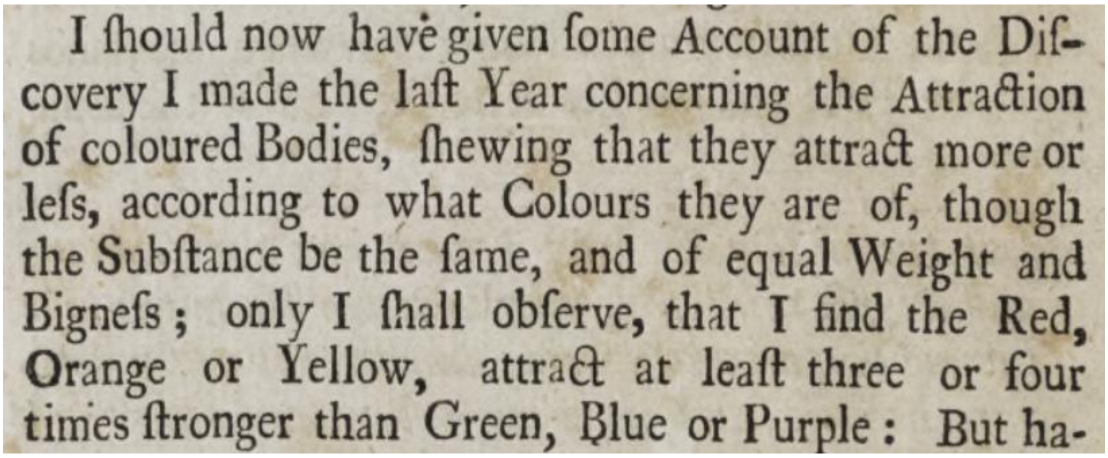


pared as uſual, the Tube being held over the Line at ſeveral Diſtances, beginning towards that End where the Ball hung, and ſo proceeding towards the farther End of the Line, the Leaf-Braſs was attracted at the Stations not exceeding two or three hundred Feet, pretty ſtrongly; but ſtill grew weaker as we came towards the farther End of the Line: Yet even at the End of the Line the Leaf-Braſs would be lifted by the Ball, when the Tube touched the Line, whoſe Length was 886 Feet.



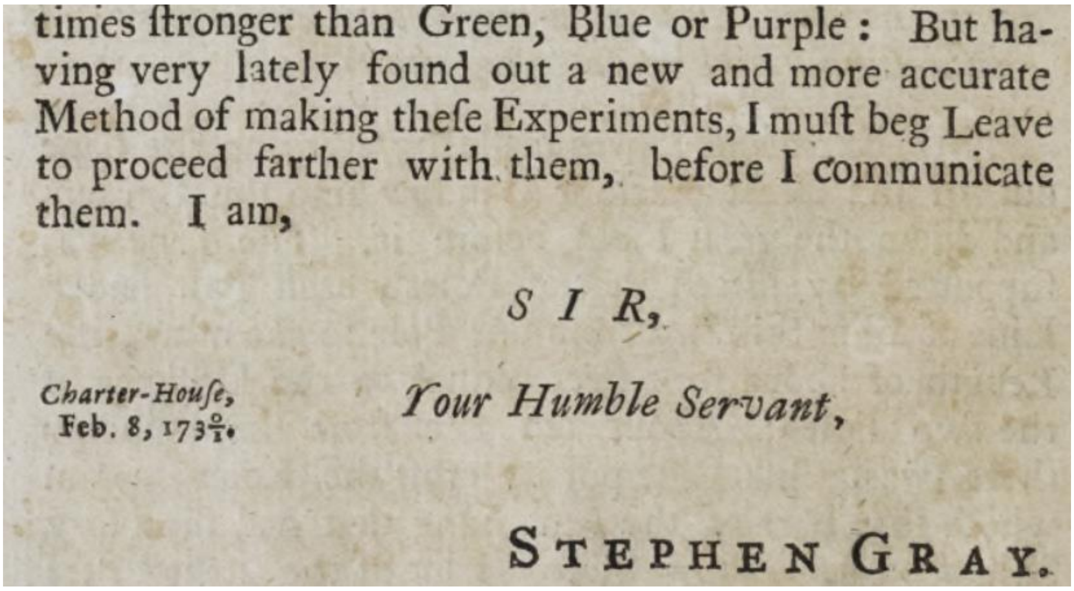
sciencesource.com

During his experiments he found that, human body is a "nice conductor". The experiment was done by a horizontally suspended boy, a charged glass tubed near his feet and a "metal thin pieces" near his face and hands in addition to an ivory ball. He observed that, charges from glass tube transmits to boy's body and his face, hands and ivory ball attract the metal pieces. This experiment was repeated and extended by him in 1732 and he found that, this effect of being electrified passed from one boy to another "if they held their hands or linked by a conductor".



I ſhould now have given ſome Account of the Diſcovery I made the laſt Year concerning the Attraction of coloured Bodies, ſhewing that they attract more or leſs, according to what Colours they are of, though the Subſtance be the ſame, and of equal Weight and Bigneſs; only I ſhall obſerve, that I find the Red, Orange or Yellow, attract at leaſt three or four times ſtronger than Green, Blue or Purple: But ha-

At the end of his initiative important paper he also considers the attraction of colored bodies. He propose that, when the substance, weight and size of bodies is the same but the difference is in colors, then this is colors which affects on the amount of attraction: "I find the Red, Orange or Yellow, attract at least three or four times stronger than Green, Blue or Purple". He was going to pursue his experiments in this respect but he died 2 years after this article.



times ſtronger than Green, Blue or Purple: But having very lately found out a new and more accurate Method of making theſe Experiments, I muſt beg Leave to proceed farther with them, before I communicate them. I am,

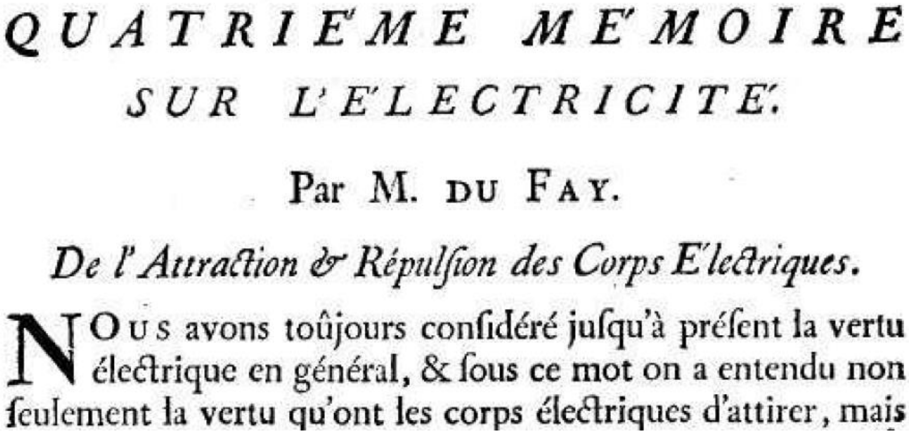
SIR,

Your Humble Servant,

Charter-Houſe,
Feb. 8, 173$\frac{0}{1}$.

STEPHEN GRAY.

A French scientist (more a chemist) Charles Du Fay (1698-1739) followed the works of Gilbert and Gray. He is now known for the discovery of electric charges. As Gilbert, he observed that "sealing-wax rubbed by fur is electrified like a glass rod but in some opposite sense". He extended Gray's discoveries and define “vitreous” (glass) and “resinous” (rosin) types of electricity (charges). It was Benjamin Franklin who named them positive and negative charges (respectively) later. Du Fay proposed that, like charges repel and opposite charges attract each other" in his paper *Quatrième mémoire sur l'électricité, De l'Attraction & Répulsion des Corps Electriques* in (1733).



QUATRIEME MEMOIRE

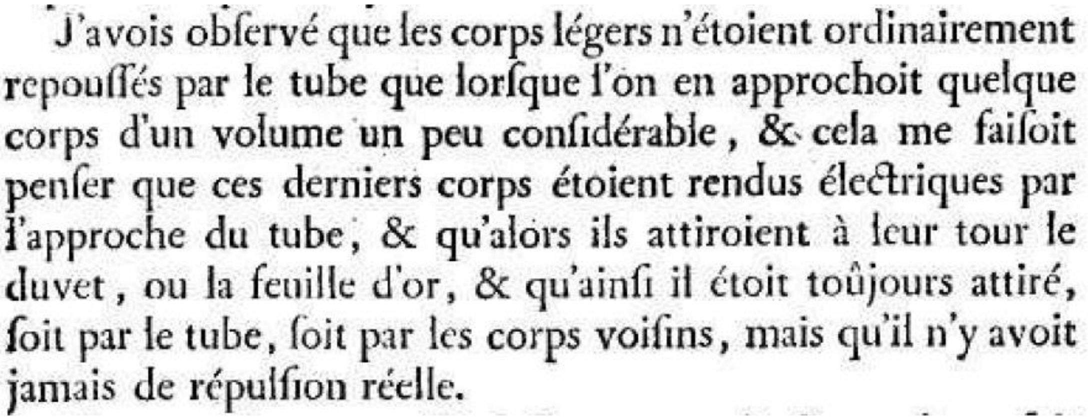
SUR L'ELECTRICITE.

Par M. DU FAY.

De l'Attraction & Répulſion des Corps Electriques.

NOUS avons toûjours conſidéré juſqu'à préſent la vertu électrique en général, & ſous ce mot on a entendu non ſeulement la vertu qu'ont les corps électriques d'attirer, mais

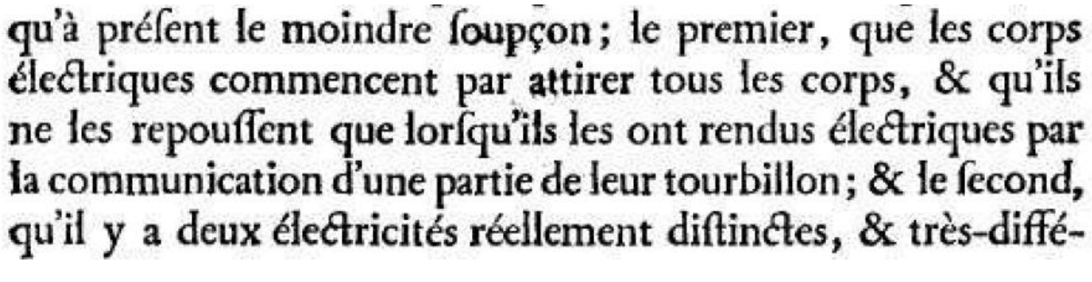
In this paper he starts with introducing Gray's "virtue electric" and that, electric bodies have either attracting and repelling properties. This pulling (pulsion) is not always constant but is subject to varieties. Theses varieties led him to examine this phenomenon of "pulsion" more carefully and reach to some "new very simple principles". Then he states that;



J'avois obſervé que les corps légers n'étoient ordinairement repouſſés par le tube que lorſque l'on en approchoit quelque corps d'un volume un peu conſidérable, & cela me faiſoit penſer que ces derniers corps étoient rendus électriques par l'approche du tube, & qu'alors ils attiroient à leur tour le duvet, ou la feuille d'or, & qu'ainſi il étoit toûjours attiré, ſoit par le tube, ſoit par les corps voiſins, mais qu'il n'y avoit jamais de répulſion réelle.

I observed that, light bodies were not ordinarily repelled by the tube only when some body of a rather considerable volume was approached and this led me to think that these latter bodies were made electric (are electrified or charged) by the approach of the tube and then they then in turn attract the down, or its gold leaf and that thus it was always attracted, either by the tube or by neighboring bodies, but that there was never any real repulsion.

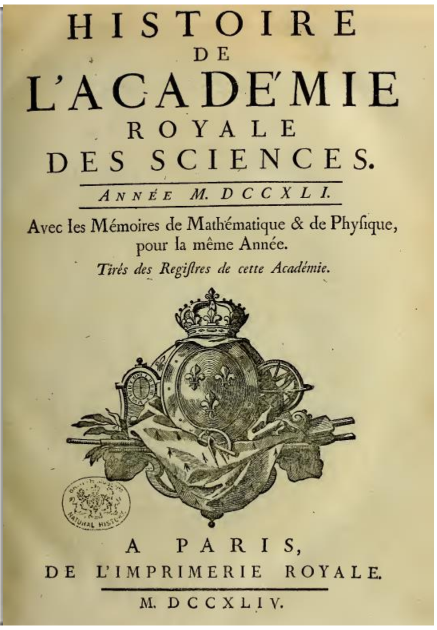
At his final remarks he concludes with two new truth or principles as;



qu'à préſent le moindre ſoupçon; le premier, que les corps électriques commencent par attirer tous les corps, & qu'ils ne les repouſſent que lorſqu'ils les ont rendus électriques par la communication d'une partie de leur tourbillon; & le ſecond, qu'il y a deux électricités réellement diſtinctes, & très-diffé-

..The first, that the bodies' electricity begin by attracting all the bodies and that they repel them only when they have made them electric by the communication (conduction) of parts of their vortex, and the second, that there are two really distinct types of electricity and very different from each other…

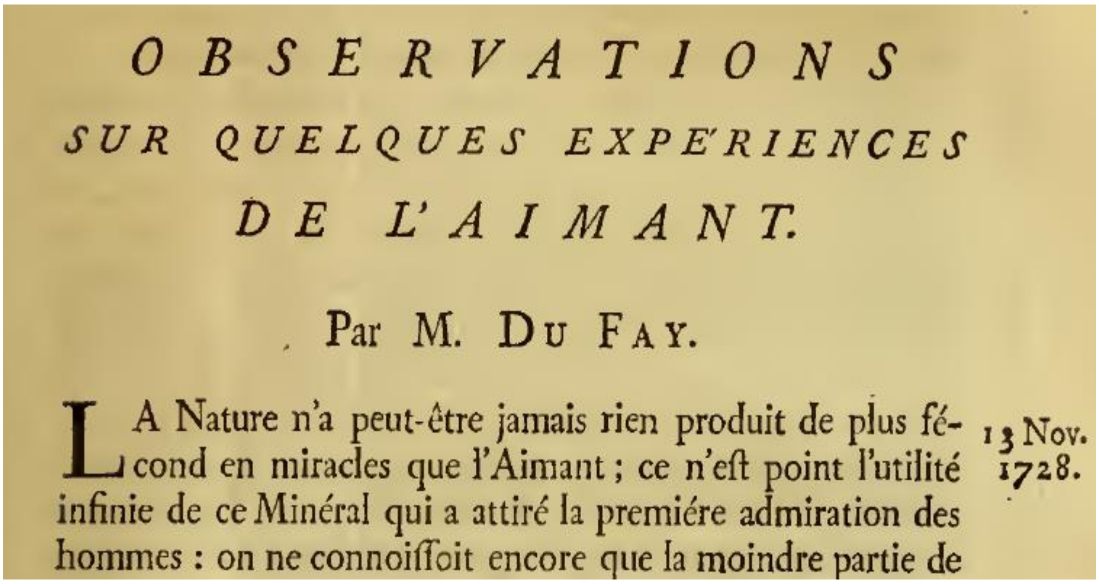
He published his papers during (1723-1745) on chemistry, Geometry and physics in journal, *Histoire de l'Académie Royale des Sciences: avec les mémoires de mathématique & de physique pour les mêmes années* (1699-1786). His publications from (1733-1745) belong to *l'électricité* (electricity) i.e. after informing about Stephen gray's experiments.



HISTOIRE
DE
L'ACADEMIE
ROYALE
DES SCIENCES.
ANNÉE M. DCCXLI.
Avec les Mémoires de Mathématique & de Phyſique, pour la même Année.
Tirés des Regiſtres de cette Académie.

A PARIS,
DE L'IMPRIMERIE ROYALE.
M. DCCXLIV.

However, he also published three papers in "observations on magnetism" (*de l'aimant*) in (1728-1731) such as, *Observations sur quelques expériences de l'aimant* in 1728.



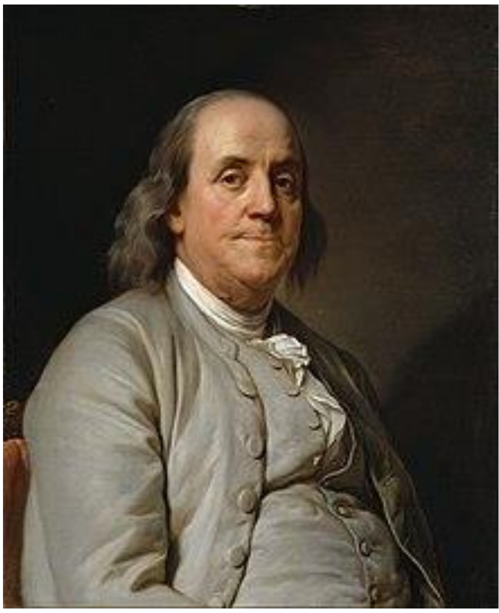
OBSERVATIONS SUR QUELQUES EXPERIENCES DE L'AIMANT.

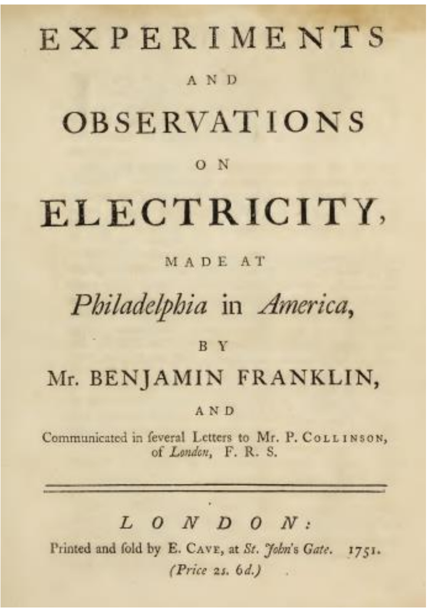
Par M. Du Fay.

LA Nature n'a peut-être jamais rien produit de plus fécond en miracles que l'Aimant; ce n'eſt point l'utilité infinie de ce Minéral qui a attiré la premiére admiration des hommes : on ne connoiſſoit encore que la moindre partie de

13 Nov. 1728.

American polymath and physicist Benjamin Franklin (1706-1790) in 1747 stated that all bodies are composed of electrical fluid in his book "experiments and observations on electricity". This book is consisting of several of Franklin's letters from Philadelphia to Peter Collinson in London as well as his essays. In these letters he describes his experiments and observations - similar to Stephen gray- on the phenomena of electricity.





EXPERIMENTS
AND
OBSERVATIONS
ON
ELECTRICITY,
MADE AT
Philadelphia in *America*,
BY
Mr. BENJAMIN FRANKLIN,
AND
Communicated in ſeveral Letters to Mr. P. COLLINSON,
of *London*, F. R. S.

LONDON:
Printed and ſold by E. CAVE, at *St. John's Gate*. 1751.
(*Price* 2*s*. 6*d*.)

In his experiments he used several experimental tools and machines. To produce static charges (electricity) he applied a machine with a fixed sphere on iron axes. "At one end of the axis-there is a small handle, with which you turn the sphere like a common grind-stone".

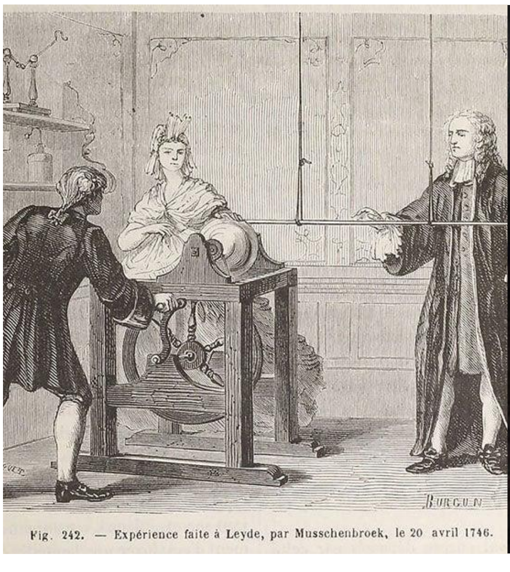
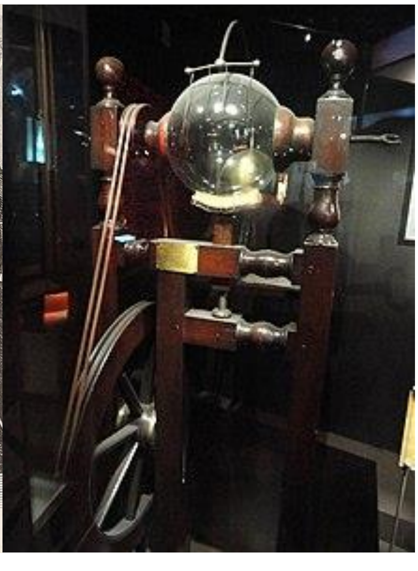
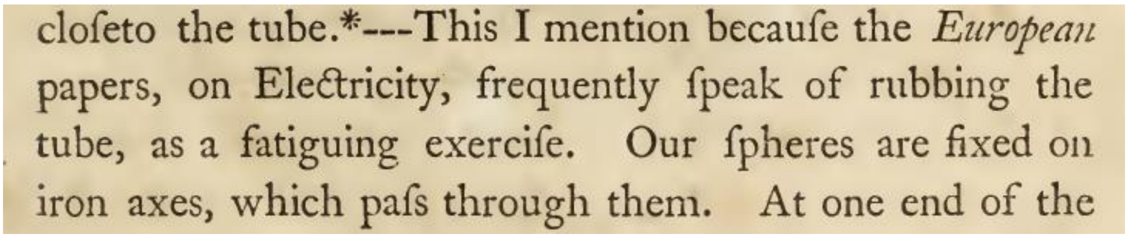


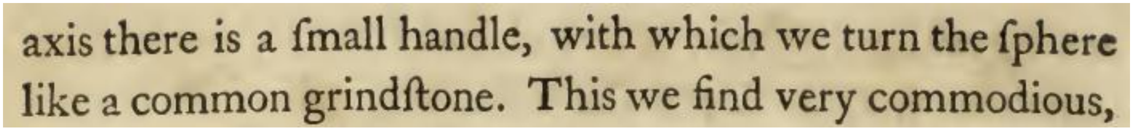
Fig. 242. — Expérience faite à Leyde, par Musschenbroek, le 20 avril 1746.



In the second letter of his book in final remarks he describes the efficiency of his electrostatic generator machine to those methods of producing static charges in Europe as below.

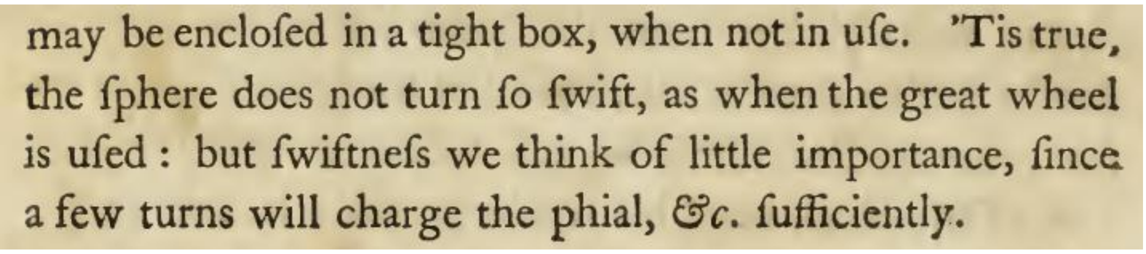


cloſeto the tube.*---This I mention becauſe the *European* papers, on Electricity, frequently ſpeak of rubbing the tube, as a fatiguing exerciſe. Our ſpheres are fixed on iron axes, which paſs through them. At one end of the



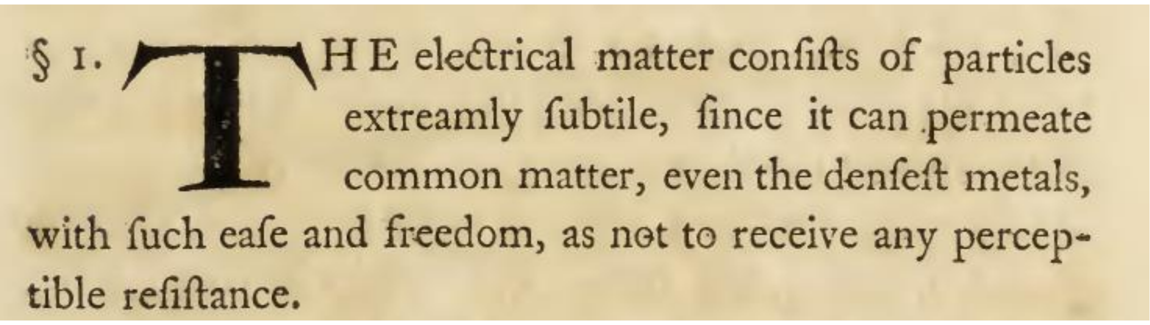
axis there is a ſmall handle, with which we turn the ſphere like a common grindſtone. This we find very commodious,

Here he also mentioned the word "charge" for the first time in his book.



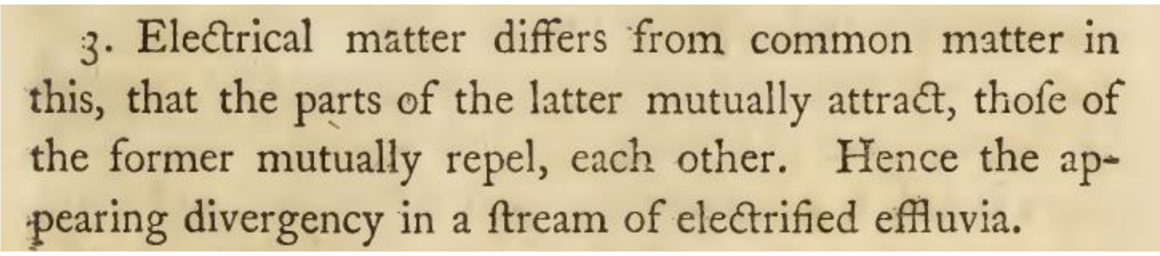
may be encloſed in a tight box, when not in uſe. 'Tis true, the ſphere does not turn ſo ſwift, as when the great wheel is uſed: but ſwiftneſs we think of little importance, ſince a few turns will charge the phial, *&c.* ſufficiently.

In "opinions and conjectures" he proposed some 36 fascinating ideas. For instance, in the first one he proposed that, electrical matters (bodies) are composed of "extremely subtile" i.e. very delicate and small, to be able to easily penetrate into different matters. Thus, he believed that, "Electrical matter is composed of particles that should be extremely small". Therefore, he was the first to suggest the atomistic, or grainy, nature of electricity.



§ 1. THE electrical matter conſiſts of particles extreamly ſubtile, ſince it can permeate common matter, even the denſeſt metals, with ſuch eaſe and freedom, as not to receive any perceptible reſiſtance.

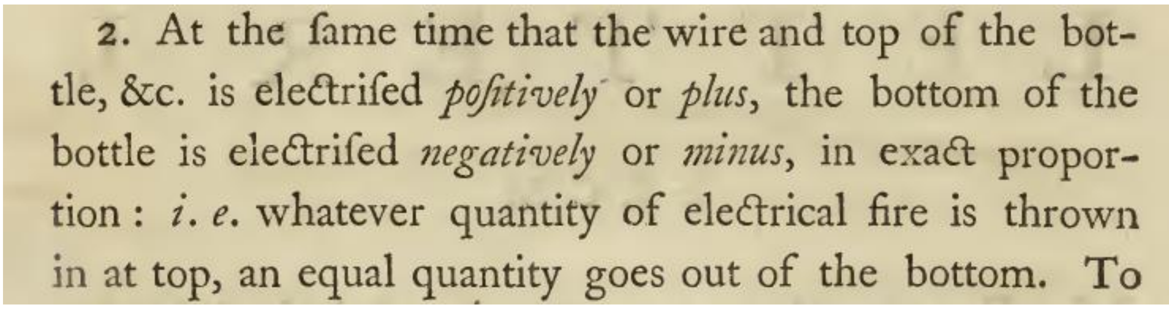
Franklin showed that for different bodies, the more electrical fluid the more positively charged and the less electrical fluid, the more negatively charged. These (independent of their matters) equal amount of positive and negative charges are existed and act simultaneously. In this respect, initiatively he states that, "electrified matters" repel each other but "common matters" attract mutually. Then he uses, gilbert's term of current i.e. effluvia and say; due to existence of these mutual attracting and repelling phenomena there exist divergency in a stream of electrified effluvia.



3. Electrical matter differs from common matter in this, that the parts of the latter mutually attract, thoſe of the former mutually repel, each other. Hence the appearing divergency in a ſtream of electrified effluvia.

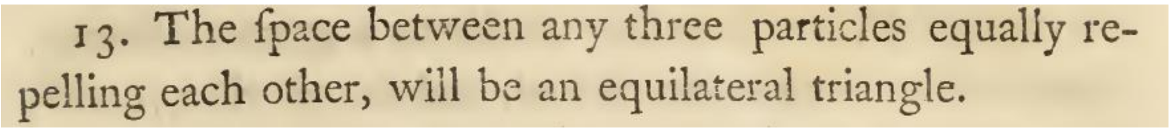
He defines common matter as a matter which behaves like a sponge. He also resembles the relations and interactions between electrified and common matters with those of water and sponge.

In the first letter using Leyden jar or as he called "M. Musschenbroek's wonderful bottle" he describes his observations. In the first note he proposed that, "electrified fire" (fluid) of the electrified matter existed into the substance when the matter is in the bottle but when it is out of the bottle this electrified fire "is accumulated on its surface and forms an electrified atmosphere" around the matter. In the second note he writes as;



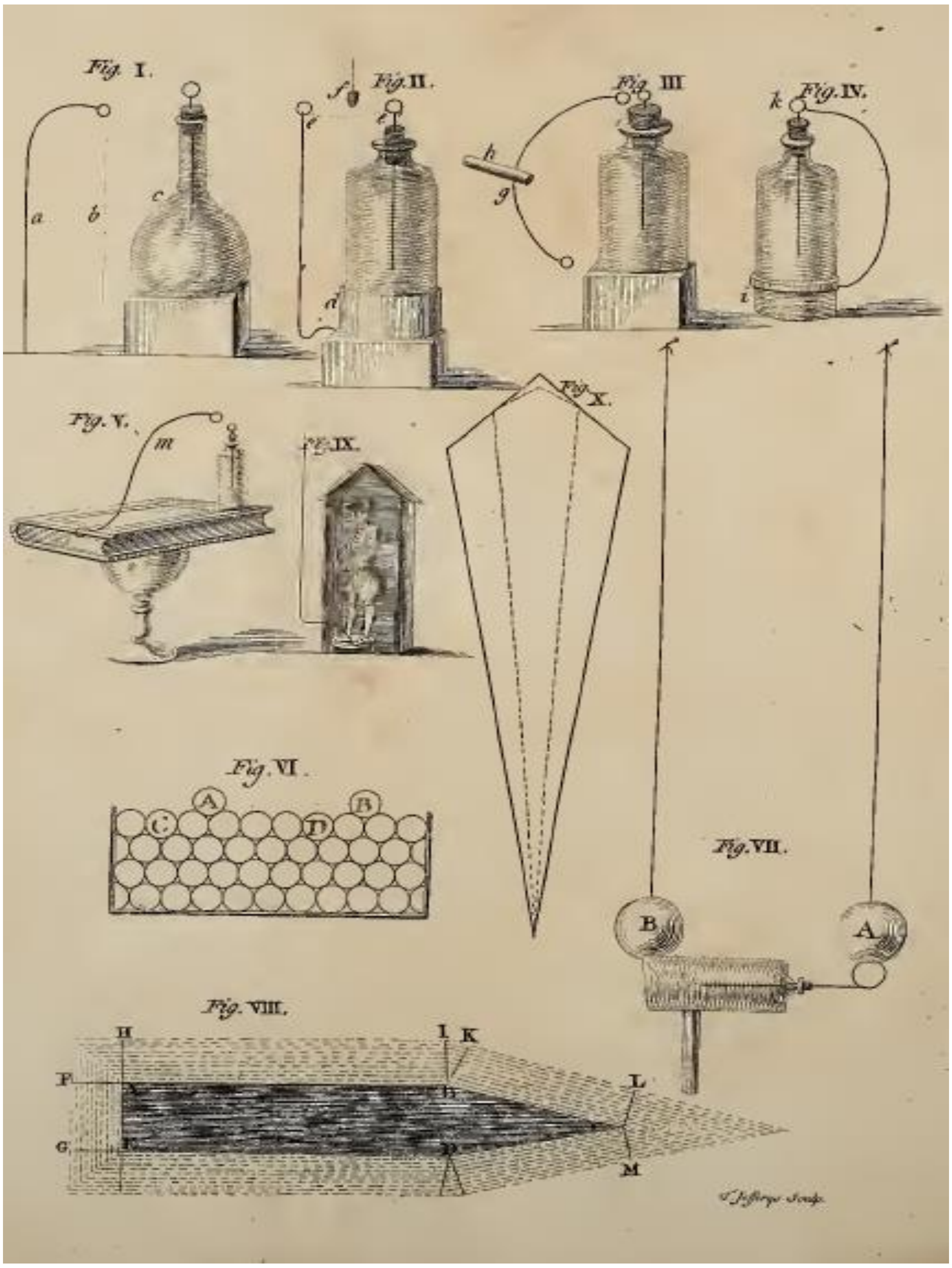
2. At the ſame time that the wire and top of the bottle, &c. is electriſed *poſitively* or *plus*, the bottom of the bottle is electriſed *negatively* or *minus*, in exact proportion: *i. e.* whatever quantity of electrical fire is thrown in at top, an equal quantity goes out of the bottom. To

He also proposed that, "Every particle of matter electrified is repelled by every other particle equally electrified" and;

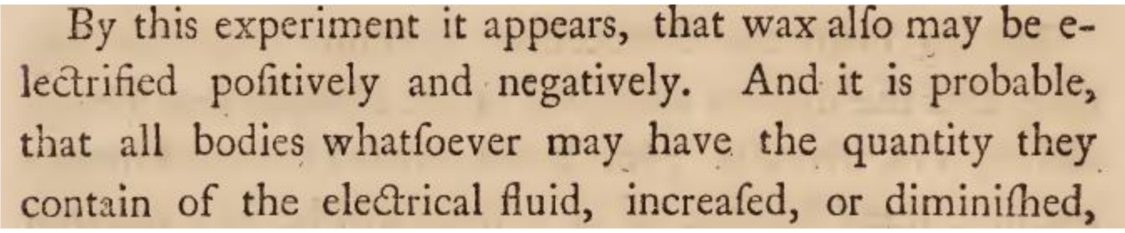


13. The ſpace between any three particles equally repelling each other, will be an equilateral triangle.

Then in describing his electrical experiments, using better experimental tools against to those used by his European contemporaries, such as Leyden jar and electrostatic generator machine, he observed some truth and extended the science of electricity. These tools are shown below in their original form.

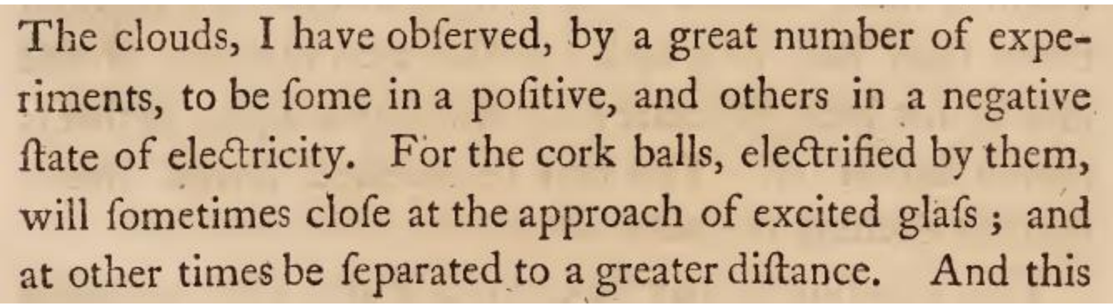


For more information about these tools the reader can refer to Benjamin Franklin's "experiments and observations on electricity" (1751). But for our discussion, we need figure VII. It is in fact the very first model of his electrostatic generator machine plus two hanging cork-balls. When half of a pretty long excited stick of wax is charged by the machine in several interval of times then; then "that half will destroy the repelling power of balls electrified by glass, and the other half will increase it". Thus;



By this experiment it appears, that wax alſo may be electrified poſitively and negatively. And it is probable, that all bodies whatſoever may have the quantity they contain of the electrical fluid, increaſed, or diminiſhed,

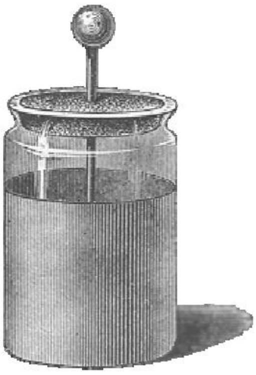
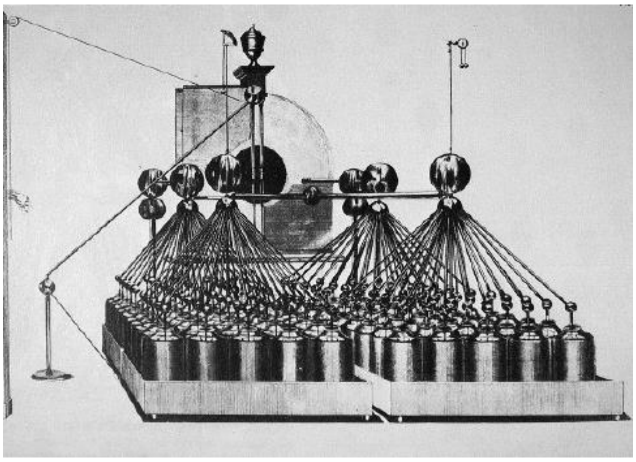
He also observed this positive and negative electricity in clouds by several experiments using a kite. He states that;



The clouds, I have obſerved, by a great number of experiments, to be ſome in a poſitive, and others in a negative ſtate of electricity. For the cork balls, electrified by them, will ſometimes cloſe at the approach of excited glaſs ; and at other times be ſeparated to a greater diſtance. And this

Here he proposed "two states of electricity" i.e. positive and negative states. This idea and his other ideas stated above shows the brilliant mind of this American polymath. In fact, he was the first American who seriously worked in science. Thus, one can say American science started with his works in 18th century.

Lots of his and his contemporaries' experiments in electricity in its sprouting days was a simple device called Leyden jar. A Leyden jar which is also Leiden or Kleistian jar is a primitive form of a capacitor. It is made up of a glass jar, covered with metal foil inside and the outside of its surfaces, and a "metal terminal projecting vertically through the jar lid to make contact with the inner foil" as shown below.

An electrostatic generator produces static electricity (charges). Then these charges transmit to the water in the bottle and accumulate in it by a suspended chain.

German cleric Ewald Georg von Kleist (1700-1748) and Dutch scientist Pieter van Musschenbroek (1692-1761) of Leiden invented it independently in the year 1745. Von Kleist’s invention did not accept by the academic societies, but three months after him Musschenbroek reported his work to Paris scientific society.

Musschenbroek observed that, a charged conductor loses its charge often very rapidly since it is wire surrounded by (conductive) air. Thus, "he enclosed the conductor in a nonconductive vessel, in order to avoid the escape of the charge". Then using a metal wire, he connected the poles of an electrostatic generator machine to a glass vessel filled with water. But he observed that the water also lost its charges very rapidly.

One day his cooperator, the amateur Andreas Cunaeus, took the glass with one of his hand and touch the wire with his other and felt a strong shock. This happened because his hand act as a coating and prevent charges to scape but transmit them to his body. With this they found that by coating the glass (jar) in it inner and outer sides they would be able to collect and preserve the electric charges.

Lots of primitive experiments on electricity is done with this device and made principles of electrostatics. It enabled scientists to accumulate, preserve and release (discharge) electric charges in large quantities.

As we said before in 1732 Stephen Gray had found that, the charges could be passed from one boy to another by hand or a conductor. After the invention of Leiden jar, French clergyman and physicist Jean Nollet (1700-1770) linked up 180 Royal guards "as a conducting path". He discharged a fully charged Leiden jar in his human path. This caused "guards leap into the air simultaneously". Nollet repeated his experiment with a line of monks in 5800 feet in length and again observed the same result. Consequently, the transmitting speed of electricity by conductors is apparently instantaneous!

in the late 18th century a profound transformation was happening in the scientific method. It was a move from a method of experiment and observation with a more qualitative explanations toward observation and explanations which leads to one of the universal laws using exact measurement.

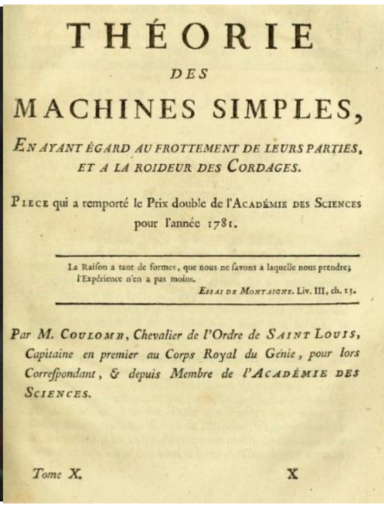
According to above discussion and qualitative explanations of the phenomena of electricity, we find that, we are dealing with an attracting force (interacting field) similar to gravity and again there was a need to experimental along with quantitative investigations. With this method a French physicist Charles Coulomb (1736-1806) discovered the law of interaction between electric charges in 1785. It was a experimental description of the electrostatic force of attraction and repulsion between two charges. Today we define his law (coulomb's law) mathematically as;

$$F = k\frac{q_1 q_2}{r^2}$$

Here F is the electrical force, q_1 & q_2 are charges and r stand for the distance between them. This is inverse square law between two charges supposed to be spheres times constant k called the coulomb (electric force/ electrostatic) constant. Again, this is a constant of proportionality.

However, "in his famous experiment on the inverse square law of electrostatics, Coulomb neither defined electric charge nor gave reliable measurements of the force–distance relation". Although the results of this experiment worked as the basis of the fundamental law of electrostatics.

Charles-Augustin de Coulomb graduated from Paris École royale du génie de Mézières in 1760. The year after he joined Army as an engineer until 1772. During this time, he also started his scientific writing in applied mechanics based on experiments in shipyards. The most important was a work in friction titled as "*Théorie des machines simples, en ayant regard au frottement de leurs parties et à la roideur des cordages" (simple machine theory, having regard with the friction of their parts and stiffness of the ropes)*. Such as most of his contemporaries, most of his paper published in HISTOIRE D EL'ACADEMIE ROYALE DES SCIENCES.



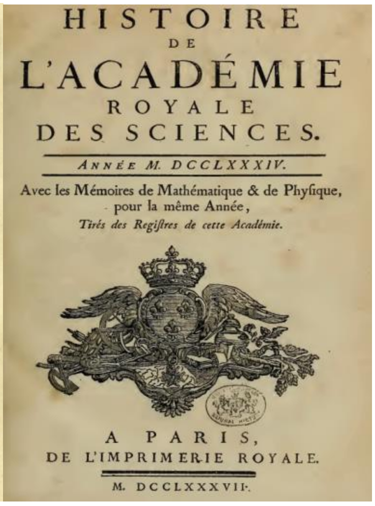
THÉORIE
DES
MACHINES SIMPLES,
EN AYANT ÉGARD AU FROTTEMENT DE LEURS PARTIES, ET A LA ROIDEUR DES CORDAGES.

PIECE qui a remporté le Prix double de l'ACADÉMIE DES SCIENCES pour l'année 1781.

La Raiſon a tant de formes, que nous ne ſavons à laquelle nous prendre; l'Expérience n'en a pas moins.
ESSAI DE MONTAIGNE. Liv. III, ch. 13.

Par M. COULOMB, Chevalier de l'Ordre de SAINT LOUIS, Capitaine en premier au Corps Royal du Génie, pour lors Correſpondant, & depuis Membre de l'ACADÉMIE DES SCIENCES.

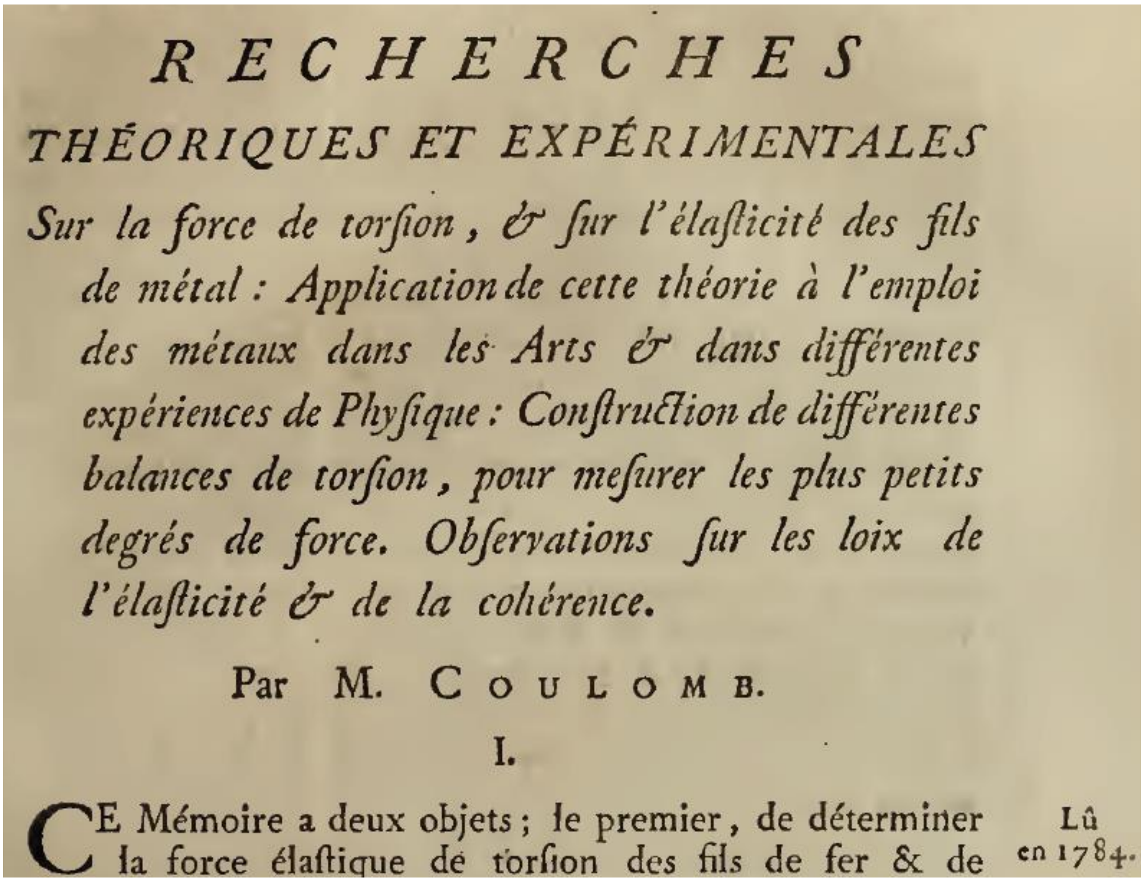
Tome X. X



HISTOIRE
DE
L'ACADÉMIE
ROYALE
DES SCIENCES.
ANNÉE M. DCCLXXXIV.
Avec les Mémoires de Mathématique & de Phyſique, pour la même Année,
Tirés des Regiſtres de cette Académie.

A PARIS,
DE L'IMPRIMERIE ROYALE.
M. DCCLXXXVII.

In 1784, his first paper on elastic force of torsion of filaments, *Recherches théoriques et expérimentales sur la force de torsion et sur l'élasticité des fils de metal* (Theoretical research and experiment on the force of torsion and the elasticity of metal wire) published in this journal. The title was: On the force of torsion, and on the elasticity of metal wires: Application of this theory to the use of metals in the Arts and in various physics experiments: Construction of different kinds of torsion balances, for measuring the smallest force levels. Observations on the laws of elasticity and of coherence.



RECHERCHES

THÉORIQUES ET EXPÉRIMENTALES

Sur la force de torſion, & ſur l'élaſticité des fils de métal : Application de cette théorie à l'emploi des métaux dans les Arts & dans différentes expériences de Phyſique : Conſtruction de différentes balances de torſion, pour meſurer les plus petits degrés de force. Obſervations ſur les loix de l'élaſticité & de la cohérence.

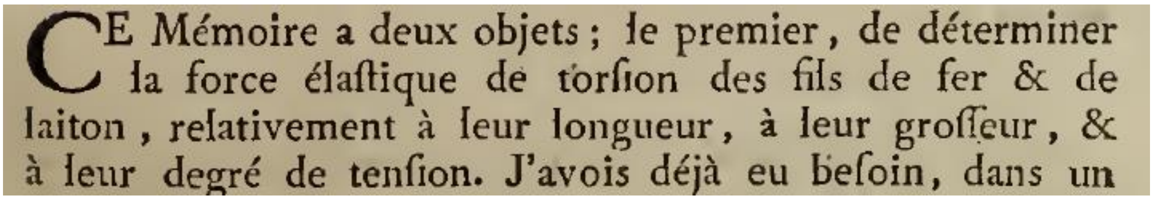
Par M. COULOMB.

I.

CE Mémoire a deux objets ; le premier, de déterminer la force élaſtique de torſion des fils de fer & de

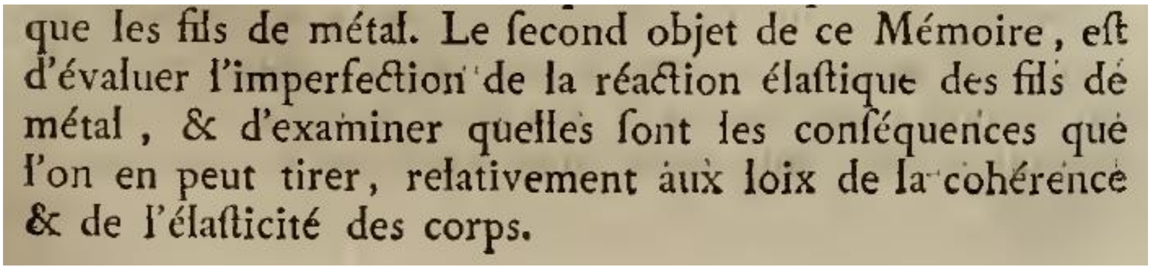
Lû en 1784.

His two objectives of this paper are: "to determine the **elastic force of torsion** of filaments (wire) of iron and of brass as a function of their length, their thickness, and their degree of tension".



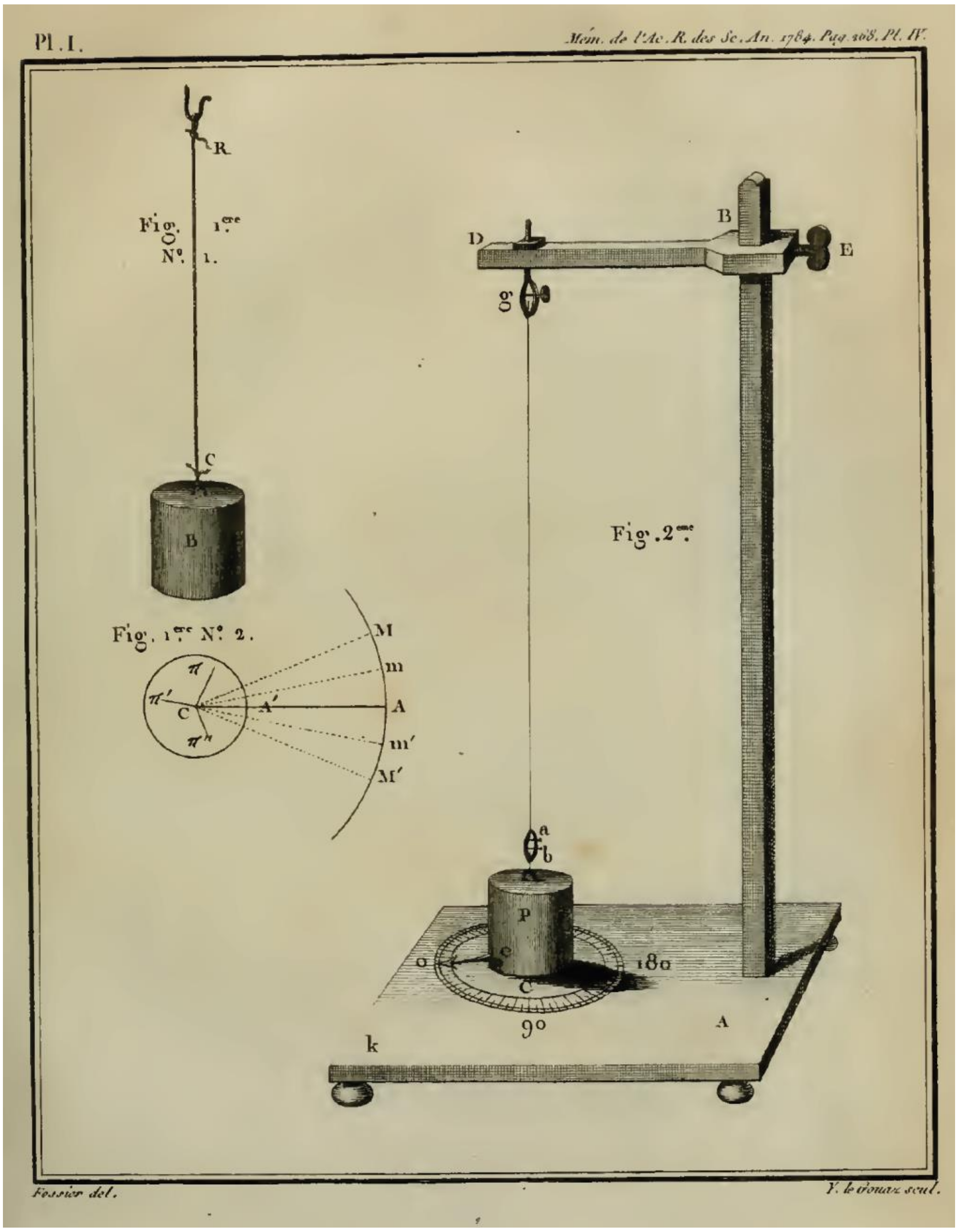
CE Mémoire a deux objets ; le premier, de déterminer la force élaſtique de torſion des fils de fer & de laiton, relativement à leur longueur, à leur groſſeur, & à leur degré de tenſion. J'avois déjà eu beſoin, dans un

And to evaluate the **deficiency of the elastic reaction** of filaments of metal, and to study the consequences that one can deduce about the laws of coherence and elasticity of bodies.

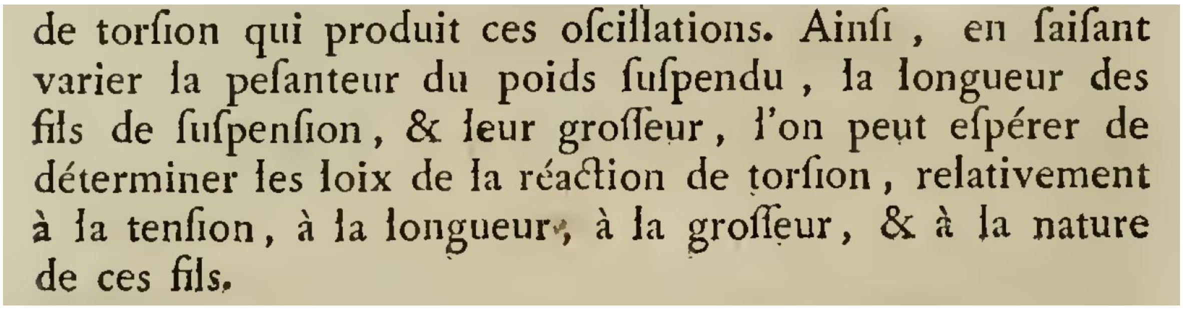


que les fils de métal. Le ſecond objet de ce Mémoire, eſt d'évaluer l'imperfection de la réaction élaſtique des fils de métal, & d'examiner quelles ſont les conſéquences que l'on en peut tirer, relativement aux loix de la cohérence & de l'élaſticité des corps.

For these objectives, he applied a device consists of, "a suspending cylindrical weight by a filament of metal in a manner such that its axis is vertical", namely in same the direction with the filament of suspension. This device is showed below.



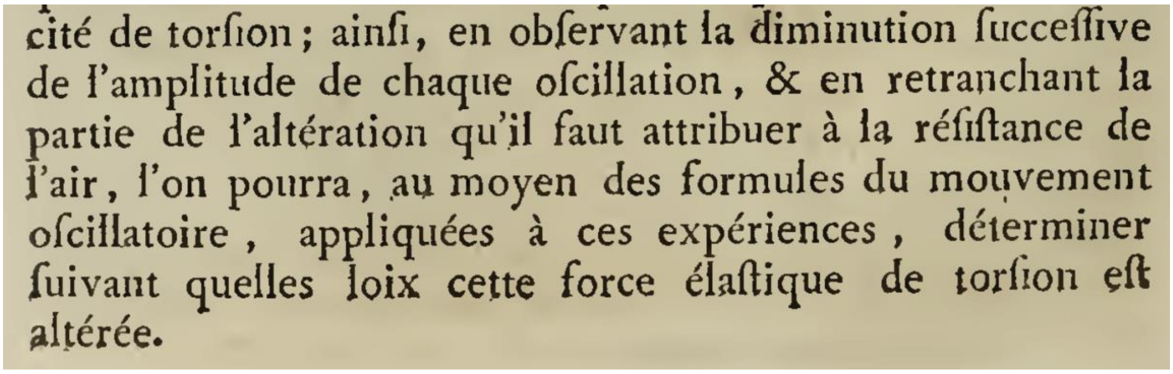
If one turns the weight about its axis, the filament twists and it will leave its rest (equilebrium) position, then will attempt to re-establish it in an interrval of time. If one push the wight horizontally, it will oscillate in terms of the amount of elastic reaction in torsion in specific time. Here one can inferred that "the motion was simple harmonic, with a torsion force proportional to the angle of twist". Thus;



de torſion qui produit ces oſcillations. Ainſi , en faiſant varier la peſanteur du poids ſuſpendu , la longueur des fils de ſuſpenſion, & leur groſſeur, l'on peut eſpérer de déterminer les loix de la réaction de torſion, relativement à la tenſion, à la longueur, à la groſſeur, & à la nature de ces fils.

By changing the amount of the suspended weight, the length and the thickness of the filament of suspension, one can expect to determine the laws of reaction of torsion with respect to the tension, the length, the thickness, and the nature of the filaments (wires).

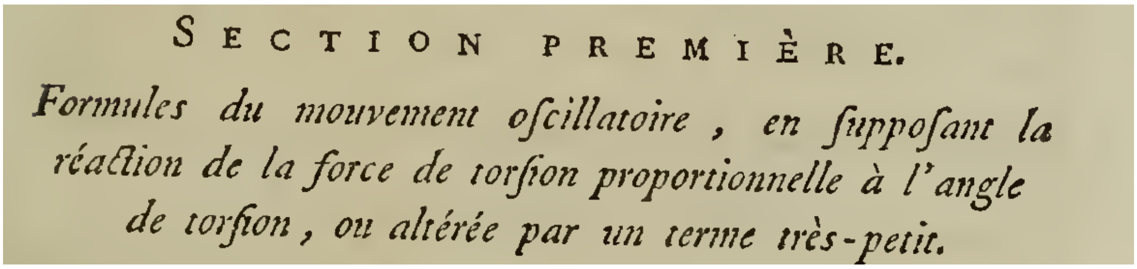
The decrese in the amplitude can be related to air resistance and deficiancy of the elasticity of torsion. Thus;



cité de torſion ; ainſi, en obſervant la diminution ſucceſſive de l'amplitude de chaque oſcillation, & en retranchant la partie de l'altération qu'il faut attribuer à la réſiſtance de l'air, l'on pourra, au moyen des formules du mouvement oſcillatoire , appliquées à ces expériences , déterminer ſuivant quelles loix cette force élaſtique de torſion eſt altérée.

In observing the successive diminution of the amplitude of each oscillation, and in taking out the part of the alteration that it is necessary to attribute to air resistance, one could, by means of the formulas of oscillatory movement, applied to these tests, determine according to which laws this force of elasticity of torsion is altered.

Thus in the first section (section premiere) , he supposed that "the reaction of the force of torsion is proportional to the angle of twist", then he drive Formulas for oscillatory movement.



SECTION PREMIÈRE.

Formules du mouvement oſcillatoire , en ſuppoſant la réaction de la force de torſion proportionnelle à l'angle de torſion , ou altérée par un terme très-petit.

In the schematic description of his torsion balance device in fig.1 we see "a horizontal section of the cylinder". Here, π, π', π'' are circular projection of the cylinder . A is the starting *angle of twist* i.e. ACM. After time t, this angle would be ACm, or reduced by the angle S *i.e.* MCm. Threfore;

$$ACm = (A - S)$$

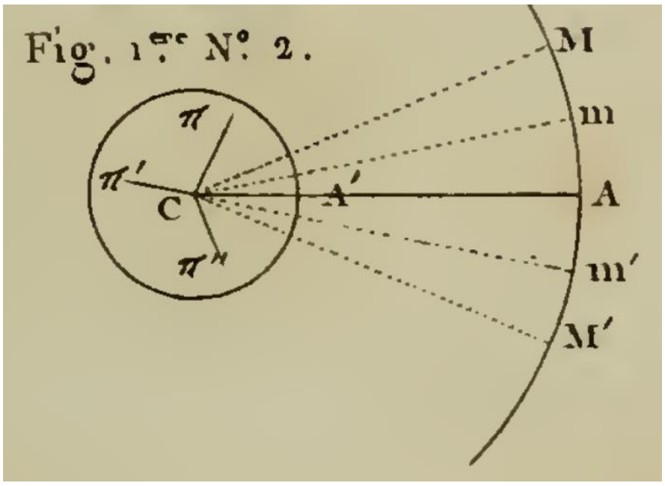
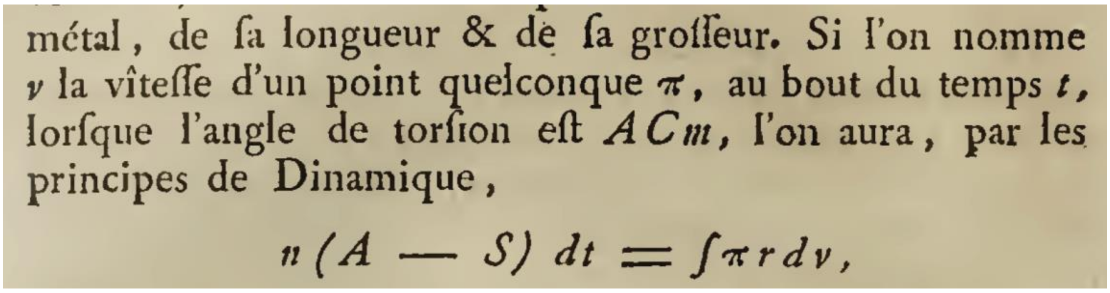


Fig. 1ère N° 2.

Now since we proposed that, the force of torsion is proportional to the angle of twist, then the momentum (n) of this force is; a constant of coefficent. Thus;

$$n\,(A - S)$$

Thus the value of n is depended to initial angle A and terminal angle S or "on the nature of the filament of metal, on its length and on its thickness". Now;


métal, de ſa longueur & de ſa groſſeur. Si l'on nomme v la vîteſſe d'un point quelconque π, au bout du temps t, lorſque l'angle de torſion eſt ACm, l'on aura, par les principes de Dinamique,

$$n\,(A - S)\,dt = \int \pi r\,dv,$$

If we call v the velocity of any point p, after a time t, when the angle of twist is ACm, we will have, by the principles of Dynamics,

$$n\,(A\ -\ S)dt = \int \pi r\; dv$$

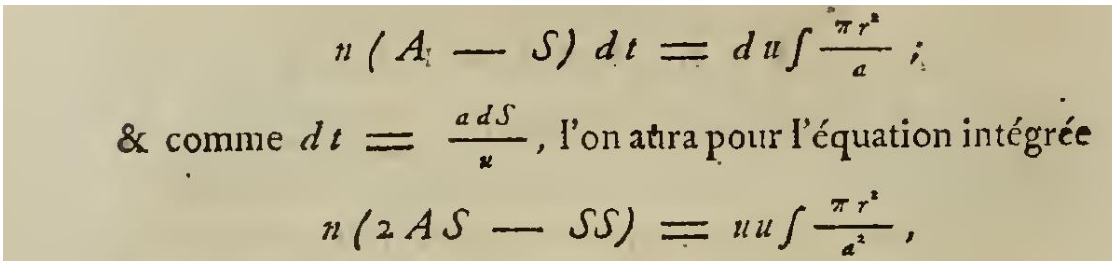
Here r is the distance from the point p to the axes of rotation C.

However, if a be the radius of the cylindrical weight, and u be the velocity of it at the point A, then; $v = ru/a$ and consequently;

$$n(A - S)\ \cdot dt = du \int \frac{\pi r^2}{a}$$

Now since $dt = a\; ds/u$ thus we can rewrite as;

$$n(2AS - S^2) = u^2 \int \frac{\pi r^2}{a^2}$$



$$n\,(A - S)\,dt = du \int \frac{\pi r^2}{a}\ ;$$

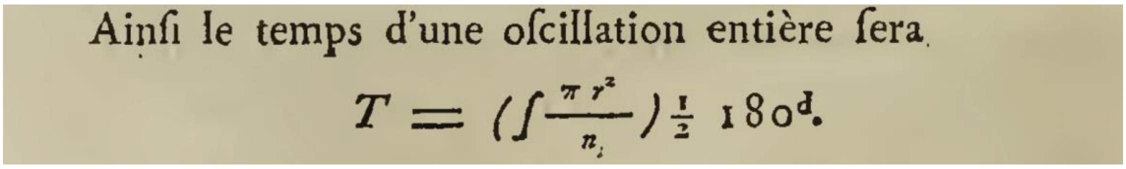
& comme $dt = \frac{a\,dS}{u}$, l'on aura pour l'équation intégrée

$$n\,(2AS - SS) = uu \int \frac{\pi r^2}{a^2}\ ,$$

Thus, we can drive *dt* as:

$$dt = \frac{dS \int \sqrt{(\pi r^2)}}{(n)^{1/2} \cdot (2AS - S^2)^{1/2}}$$

Here, ${}^{dS}/_{(2AS\ -\ SS)^{1/2}}$ is the angle of which A is the radius and S the section sine (le sinus verse'), which disappears when $S = 0$, and which becomes equal to 90 degrees when $S = A$. Therefore the time of an complete oscillation will be



Ainſi le temps d'une oſcillation entière ſera

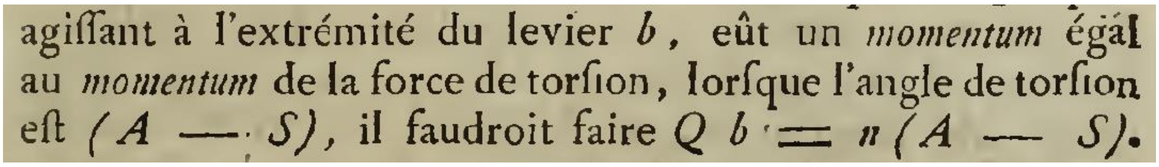
$$T = \left(\int \frac{\pi r^2}{n}\right)^{\frac{1}{2}} 180^{d}.$$

$$T = \sqrt{\left(\int \frac{\pi r^2}{n}\right)} \cdot 180^{deg}$$

Comparing the force of torsion with the force of gravity in a pendulum, we remember that, the time T of a complete oscillation in the pendulum with *lambda* as the length of the pendulum and g as the force of gravity is; $\sqrt{\lambda/g}\cdot 180^{deg}$. Therefore a pendulum which is isochronous (equal in duration or interval) to the oscillations of the cylinder gives;

$$\int\frac{\pi r^2}{n}=\frac{\lambda}{g}$$

Now since the dimensions and the weight of the cylinder are given, using this formula one can easily find the value of n from the experiment. And also; the time of one oscillation, which determines the value of lambda, to find the weight Q which;



agiſſant à l'extrémité du levier *b*, eût un *momentum* égal au *momentum* de la force de torſion, lorſque l'angle de torſion eſt *(A — S)*, il faudroit faire $Q\,b = n\,(A - S)$.

acting at the extremity of the lever *b*,would have a *momentum* equal to the *momentum* of the force of torsion, when the angle of twist is *(A - S)*, it requires setting *Qb = n(A - S)*.

Now we consider the volume of the cylinder such that;

$$\int\pi r^2=\frac{\varphi\delta L\cdot a^4}{4}$$

Here φ is ratio of the cercumference to the radius, δ is the density of the cylinder and a is the radius.

Now since the mass M of the cylender is; $M=\frac{\varphi\delta L\cdot a^2}{2}$ then;

$$\int\pi r^2=\frac{Ma^2}{2}$$

Now using $T=\sqrt{(\int\frac{\pi r^2}{n})}\cdot 180^{deg}$ we can rewrite the time for complete oscillation as;

$$T=\sqrt{\frac{Ma^2}{2n}}\;180^{deg}$$

Consequently;

$$\frac{\lambda}{g} = \frac{Ma^2}{2n}$$

And since, $W = Mg$ is the weight of the cylender then;

$$n = \frac{pa^2}{2\lambda}$$

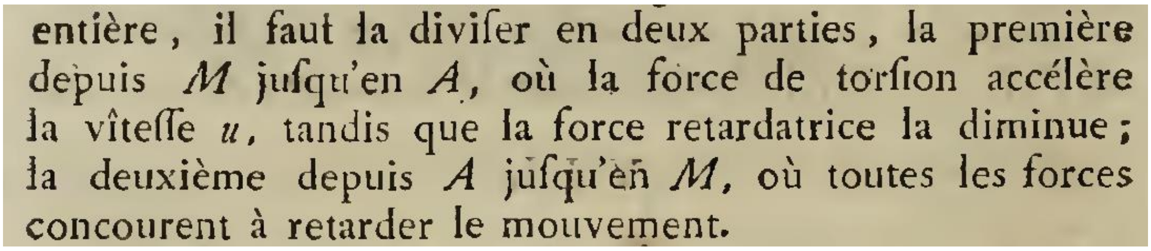
and finally, If the force of torsion, i.e. *n(A - S)*, changed by a quantity R, the formula (law) of oscillatory motion would be;

$$[n(A - S) - R] \cdot \partial t = \partial u \int \frac{\pi r^2}{a}$$

Now since $\partial t = ads/u$ then;

$$n(2AS - S^2) - 2 \int Rds = u^2 \int \frac{\pi r^2}{a^2}$$

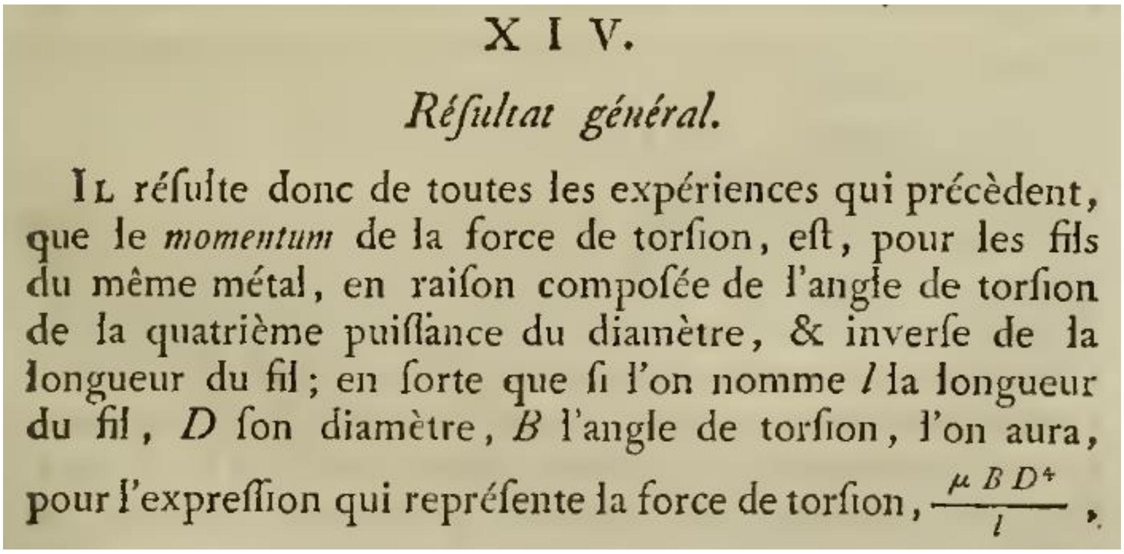
To extend this integration to a complete oscillation it needs to;



entière, il faut la diviſer en deux parties, la première depuis *M* juſqu'en *A*, où la force de torſion accélère la vîteſſe *u*, tandis que la force retardatrice la diminue; la deuxième depuis *A* juſqu'en *M*, où toutes les forces concourent à retarder le mouvement.

divide it into two parts, the first from *M* until *A*, where the force of torsion accelerates the velocity *u*, while the force of retardation diminishes; the second from *A* unto *M*, where all the forces together retard the motion.

The general results of his experiments using the mathematical method and formulation we considered above gaved him a general result (*resultat general*) which is also valuable for our main discusssion, namely, the constant of the law governing between to charges. We bring all here.



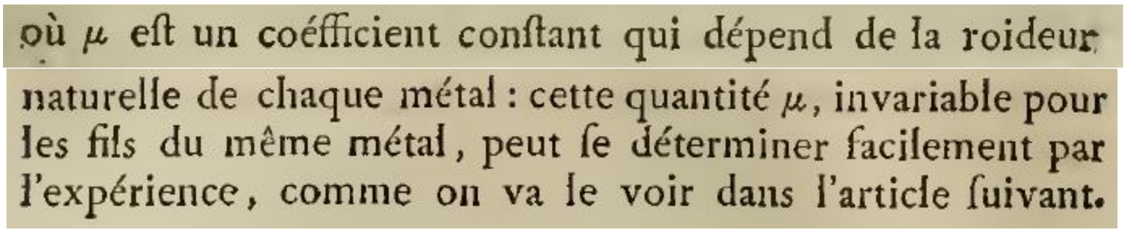
X I V.

Réſultat général.

Il réſulte donc de toutes les expériences qui précèdent, que le *momentum* de la force de torſion, eſt, pour les fils du même métal, en raiſon compoſée de l'angle de torſion de la quatrième puiſſance du diamètre, & inverſe de la longueur du fil; en ſorte que ſi l'on nomme *l* la longueur du fil, *D* ſon diamètre, *B* l'angle de torſion, l'on aura, pour l'expreſſion qui repréſente la force de torſion, $\frac{\mu B D^4}{l}$,

It results thus from all the preceding experiments that, the *moment* (momentum) of the force of torsion for filaments of the same metal is, proportional to the angle of twist, the fourth power of the diameter, and inversely proportional to the length of the filament; so that, if we let *l* be the length of the filament, *D* its diameter, *B* the angle of twist, we will have for the expression which represents the torque;

$$\mu B D^4/l$$

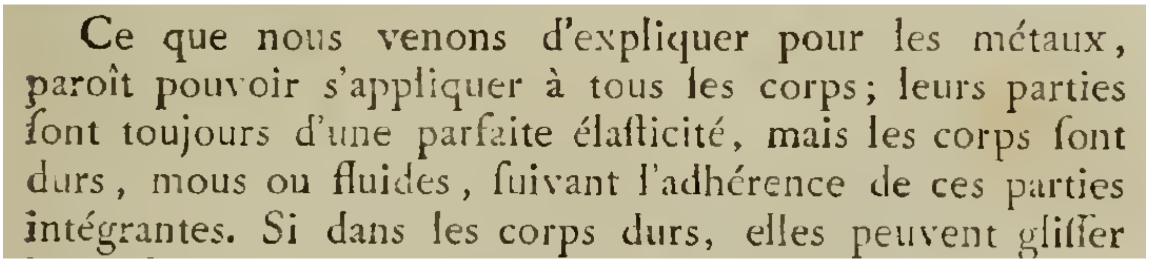


où μ eſt un coéfficient conſtant qui dépend de la roideur naturelle de chaque métal : cette quantité μ, invariable pour les fils du même métal, peut ſe déterminer facilement par l'expérience, comme on va le voir dans l'article ſuivant.

where $\boldsymbol{\mu}$ is a *constant coefficient* which depends on the natural stiffness of each metal: this quantity $\boldsymbol{\mu}$, is invariable for filaments of the same metal, can be easily determined from experiment, as we see in the following article.

Thus, similar to the case of gravitational constant and the using of a torsion balance by Henry Cavendish to weigh the (mass of) earth, Coulomb also made several kinds of mechanical torsion balance (this is against electrical torsion balance which we consider so soon) to measure the smallest levels of the torsion force by observations on the laws of elasticity and of coherence of metal wires.

In his final remarks he adds this point that,

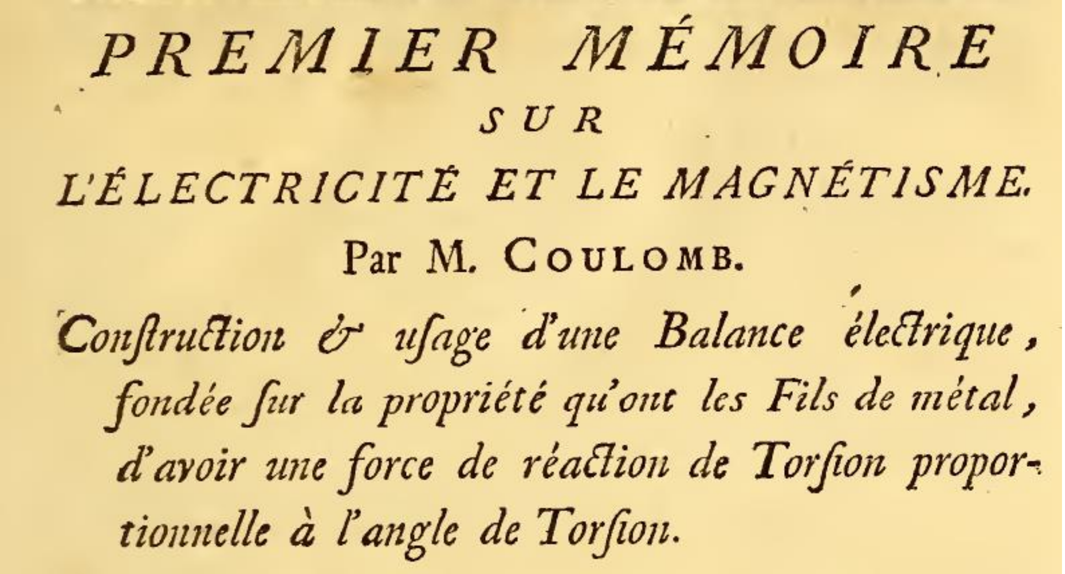


Ce que nous venons d'expliquer pour les métaux, paroît pouvoir s'appliquer à tous les corps; leurs parties ſont toujours d'une parfaite élaſticité, mais les corps ſont durs, mous ou fluides, ſuivant l'adhérence de ces parties intégrantes. Si dans les corps durs, elles peuvent gliſſer

This that we have come to explain for metals appears to be able to be applied to all bodies; their parts are always of a perfect elasticity, but the bodies made hard, tender or fluids, follow the adhesion of the integral parts.

His discovery of inverse square (of the distance) force-relationship between electric charges as well as magnetic poles namely Coulomb's law happened after this work and also after outbreak of the Revolution in 1789 caused him to resigned his position at royal state.

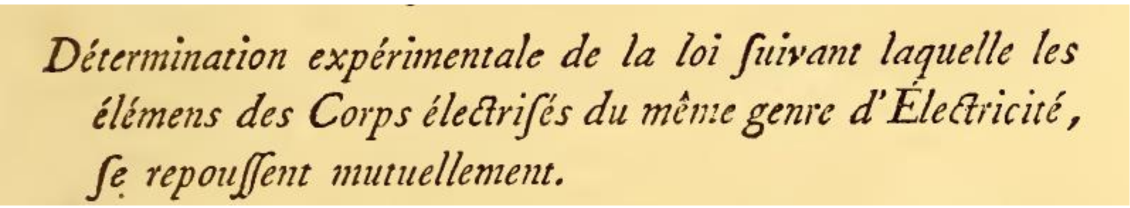
Now we go on with the constant of proportionality between static charges. As it is mentioned above, the results of theoretical research and experiment on the force of torsion and the elasticity of metal is be able to be applied to all bodies. Thus, Coulomb extended his mechanical results to world of electrostatic charges. He did a series of experiments on the *electric torsional force for metal wires* (within an electric torsion balance) and the results of them appeared in his paper in 1785".



PREMIER MÉMOIRE
SUR
L'ÉLECTRICITÉ ET LE MAGNÉTISME.
Par M. COULOMB.

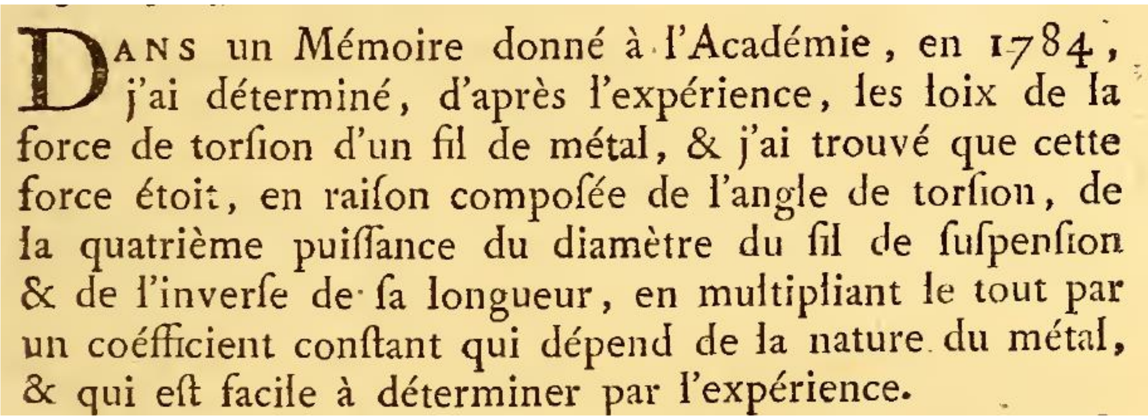
Conſtruction & uſage d'une Balance électrique, fondée ſur la propriété qu'ont les Fils de métal, d'avoir une force de réaction de Torſion proportionnelle à l'angle de Torſion.

In the title, he mentions the application of his theory as, the "Construction and usage of an electric balance based on property of the metal wire to have a torsion reaction force proportional to the angle of torsion". The aim of the paper is described by him as;



Détermination expérimentale de la loi ſuivant laquelle les élémens des Corps électriſés du même genre d'Électricité, ſe repouſſent mutuellement.

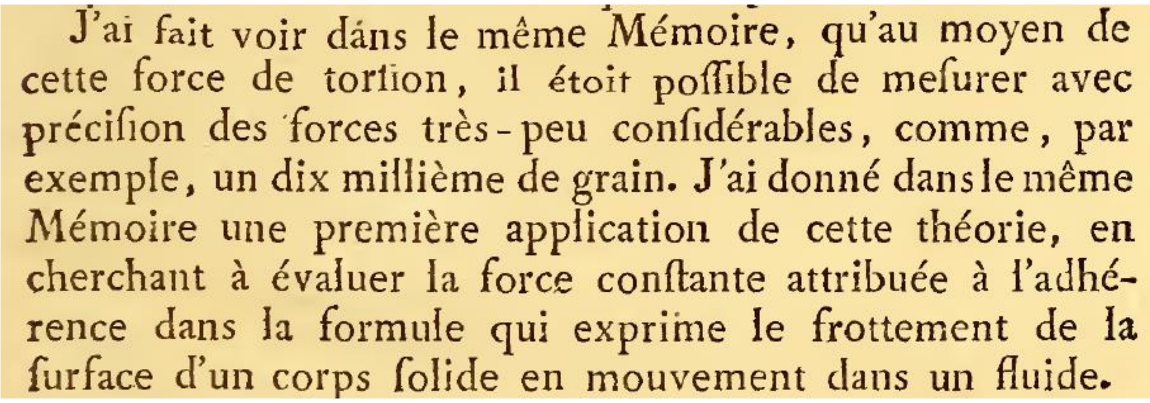
"experimental determination of the law according to which the elements of bodies, electrified with the same kind of electricity mutually repel each other". Then he started as;



DANS un Mémoire donné à l'Académie, en 1784, j'ai déterminé, d'après l'expérience, les loix de la force de torſion d'un fil de métal, & j'ai trouvé que cette force étoit, en raiſon compoſée de l'angle de torſion, de la quatrième puiſſance du diamètre du fil de ſuſpenſion & de l'inverſe de ſa longueur, en multipliant le tout par un coéfficient conſtant qui dépend de la nature du métal, & qui eſt facile à déterminer par l'expérience.

In a memoir given to the Academy in 1784, I determined from the experience, the laws of torsion force of a metal wire and I found that this force was ,due to the *composition* of the angle of torsion , the fourth power of the diameter of the suspending wire and the inverse of its length, by multiplying the whole by a constant coefficient which depends on the nature (stiffness) of the metal , and which is easy to determine by experience.

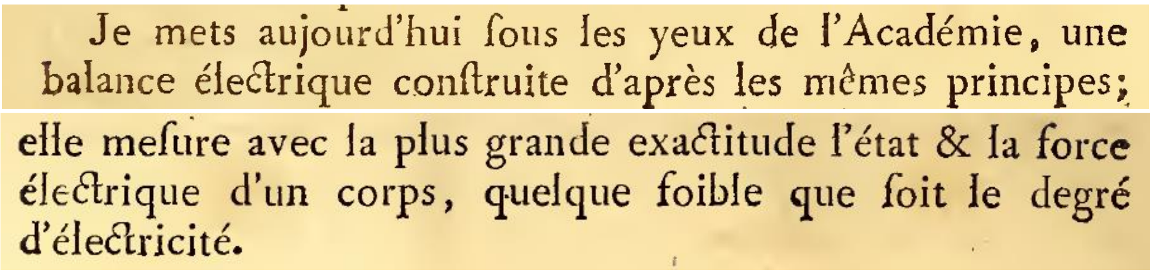
Thus, as a law this can be expressed as; "the moment of the torque is, for wires of the same metal, proportional to the torsional angle, the fourth power of the diameter and the inverse of the length of the wire". Then, he continues as;



J’ai fait voir dans le même Mémoire, qu’au moyen de cette force de torſion, il étoit poſſible de meſurer avec préciſion des forces très-peu conſidérables, comme, par exemple, un dix millième de grain. J’ai donné dans le même Mémoire une première application de cette théorie, en cherchant à évaluer la force conſtante attribuée à l’adhérence dans la formule qui exprime le frottement de la ſurface d’un corps ſolide en mouvement dans un fluide.

I showed in the same Memoir that by means of this torsion force, it was possible to measure with precision forces very inconsiderable, as for example a tenth-thousandth of a grain (1 grain = 0,053 g). I gave in the same Memoir a first application of this theory, by seeking to evaluate the constant force attributed to adhesion in the formula which expresses the friction of the surface of a solid body moving in a fluid.

Then he connects his previous paper with the current one as;

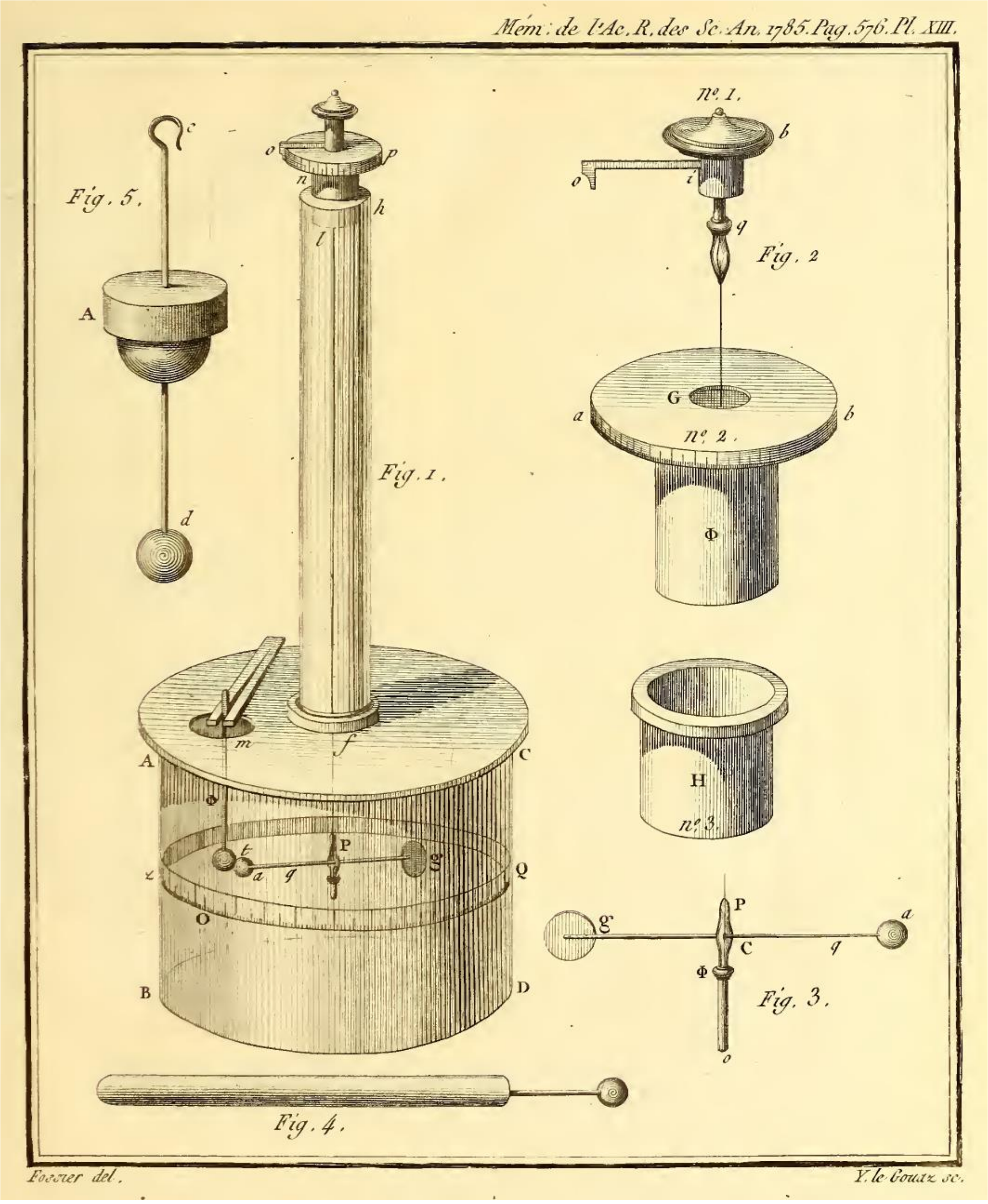


Je mets aujourd’hui ſous les yeux de l’Académie, une balance électrique conſtruite d’après les mêmes principes; elle meſure avec la plus grande exactitude l’état & la force électrique d’un corps, quelque foible que ſoit le degré d’électricité.

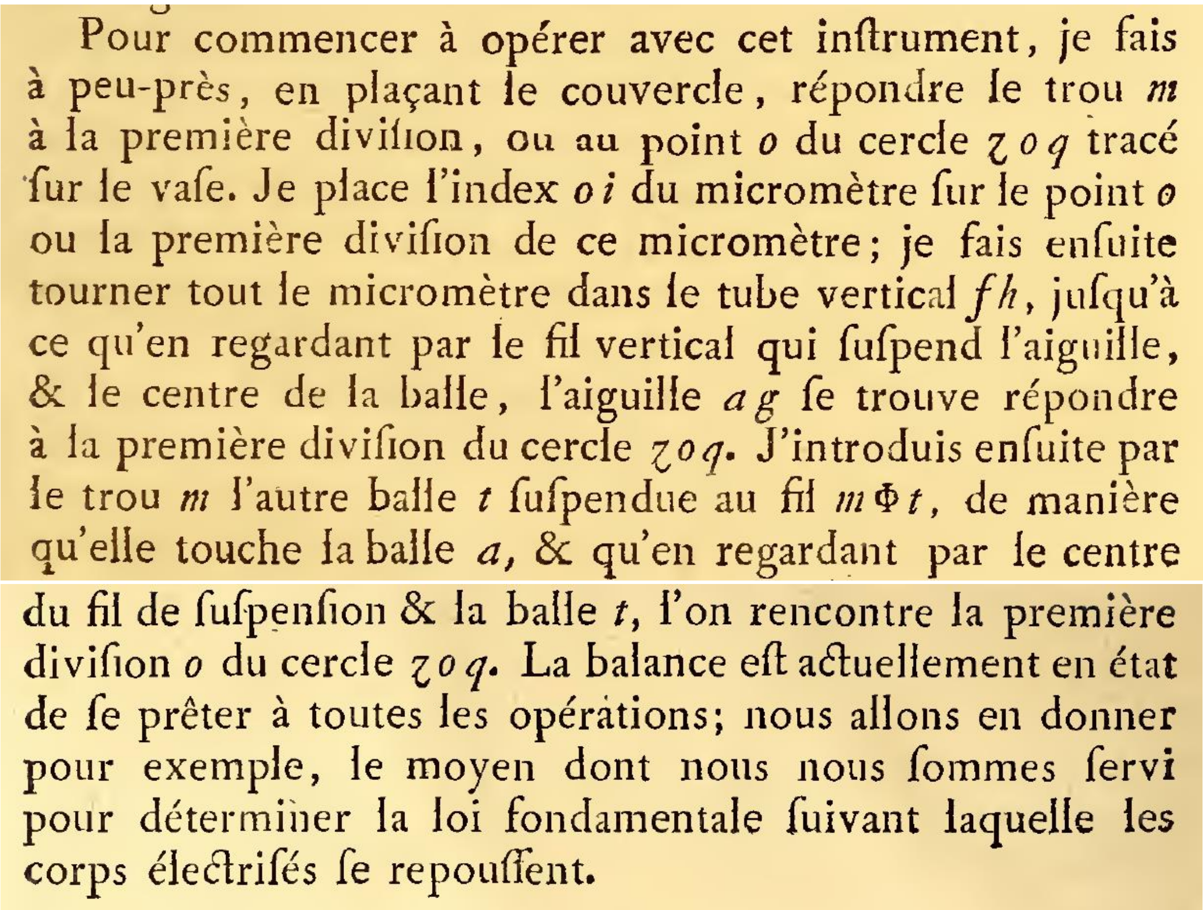
I put today before the eyes of the Academy, an electric balance constructed according to the same principles; it measures with the greatest accuracy the state and electric force of a body, however weak the degree of electricity.

Before to continue it is necessary to note that, "much of the modern physical description of electrical forces comes from these experiments". To execute several electric experiments in a convenient way Coulomb had to alter and improve previous torsion balance according to the nature of the new experiments.

The original figure below presents his electric torsion balance in its perspective. "It consists of a light insulating needle A suspended by a fine metal wire F and carrying at one of its ends a small ball B balanced by a counterweight P provided at the other end. The upper end of the thread F is fixed to the rotating head of the device, allowing the angle of torsion of the filament to be measured with precision. Inside the apparatus one can introduce another ball B' having the same diameter as the ball B. The needle is enclosed in a large glass cage which protects the sensitive parts against air movement.



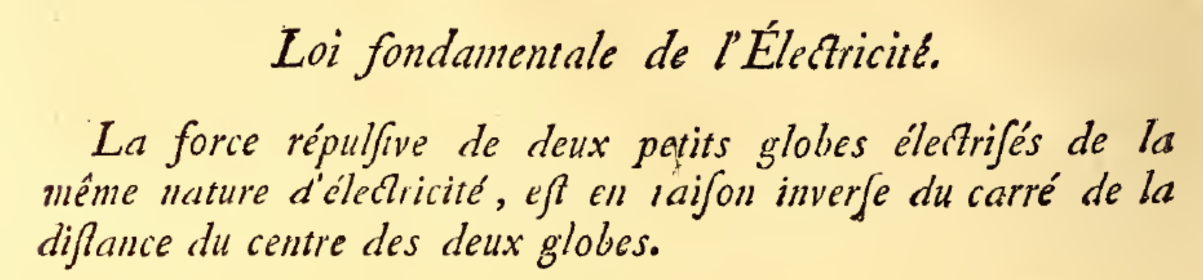
We leave here the small details of his device and just consider the way he operated the device. He himself describe this as;



Pour commencer à opérer avec cet inſtrument, je fais à peu-près, en plaçant le couvercle, répondre le trou *m* à la première diviſion, ou au point *o* du cercle *z o q* tracé ſur le vaſe. Je place l'index *o i* du micromètre ſur le point *o* ou la première diviſion de ce micromètre; je fais enſuite tourner tout le micromètre dans le tube vertical *f h*, juſqu'à ce qu'en regardant par le fil vertical qui ſuſpend l'aiguille, & le centre de la balle, l'aiguille *a g* ſe trouve répondre à la première diviſion du cercle *z o q*. J'introduis enſuite par le trou *m* l'autre balle *t* ſuſpendue au fil *m* Φ *t*, de manière qu'elle touche la balle *a*, & qu'en regardant par le centre du fil de ſuſpenſion & la balle *t*, l'on rencontre la première diviſion *o* du cercle *z o q*. La balance eſt actuellement en état de ſe prêter à toutes les opérations; nous allons en donner pour exemple, le moyen dont nous nous ſommes ſervi pour déterminer la loi fondamentale ſuivant laquelle les corps électriſés ſe repouſſent.

To begin to operate with this instrument, in placing the cover, I placed the hole ***m*** approximately at the first division or at the point **O** of the circle ***Z O Q*** traced on the vase. I place the pointer ***o i*** of the micrometer on the point **O**, at the first division of this micrometer. I then turn the micrometer within the vertical tube ***f h*** until, keeping in view the vertical filament which suspends the needle, and the center of the ball, the needle ***a g*** is directed towards the first division of the circle ***Z O Q***. I then introduce through the hole ***m*** the other ball *t* suspended by the filament ***m Φ t*** such that it touches the ball ***a*** and that, in keeping in view the center of the filament of suspension and the ball ***t***, one encounters the first division ***o*** of the circle ***Z O Q***. The balance is now in a state ready for all operations; we go on to give as an example, the means by which we are able to determine the fundamental law according to which *electrified bodies repel themselves*.

Then he defines his fundamental law of electricity between two globes since he supposed electric charges as a shape of sphere.



Loi fondamentale de l'Électricité.

La force répulſive de deux petits globes électriſés de la même nature d'électricité, eſt en raiſon inverſe du carré de la diſtance du centre des deux globes.

The repulsive force of two small globes electrified with the same kind of electricity, is inversely proportional to the square of the distance between the centers of the two globes.

From now on we leave the original paper since there exist space-ban here and also there is no new mathematical method here but it composes of descriptions of experiments on his electric torsion device to prove the above descriptive law for "repulsive force of two small globes electrified".

We know that, "electric charges are always distributed in a volume". Thus, in a mathematical point any "electric charge of finite value" cannot be exist. Therefore, when we use the term "point charge" we mean "an extended charged body whose dimensions are very small compared to the distance which separates this body from the other charges".

More clearly coulomb applied his electrical torsion balance to establish proportionality of the exerted force between the charges to their distance.

If one electrified (charged) the balls B and B', using a third charged ball C kept by an insulating holder, the balls repel each other and halt at a certain distance that is measured using the graded dial of the device. If one turns the head of the device, according to well-determined angles of torsion on the suspension wire then he/she can measure the distances to which the balls approach for these different angles.

As we know from the mechanics course, in the event of a torsion deformation (in the field of reversible elastic deformations) the angle of torsion is proportional to the moment of the torsion force; knowing the ratio in which we have increased the angle of torsion of the thread, we can thereby determine the ratio in which the moment of the force has increased and therefore find the force which is exerted on the ball fixed at the end of the needle. Following his experiments Coulomb came to the conclusion that the force which is exerted between two-point charges is directed along the line which joins them and is inversely proportional to the square of their distance i.e.

$$F \approx 1/r^2$$

The force of interaction between the balls still depends on the value of the charges carried by these balls. This dependence can be demonstrated with the help of the following experiment. If one touches, for a short time, one of the charged balls B or B' with another ball having the same dimensions but not charged, the charge is distributed, because of the identity of the two balls put in contact, in two equal parts. It then turns out that for the same distance the force which is exerted between the balls B and B'. one of which now only carries half the initial load, is reduced by half. By using this process, one can reduce the load of the ball by two times, four times, etc., and be convinced that the force which is exerted between them is proportional to the load of each of the balls.

Now let us connect to the balls B and B' any rate of charges and let us measure the force $F_{BB'}$ which is exerted between them for a certain determined distance. Then replace the ball B' by another charged ball C and measure again the force F_{BC} which is exerted between the balls B and C.

the distance being the same as in the first case. If, now, we modify in an arbitrary way the charge of the ball B and we measure again the forces which are exerted between the ball B on the one hand and the balls B' and C on the other hand, the experience shows that the ratio of forces $F_{BB'}$:F_{BC} is the same in both cases. that is to say that it does not depend on the value of the charge carried by the ball B. This means that this ratio of forces depends only on the charges of the balls B' and C; it can therefore be taken equal to the ratio of the charges of the two balls:

$$\frac{F_{BB\prime}}{F_{BC}} = \frac{q_{B\prime}}{q_c}$$

This relationship is the definition of the quantitative ratio of two charges and provides a method for comparing the charges.

The result obtained shows that the force exerted between two charged balls is proportional to the charge of one of them (ball B). Since the two balls intervene in these experiments on an equal footing, it follows that the force which is exerted between them is proportional to each of the charges q_1 and q_2. Thus, the force acting between two electric charge points is equal to the famous formula:

$$F = \boldsymbol{k}\frac{q_1 q_2}{r^2}$$

Here constant $\boldsymbol{k}$ is the electric form of the constant $\boldsymbol{\mu}$ which was depended on the natural stiffness of each metal wires. From De Magnet and other primitive works, we considered above, those scientific men proposed that this attracting force have directions and divergences. Thus, it is reasonable the change the first relation as;

$$F = K_e \frac{q_1 q_2}{r^2}\hat{e}$$

Here, $\hat{\boldsymbol{e}}$ is a unit vector in the r direction i.e. the distance between two charges. The value of constant of inverse proportionality K_e depends only on the *units of measurement chosen for the charge*, the *distance* and the *force*. Thus, in SI we write;

$$K_e = \frac{1}{4\pi\varepsilon_0}$$

The note is that, 4π is a common factor, in any system of units, in the formulas of electromagnetism e.g. Maxwell equations and this coulomb's law. Also, one of the differences between the SI system and the C.G.S. is that for writing the laws of electricity the SI system uses a so-called "rationalized" form. This means that, in SI units the factor of 4π would be disappeared. To do this we introduce from the beginning, in the expression of the electrostatic Coulomb's law, the factor $1/4\pi$ adjusting with the SI system of units. For this reason, Coulomb's electrostatic law is written in the SI system as:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{e}$$

Thus, the proportionality coefficient k i.e. $\frac{1}{4\pi\varepsilon_0}$ called **coulomb constant** is a certain constant whose value depends on the choice of units. Here ε_0 is a constant as well. It is called dielectric constant or specific inductive power of (classical) vacuum; thus, it is also called permittivity (distributed capacitance) of vacuum i.e. a free space. Therefore, it is a "measure of the electric polarizability of a dielectric". The permittivity is a quantity which characterizes the electrical properties of bodies and depends on their nature and their state (temperature, pressure, etc.).

But why it is defined here?

As we know, coulomb C is a derived unit for electric charge. In fact, it is the quantity of electricity transported in 1 second by a constant current of 1 ampere, namely $1C = A.s$. According to this the constant k cannot be reduced to unity. This is why, unlike the C.G.S. system, the SI system involves in the laws of electricity expressed in units of this system a new electric constant ε_0 having well-determined physical dimensions.

To calculate the value of ε_0 in the SI system, we suppose two points charges such that, $q_1 = q_2 = 1C = 3 \cdot 10^9$ (speed of light) at a distance of $1\,m = 10^2 cm$ from each other in vacuum.

According to coulomb's law the force acting between these charges has the value of;

$$F = \frac{(3 \cdot 10^9)^2}{(10^2)^2} = 9 \cdot 10^{14}\, dynes = 9.10^9 N$$

Here dyne is CGS unit of force, proposed in 1873. One dyne is equal to ten micronewtons, 10^{-5} N namely;

$$1\, dyn = 1\, g \cdot \frac{cm}{s^2} = 10^{-5} kg \cdot \frac{m}{s^2} = 10^{-5} N$$

And again, according to coulomb's law the same force would be;

$$F = \frac{1}{4\pi\varepsilon_0} \frac{1 \cdot 1}{1^2} = \frac{1}{4\pi\varepsilon_0}\, N$$

Such that in SI unit;

$$\varepsilon_0 = \frac{1}{4\pi \cdot 9 \times 10^9} = 8.85\ \times 10^{-12}\, N$$

Finally,

$$K_e = \frac{1}{4\pi\varepsilon_0} = 8.98\ \times 10^9\, N.m^2.C^{-2}$$

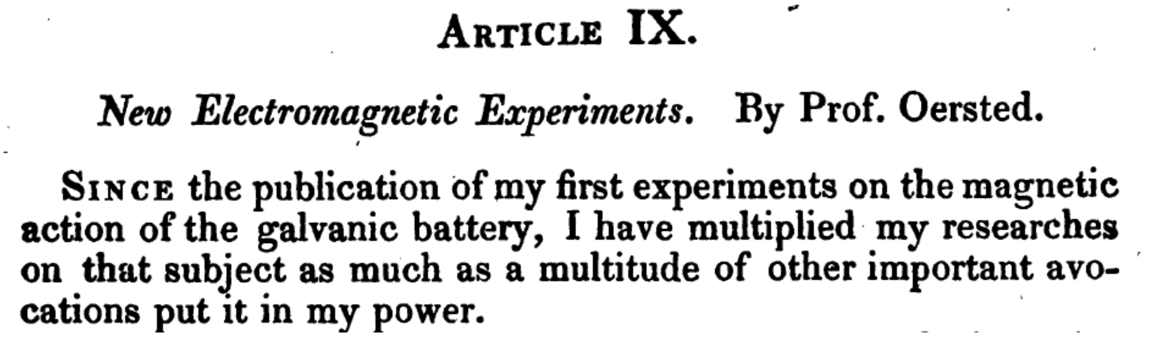
Of course, Coulomb's experiments are not the only process for demonstrating the validity of the law of the inverse square of the distance. Currently, we have a large number of experimental data which prove that Coulomb's law is verified very exactly and this both for very large distances and for very small distances. In particular, studies of atomic phenomena lead to the conclusion that it remains valid at least up to distances of the order of 10^{-13} cm.

The story of electron (cont.1)

We mentioned that, in Faraday's idea, the responsible for transferring electromagnetic interactions was something called "material medium". In this medium, these interactions are being transferred from one point to another.

Before 1819 people believe that electricity and magnetism have nothing to do with each other. But in this year, a Danish physicist and chemist, Hans Christian Oersted (1777-1851) in Copenhagen, showed that there exists a relationship between electricity and magnetism by some experiments. He observed that "a magnetized needle could be deflected by an electric current". It was "the first indication of a definite relation existing between magnetism and electricity".

In describing one of his first experiments he starts as;

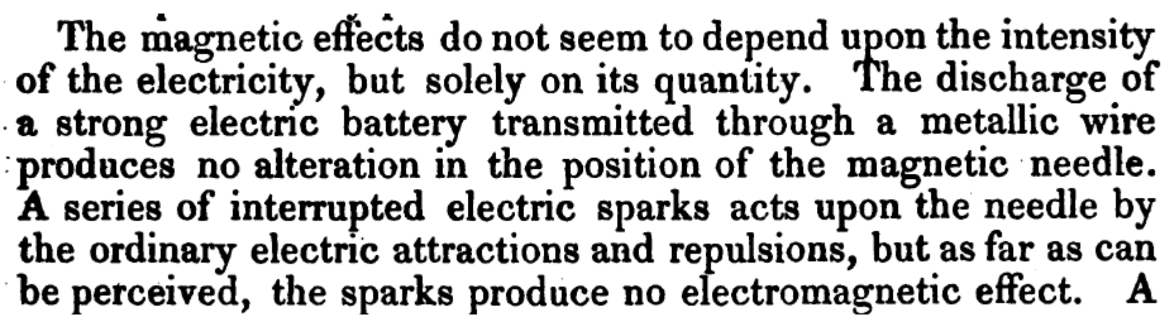


ARTICLE IX.

New Electromagnetic Experiments. By Prof. Oersted.

SINCE the publication of my first experiments on the magnetic action of the galvanic battery, I have multiplied my researches on that subject as much as a multitude of other important avocations put it in my power.

Then he continues as;

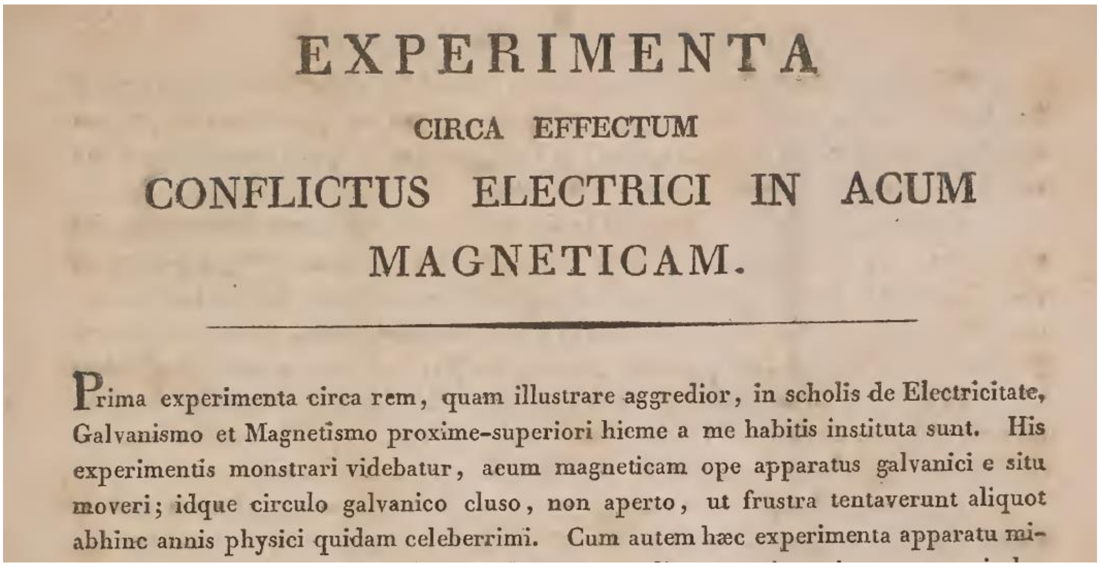


The magnetic effects do not seem to depend upon the intensity of the electricity, but solely on its quantity. The discharge of a strong electric battery transmitted through a metallic wire produces no alteration in the position of the magnetic needle. A series of interrupted electric sparks acts upon the needle by the ordinary electric attractions and repulsions, but as far as can be perceived, the sparks produce no electromagnetic effect. A

The problem he mentioned in the last sentence above later will explain by A. Ampere which we will consider as well. The main tools and material in his experiments were used to be a galvanic pile, a compass and pieces of wire. Then he altered these materials and observed the results as below

"A galvanic pile composed of 100 discs of two inches square each metal, and of paper moistened with salt water to serve as a fluid conductor, is likewise destitute of sensible effect 11 on the needle. On the other hand, we obtain the effect by a single galvanic arc of zinc and copper having for a conductor a liquid possessed of great conducting power; for example, of one-part Sulphur acid, as much of nitric acid, and 60 parts of water. We may even double the quantity of water without much diminishing the effect".

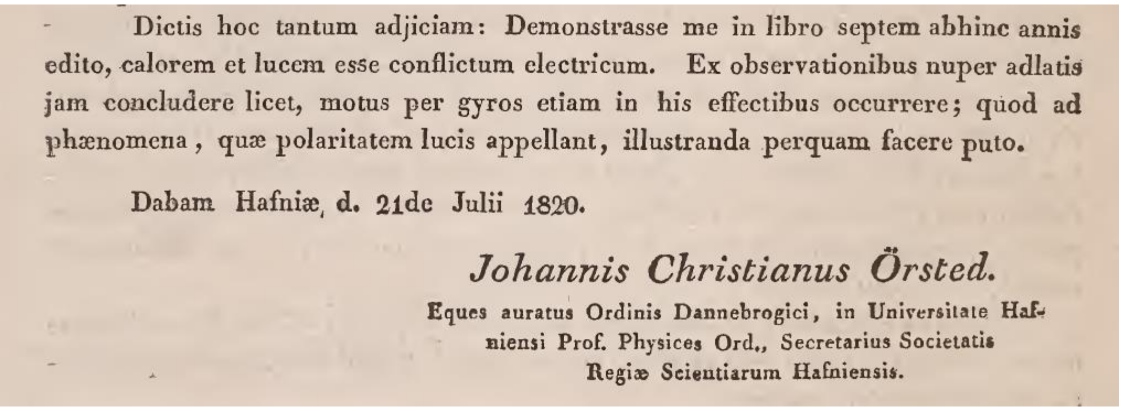
The first description of his experiments in the phenomena electromagnetoism printed in 1820. It was a 4 pages report in Latin titled in "experiments on the effect of electrical conflict on a magnetic needle".



EXPERIMENTA
CIRCA EFFECTUM
CONFLICTUS ELECTRICI IN ACUM
MAGNETICAM.

Prima experimenta circa rem, quam illustrare aggredior, in scholis de Electricitate, Galvanismo et Magnetismo proxime-superiori hieme a me habitis instituta sunt. His experimentis monstrari videbatur, acum magneticam ope apparatus galvanici e situ moveri; idque circulo galvanico cluso, non aperto, ut frustra tentaverunt aliquot abhinc annis physici quidam celeberrimi. Cum autem hæc experimenta apparatu mi-

Then its English and German translation was also become available. He ended this report as below.



Dictis hoc tantum adjiciam: Demonstrasse me in libro septem abhinc annis edito, calorem et lucem esse conflictum electricum. Ex observationibus nuper adlatis jam concludere licet, motus per gyros etiam in his effectibus occurrere; quod ad phænomena, quæ polaritatem lucis appellant, illustranda perquam facere puto.

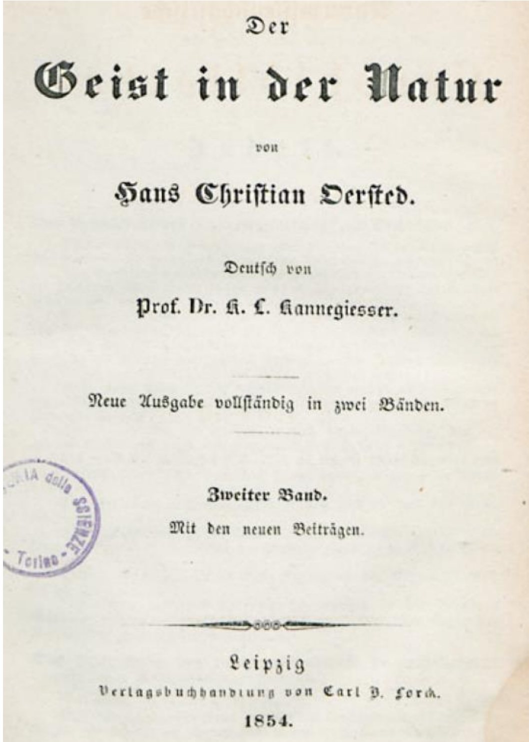
Dabam Hafniæ, d. 21de Julii 1820.

Johannis Christianus Örsted.

Eques auratus Ordinis Dannebrogici, in Universitate Hafniensi Prof. Physices Ord., Secretarius Societatis Regiæ Scientiarum Hafniensis.

Here we consider its German translation. He was a scientist and a poet as well such that his writings are a combination of both.

Oersted published in 1854 his articles in a book in two volumes under the title *Der Geist in der Natur* "The Soul in Nature" in German.



Der

Geist in der Natur

von

Hans Chriſtian Oerſted.

Deutſch von

Prof. Dr. K. L. Kannegiesser.

Neue Ausgabe vollſtändig in zwei Bänden.

Zweiter Band.

Mit den neuen Beiträgen.

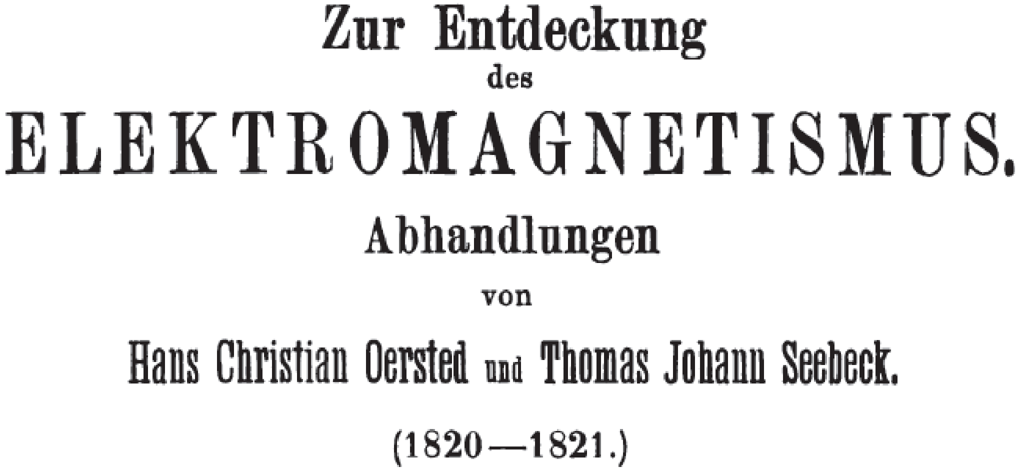
Leipzig

Verlagsbuchhandlung von Carl B. Lorck.

1854.

IA delle SCIENZE - Torino -

In 1895 his and Thomas Johann Seebeck's essays on electromagnetism published under the title *Zur Entdeckung des Electromagnetismus* " Discovering Electromagnetism".



Zur Entdeckung

des

ELEKTROMAGNETISMUS.

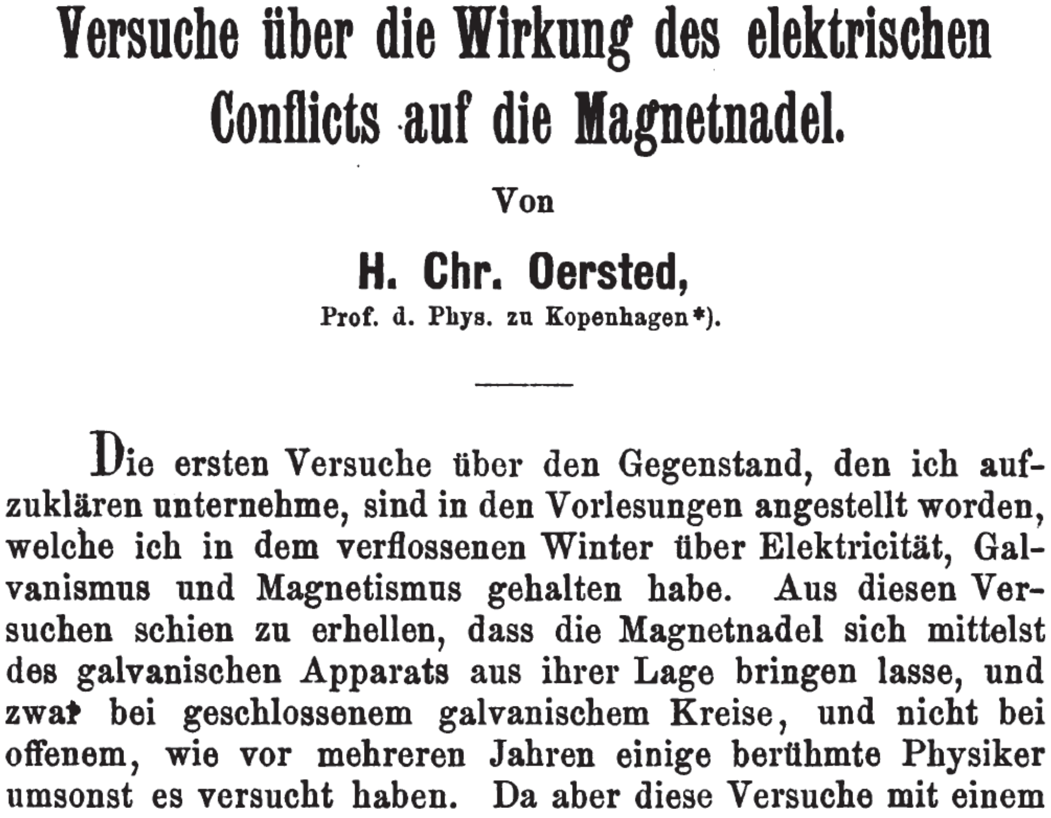
Abhandlungen

von

Hans Christian Oersted und Thomas Johann Seebeck.

(1820—1821.)

In the first article *Versuche über die Wirkung des elektrischen conflicts auf die magnetnadel* "Experiments on the effect of electrical conflict on the magnetic needle" which is the German translation of the Latin report he describe his experiments, observations and conclusions.



Versuche über die Wirkung des elektrischen Conflicts auf die Magnetnadel.

Von

H. Chr. Oersted,

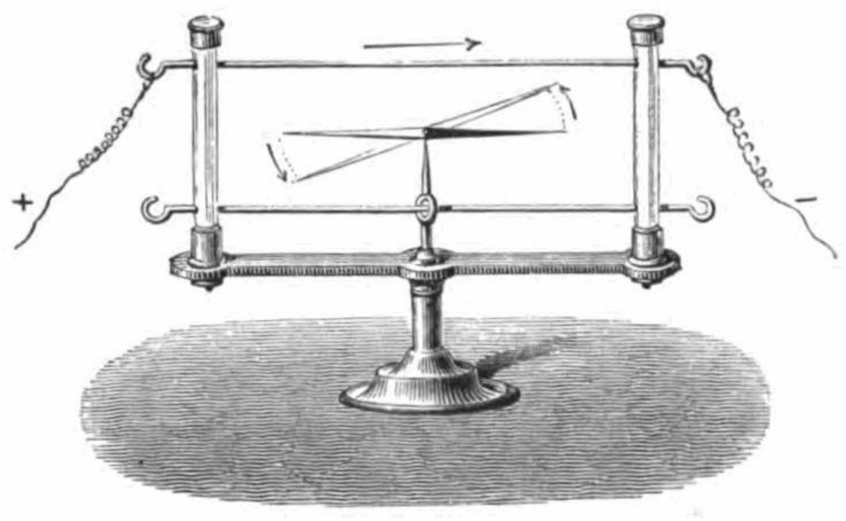
Prof. d. Phys. zu Kopenhagen*).

Die ersten Versuche über den Gegenstand, den ich aufzuklären unternehme, sind in den Vorlesungen angestellt worden, welche ich in dem verflossenen Winter über Elektricität, Galvanismus und Magnetismus gehalten habe. Aus diesen Versuchen schien zu erhellen, dass die Magnetnadel sich mittelst des galvanischen Apparats aus ihrer Lage bringen lasse, und zwar bei geschlossenem galvanischem Kreise, und nicht bei offenem, wie vor mehreren Jahren einige berühmte Physiker umsonst es versucht haben. Da aber diese Versuche mit einem

The first experiments on the subject which I undertake to explain were made in the lectures which I gave last winter on electricity, galvanism and magnetism. From these experiments it seemed clear that the magnetic needle could be moved from its position by means of the galvanic apparatus, and that with the galvanic circuit closed, and not with the circuit open, as some famous physicists tried in vain several years ago.

Again, his last sentence has a point to notice i.e. the closed and open circuit. He observed that, "the magnetic needle could be moved from its position by means of the galvanic apparatus, and that with the galvanic circuit closed". This is the point we consider later on.

His experiment is described by A. Privat Deschanel in part III of Elementary *Treaties on natural philosophy* (electricity and magnetism) published in 1876. Deschanel states that, "Oersted's experiment can be repeated as in Figure below. Two insulated metallic wires are placed in the magnetic meridian, one of them above, and the other below a magnetized needle.



"If a current be sent through one of these wires, the needle will be deflected; and if the current be strong, the deflection will nearly amount to a right angle. The direction of the deflection will be re versed if the current be passed through the lower instead of the upper wire. It will also be reversed by reversing the direction of the current. In the figure, the current is supposed to be passing above the needle from south to north. In this case the north end of the needle moves to the west, and the south end to the east. On making the current pass in various directions, either horizontally, vertically, or obliquely, near one pole of the needle, it will be found that deviation is always produced except when the plane containing the pole and current is perpendicular to the length of the needle".

Therefore, "by setting up a compass through a wire carrying an electric current, Oersted showed that moving electrons can create a magnetic field. The magnetic field created by the current goes in circles around the wire".

In *Versuche über die Wirkung des elektrischen conflicts auf die magnetnadel* he concludes that;

> Dass der elektrische Conflict nicht in dem leitenden Drahte eingeschlossen, sondern, wie gesagt, zugleich in dem umgebenden Raume ziemlich weithin verbreitet ist, ergiebt sich aus den angeführten Beobachtungen hinlänglich.

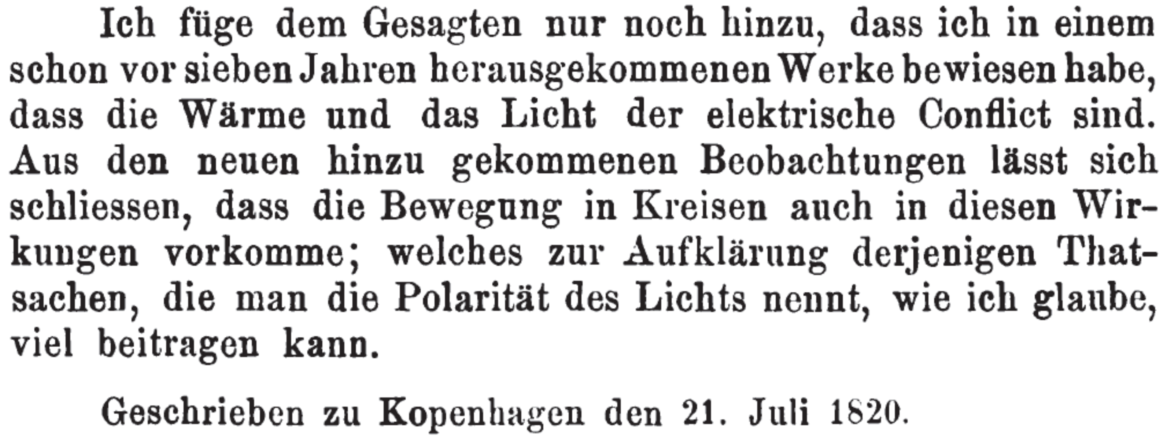
That the electrical conflict is not confined in the conducting wire, but, as has been said, is at the same time fairly widespread in the surrounding space, is sufficiently evident from the observations quoted.

> Es lässt sich auch aus dem, was beobachtet worden, schliessen, dass dieser Conflict in Kreisen fortgehe; denn es scheint ohne diese Annahme nicht zu begreifen zu sein, wie derselbe Theil des verbindenden Drahtes, der unter einem Pole der Magnetnadel gestellt, diese nach Osten treibt, sie nach Westen bewegen sollte, wenn er sich über diesem Pole befindet; eine Kreisbewegung geht aber in den beiden entgegengesetzten Enden eines Durchmessers nach entgegengesetzten Richtungen vor sich. Es scheint überdem, es müsse die Kreisbewegung, verbunden mit der fortschreitenden Bewegung nach der Länge des Leiters, eine Schneckenlinie oder Spirale beschreiben, welches jedoch, wenn ich nicht irre, zur Erklärung der bisher beobachteten Erscheinungen nichts beiträgt.8)

It may also be concluded from what has been observed that this conflict is going on in circles; for without this assumption it seems incomprehensible how the same part of the connecting wire, placed under one pole of the magnetic needle, driving it eastward, should move it westward when it is over that pole; but a circular motion takes place in opposite directions at the two opposite ends of a diameter. It also seems that the circular movement, combined with the progressive movement along the length of the conductor, must describe a helix or spiral, which, however, if I am not mistaken, contributes nothing to the explanation of the phenomena observed so far.

> Alle hier angegebene Wirkungen auf den Nordpol der Nadel lassen sich leicht verstehen, wenn man annimmt, dass die negativ elektrische Kraft oder Materie eine rechts gewundene Spirale durchläuft, und den Nordpol fortstösst, auf den Südpol aber nicht [304] wirkt; und eben so alle Wirkungen auf den Südpol, wenn man der positiv elektrischen Kraft oder Materie eine Bewegung in entgegengesetzter Richtung, und das Vermögen auf den Südpol und nicht auf den Nordpol der Nadel zu wirken, zuschreibt. Von der Uebereinstimmung dieses Gesetzes mit der Natur überzeugt man sich besser durch Wiederholen der Versuche, als durch eine lange Erklärung. Die Beurtheilung der Versuche würde aber durch Figuren sehr erleichert werden, welche den Weg, den die elektrischen Kräfte in dem verbindenden Drahte gehen, zeigen.9)

All the effects here given on the north pole of the needle can be easily understood, if it be supposed that the negative electric force or matter runs through a right-hand helix and pushes away the north pole, but does not act on the south pole; and likewise all the effects on the south pole, ascribing to the positive electric force or matter a motion in the opposite direction, and the power of acting on the south pole and not on the north pole of the needle. It is better to convince oneself of the agreement of this law with nature by repeating the experiments than by a long explanation. Judging the experiments would, however, be greatly facilitated by figures showing the path taken by the electrical forces in the connecting wire.



Ich füge dem Gesagten nur noch hinzu, dass ich in einem schon vor sieben Jahren herausgekommenen Werke bewiesen habe, dass die Wärme und das Licht der elektrische Conflict sind. Aus den neuen hinzu gekommenen Beobachtungen lässt sich schliessen, dass die Bewegung in Kreisen auch in diesen Wirkungen vorkomme; welches zur Aufklärung derjenigen Thatsachen, die man die Polarität des Lichts nennt, wie ich glaube, viel beitragen kann.

Geschrieben zu Kopenhagen den 21. Juli 1820.

only add to what has been said that I proved in a work published seven years ago that heat and light are the electrical conflict. From the new observations which have been added it can be concluded that the movement in circles also occurs in these actions; which can do much to explain that fact which I believe is called the polarity of light.

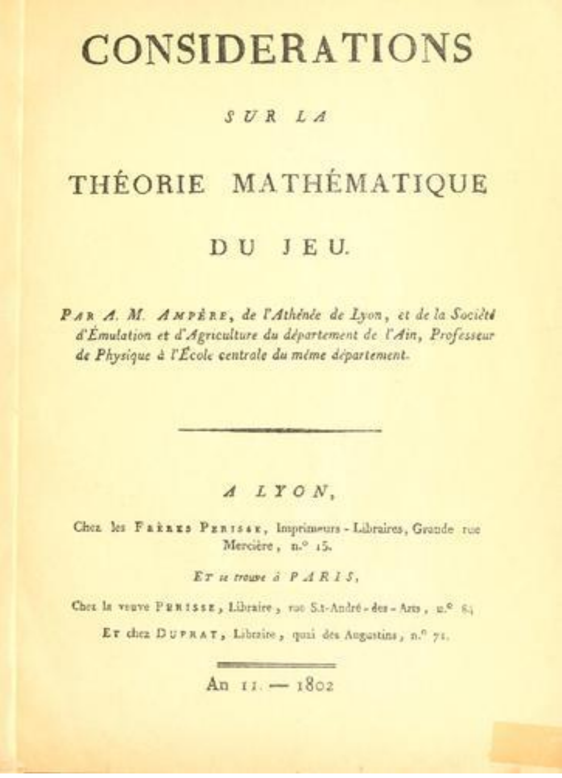
Written at Copenhagen, July 21, 1820.

Knowing the results of Oersted experiments a French physicist André-Marie Ampère (1775-1836) developed one of the first rigorous mathematical and physical theory of electromagnetism to understand the relationship between electricity and magnetism. He discovered that there exist any "interaction between electricity and magnetism" if and only if there exist a moving array of electrons i.e. an electric current. He showed that in the case of static electricity there exist no effect of this kind.

For his efforts he is known as one of the founding fathers of classical electromagnetism. He was a prolific inventor. For example, he invented one of the first electromagnet called it solenoid and a telecommunication device called electrical telegraph. The famous SI unit for electric current namely *Ampere* is also named after him. Since similar to newton who mathematicised foundations of classical mechanics, Ampere did this for classical electromagnetism thus it is fair to relay on James Clark Maxwell and call him the Newton of electromagnetism.

From starting points of his more self-teach education path, he was affected by Rousseau nature-affected approach of education and the works of great Leonard Euler and Daniel Bernoulli. For instance, when around 1802 he began to publish scientific works, he released a treatise on mathematical probability, *Considérations sur la théorie mathématique de jeu* "Considerations on the Mathematical Theory of Games".

Dessiné d'après nature en 1825, et Gravé par Ambroise Tardieu.



CONSIDERATIONS

SUR LA

THÉORIE MATHÉMATIQUE

DU JEU.

Par A. M. Ampère, de l'Athénée de Lyon, et de la Société d'Émulation et d'Agriculture du département de l'Ain, Professeur de Physique à l'École centrale du même département.

A LYON,

Chez les Frères Perisse, Imprimeurs-Libraires, Grande rue Mercière, n.° 15.

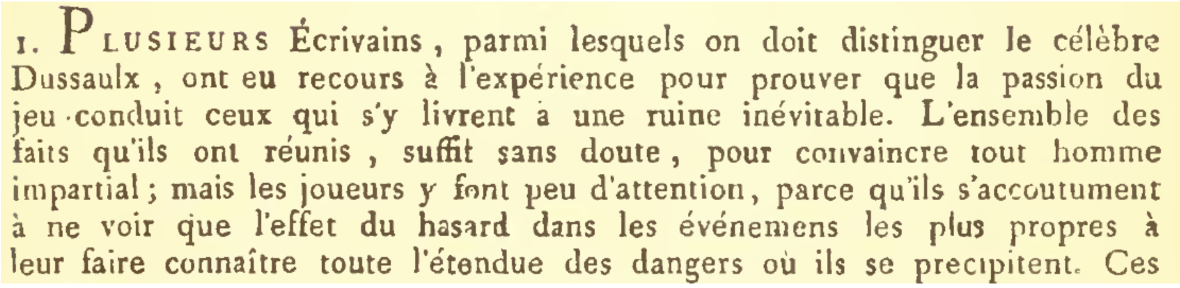
Et se trouve à PARIS,

Chez la veuve Perisse, Libraire, rue St-André-des-Arts, n.° 64;

Et chez Duprat, Libraire, quai des Augustins, n.° 71.

An 11. — 1802

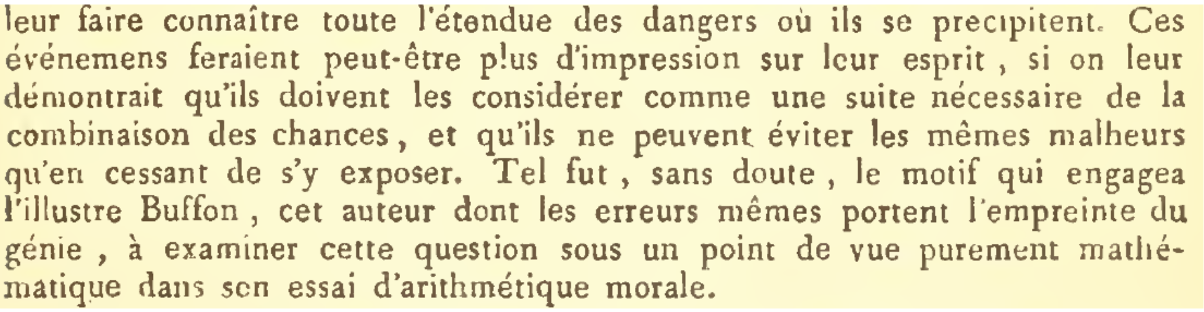
In the first paragraph he states that how much the factor of chance is important in gambling and there is a need to describe it mathematically. He writes;



1. Plusieurs Écrivains, parmi lesquels on doit distinguer le célèbre Dussaulx, ont eu recours à l'expérience pour prouver que la passion du jeu conduit ceux qui s'y livrent à une ruine inévitable. L'ensemble des faits qu'ils ont réunis, suffit sans doute, pour convaincre tout homme impartial; mais les joueurs y font peu d'attention, parce qu'ils s'accoutument à ne voir que l'effet du hasard dans les événemens les plus propres à leur faire connaître toute l'étendue des dangers où ils se précipitent. Ces

Several writers, among whom we must distinguish the famous Dussaulx, have had recourse to experience to prove that the passion for gambling leads those who indulge in it to inevitable ruin. The whole of the facts which they have brought together, doubtless suffices to convince any impartial man; but the gamblers pay little attention to it, because they are accustomed to seeing only the effect of chance in the events most calculated to make them aware of the full extent of the dangers into which they are precipitating.

Then he continues as;



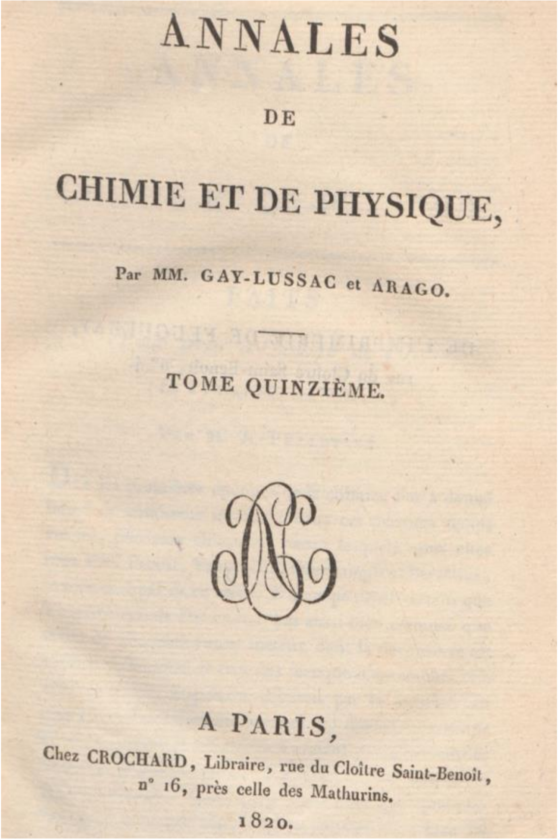
leur faire connaître toute l'étendue des dangers où ils se précipitent. Ces

événemens feraient peut-être plus d'impression sur leur esprit, si on leur démontrait qu'ils doivent les considérer comme une suite nécessaire de la combinaison des chances, et qu'ils ne peuvent éviter les mêmes malheurs qu'en cessant de s'y exposer. Tel fut, sans doute, le motif qui engagea l'illustre Buffon, cet auteur dont les erreurs mêmes portent l'empreinte du génie, à examiner cette question sous un point de vue purement mathématique dans son essai d'arithmétique morale.

These events would perhaps make a greater impression on their minds, if it were demonstrated to them that they must consider them as a necessary consequence of the combination of chances, and that they can avoid the same misfortunes only by ceasing to exposing to it. Such was, no doubt, the motive which induced the illustrious Buffon, that author whose very errors bear the mark of genius, to examine this question from a purely mathematical point of view in his essay on moral arithmetic.

Then later on he describes the probability procedure in a game as; "this probability is the same for all the arrangements that can be imagined between these won and lost parts, and as they are absolutely independent of each other, it is obvious that the probability which we have just found must be multiplied by the number of these arrangements".

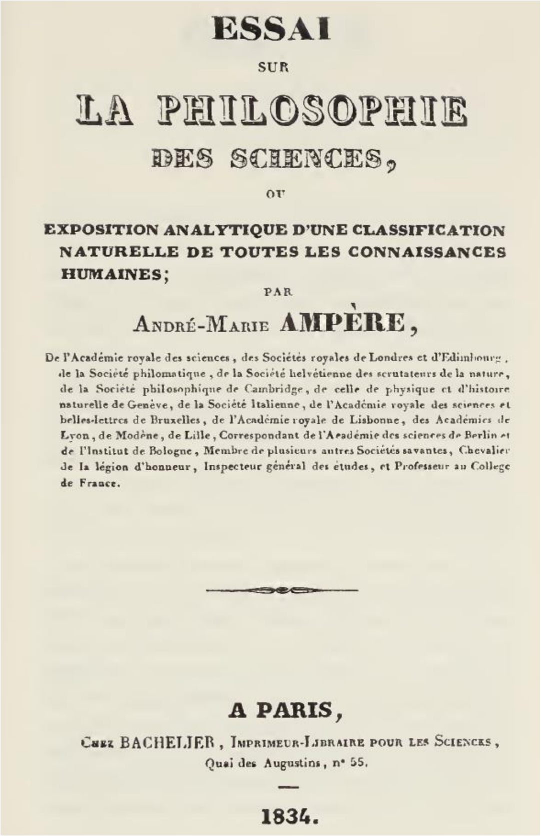
In 1804 he started his career as a tutor and professor of mathematics, philosophy, astronomy and experimental physics in Paris in new stablished École Polytechnique, university of Paris and some other institutes. During this time, he also started to published his scientific papers in different journal such as *The Philosophical Magazine and Journal, Annales de Chimie et de Physique* and *Annales Generales des SciencesPhysiques*.



ANNALES
DE
CHIMIE ET DE PHYSIQUE,
Par MM. GAY-LUSSAC et ARAGO.

TOME QUINZIÈME.

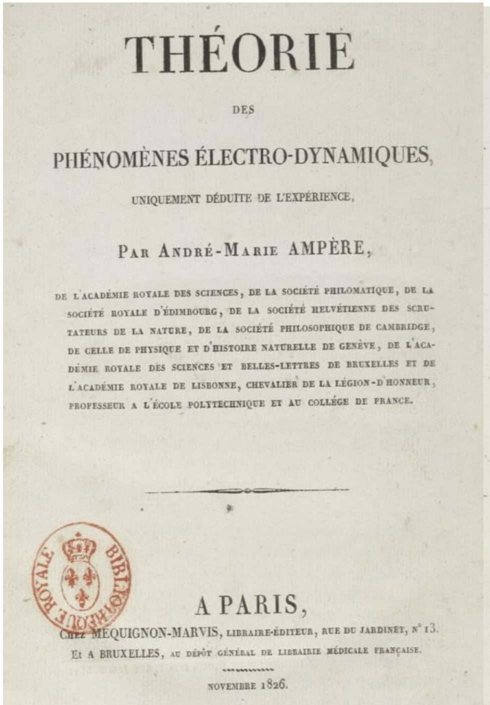
A PARIS,
Chez CROCHARD, Libraire, rue du Cloître Saint-Benoît,
n° 16, près celle des Mathurins.
1820.



ESSAI
SUR
LA PHILOSOPHIE
DES SCIENCES,
OU
EXPOSITION ANALYTIQUE D'UNE CLASSIFICATION
NATURELLE DE TOUTES LES CONNAISSANCES
HUMAINES;
PAR
ANDRÉ-MARIE AMPÈRE,
De l'Académie royale des sciences, des Sociétés royales de Londres et d'Edimbourg, de la Société philomatique, de la Société helvétienne des scrutateurs de la nature, de la Société philosophique de Cambridge, de celle de physique et d'histoire naturelle de Genève, de la Société Italienne, de l'Académie royale des sciences et belles-lettres de Bruxelles, de l'Académie royale de Lisbonne, des Académies de Lyon, de Modène, de Lille, Correspondant de l'Académie des sciences de Berlin et de l'Institut de Bologne, Membre de plusieurs autres Sociétés savantes, Chevalier de la légion d'honneur, Inspecteur général des études, et Professeur au College de France.

A PARIS,
CHEZ BACHELIER, IMPRIMEUR-LIBRAIRE POUR LES SCIENCES,
Quai des Augustins, n° 55.
—
1834.

In 1826 he published his stunning success, *Mémoire sur la théorie mathématique des phénomènes électrodynamiques uniquement déduite de l'experience* "Memoir on the Mathematical Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience". After this work the term "electrodynamics" become common in physics. It was deduced from his four experiments in electricity and magnetism. In this book he represented a mathematical derivation of the electrodynamic force law based on experiments for the first time.



THÉORIE

DES

PHÉNOMÈNES ÉLECTRO-DYNAMIQUES,

UNIQUEMENT DÉDUITE DE L'EXPÉRIENCE,

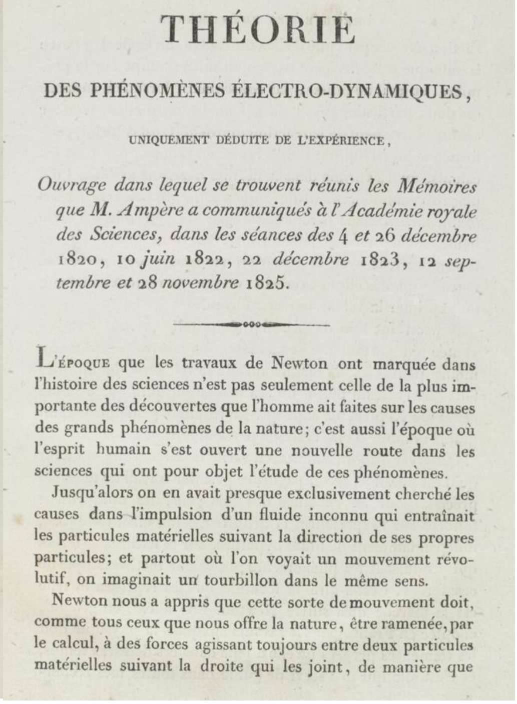
PAR ANDRÉ-MARIE AMPÈRE,

DE L'ACADÉMIE ROYALE DES SCIENCES, DE LA SOCIÉTÉ PHILOMATIQUE, DE LA SOCIÉTÉ ROYALE D'ÉDIMBOURG, DE LA SOCIÉTÉ HELVÉTIENNE DES SCRUTATEURS DE LA NATURE, DE LA SOCIÉTÉ PHILOSOPHIQUE DE CAMBRIDGE, DE CELLE DE PHYSIQUE ET D'HISTOIRE NATURELLE DE GENÈVE, DE L'ACADÉMIE ROYALE DES SCIENCES ET BELLES-LETTRES DE BRUXELLES ET DE L'ACADÉMIE ROYALE DE LISBONNE, CHEVALIER DE LA LÉGION-D'HONNEUR, PROFESSEUR A L'ÉCOLE POLYTECHNIQUE ET AU COLLÉGE DE FRANCE.

BIBLIOTHÈQUE ROYALE

A PARIS,
Chez MÉQUIGNON-MARVIS, LIBRAIRE-ÉDITEUR, RUE DU JARDINET, N° 13.
Et A BRUXELLES, AU DÉPÔT GÉNÉRAL DE LIBRAIRIE MÉDICALE FRANÇAISE.

NOVEMBRE 1826.



THÉORIE

DES PHÉNOMÈNES ÉLECTRO-DYNAMIQUES,

UNIQUEMENT DÉDUITE DE L'EXPÉRIENCE,

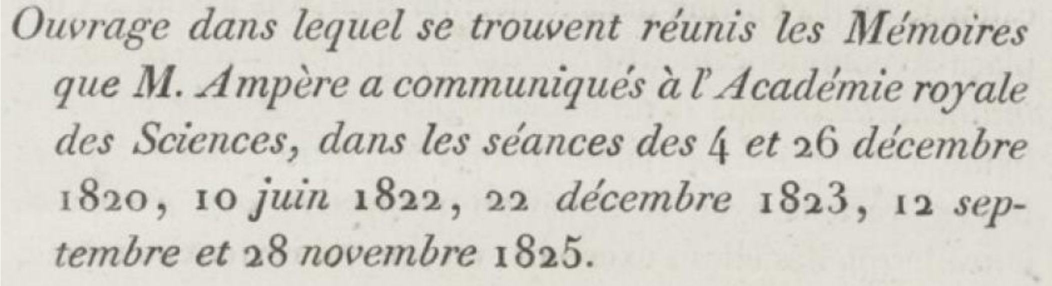
Ouvrage dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances des 4 *et* 26 *décembre* 1820, 10 *juin* 1822, 22 *décembre* 1823, 12 *septembre et* 28 *novembre* 1825.

L'ÉPOQUE que les travaux de Newton ont marquée dans l'histoire des sciences n'est pas seulement celle de la plus importante des découvertes que l'homme ait faites sur les causes des grands phénomènes de la nature; c'est aussi l'époque où l'esprit humain s'est ouvert une nouvelle route dans les sciences qui ont pour objet l'étude de ces phénomènes.

Jusqu'alors on en avait presque exclusivement cherché les causes dans l'impulsion d'un fluide inconnu qui entraînait les particules matérielles suivant la direction de ses propres particules; et partout où l'on voyait un mouvement révolutif, on imaginait un tourbillon dans le même sens.

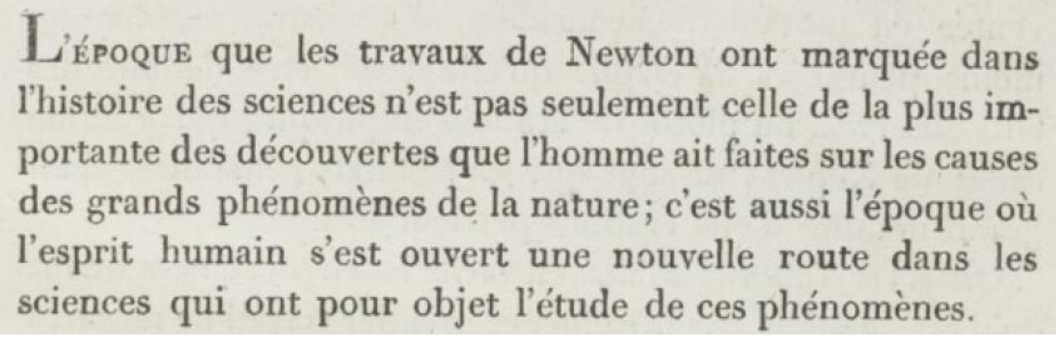
Newton nous a appris que cette sorte de mouvement doit, comme tous ceux que nous offre la nature, être ramenée, par le calcul, à des forces agissant toujours entre deux particules matérielles suivant la droite qui les joint, de manière que

This was in fact, a mathematical description of his " gathered Memoirs that communicated to the Royal Academy of Sciences, in the sessions of December 4 and 26, 1820, June 10, 1822, December 22, 1823, September 12 and November 28, 1825".



Ouvrage dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances des 4 *et* 26 *décembre* 1820, 10 *juin* 1822, 22 *décembre* 1823, 12 *septembre et* 28 *novembre* 1825.

Here he starts with Newton and says;



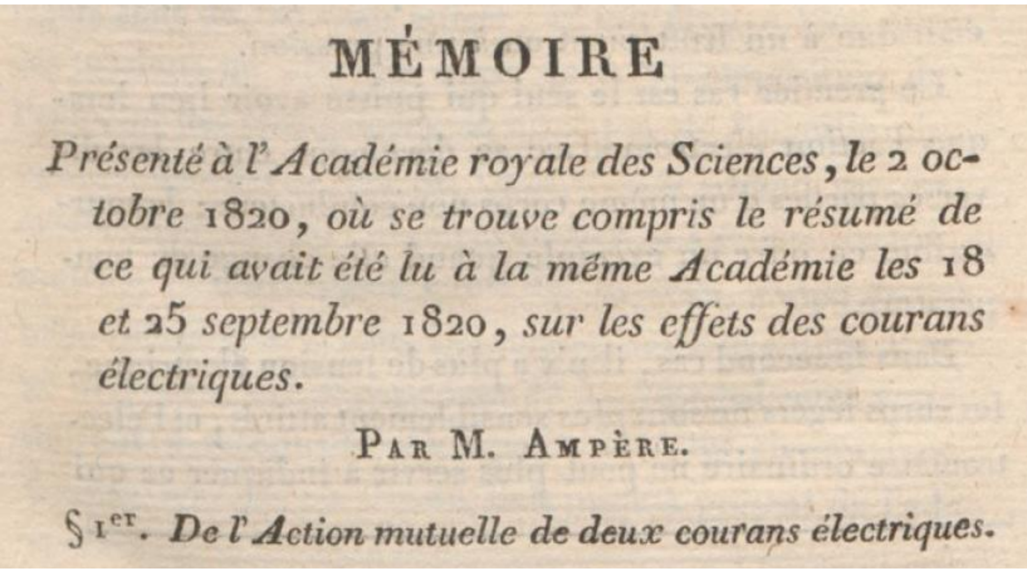
L'ÉPOQUE que les travaux de Newton ont marquée dans l'histoire des sciences n'est pas seulement celle de la plus importante des découvertes que l'homme ait faites sur les causes des grands phénomènes de la nature; c'est aussi l'époque où l'esprit humain s'est ouvert une nouvelle route dans les sciences qui ont pour objet l'étude de ces phénomènes.

> The time marked by Newton's work in the history of the sciences is not only that of the most important of the discoveries that man has made on the causes of the great phenomena of nature; it is also the time when the human mind opened up a new path in the sciences which have as their object the study of these phenomena.

He was a "key contributor to modern physics". In fact, he knotted experiments, theory and mathematics in electromagnetism. In his *A Treatise on Electricity and Magnetism* J. C. Maxwell, describe Ampere's works as;

> The experimental investigation by which Ampere established the law of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the “Newton of Electricity”. It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics.

Chasing Oersted's works, he showed that "two parallel wires carrying electric currents attract or repel each other, depending on whether the currents flow in the same or opposite directions, respectively". This is the most fundamental foundation of electrodynamics. In *Annales de Chimie et de Physique* 1820 he released a work on "effects of electric currents" which was presented before to *à l'Académie royale des Sciences*.



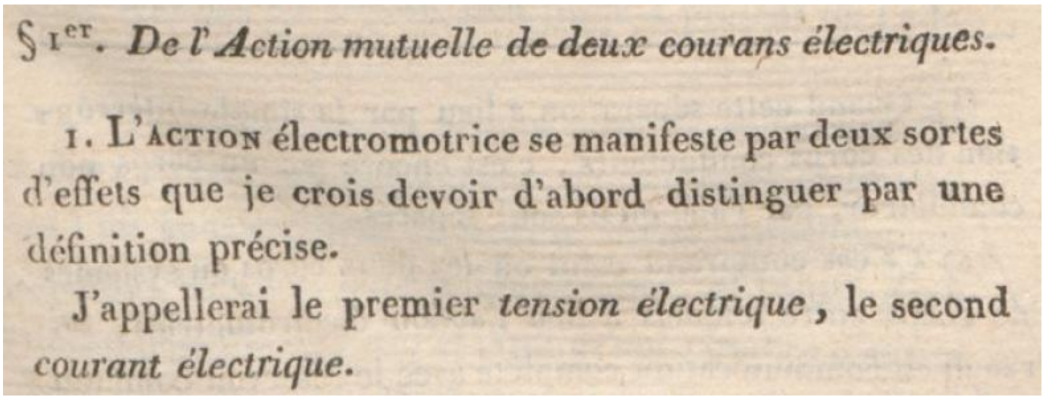
MÉMOIRE

Présenté à l'Académie royale des Sciences, le 2 octobre 1820, où se trouve compris le résumé de ce qui avait été lu à la même Académie les 18 et 25 septembre 1820, sur les effets des courans électriques.

PAR M. AMPÈRE.

§ 1er. *De l'Action mutuelle de deux courans électriques.*

He starts as;



§ 1er. *De l'Action mutuelle de deux courans électriques.*

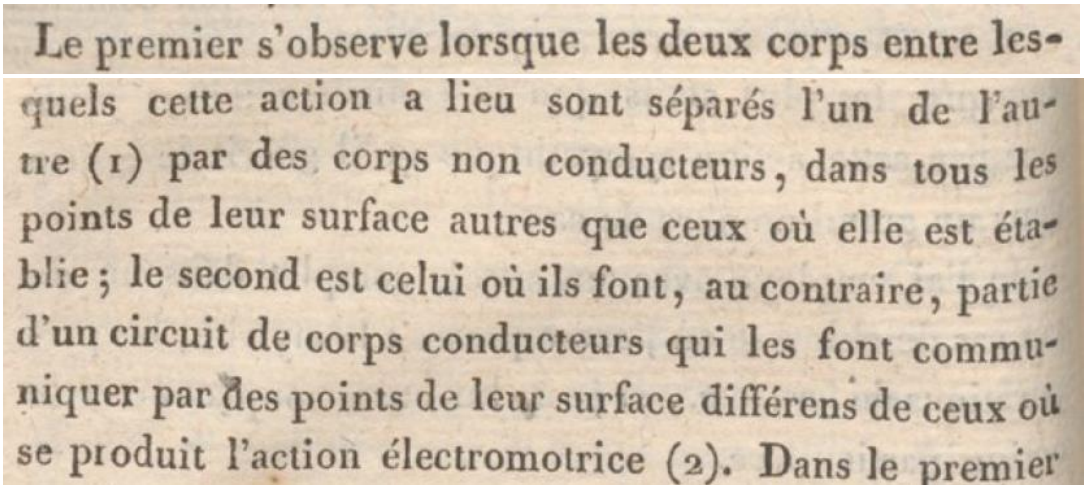
1. L'Action électromotrice se manifeste par deux sortes d'effets que je crois devoir d'abord distinguer par une définition précise.

J'appellerai le premier *tension électrique*, le second *courant électrique*.

Of the mutual action of two electric currents.

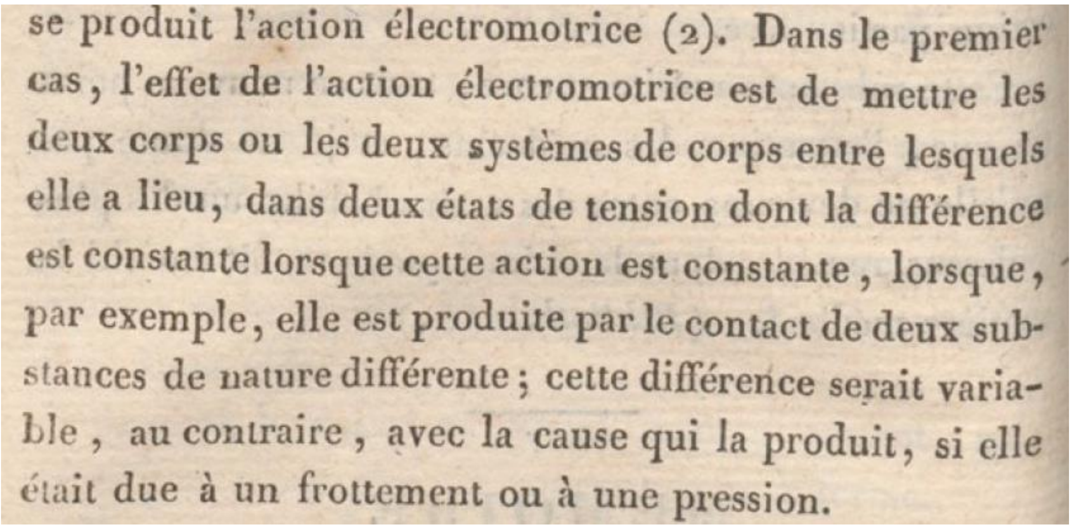
1. The electromotive action is manifested by two sorts of effects which I think I should first distinguish by a precise definition. I will call the first *electric voltage*, the second *electric current*.

Then he continues to describe and summarizes his observations as;



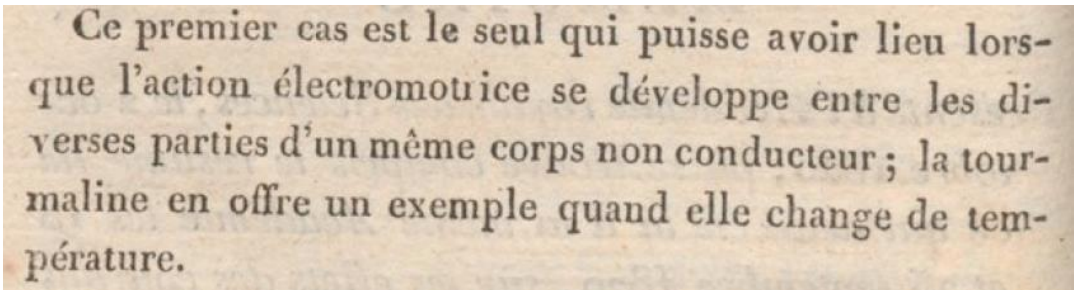
Le premier s'observe lorsque les deux corps entre lesquels cette action a lieu sont séparés l'un de l'autre (1) par des corps non conducteurs, dans tous les points de leur surface autres que ceux où elle est établie; le second est celui où ils font, au contraire, partie d'un circuit de corps conducteurs qui les font communiquer par des points de leur surface différens de ceux où se produit l'action électromotrice (2). Dans le premier

The first is observed when the two bodies between which this action takes place are separated from each other (1; When this separation takes place by the simple interruption of the conducting bodies, it is still by a conducting body, not by the air, that they are separated) by non-conductive bodies, in all points of their surface other than those where it is established; the second is that where they form, on the contrary, part of a circuit of conductive bodies which make them communicate by points of their surface different from those where the electromotive action is produced(2; This case includes that in which the two bodies or systems of bodies between which the electromotive action takes place would be in complete communication with the commutated reservoir which would then form part of the circuit).



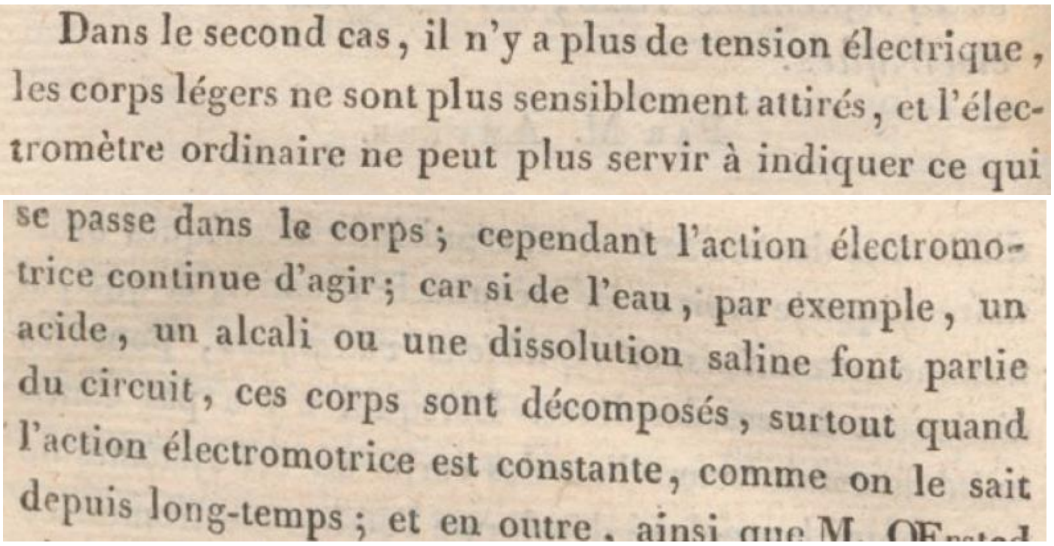
se produit l’action électromotrice (2). Dans le premier cas, l’effet de l’action électromotrice est de mettre les deux corps ou les deux systèmes de corps entre lesquels elle a lieu, dans deux états de tension dont la différence est constante lorsque cette action est constante, lorsque, par exemple, elle est produite par le contact de deux substances de nature différente ; cette différence serait variable, au contraire, avec la cause qui la produit, si elle était due à un frottement ou à une pression.

In the first case, the effect of the electromotive action is to put the two bodies or the two systems of bodies between which it takes place, in two states of tension whose difference is constant when this action is constant, when, for example, it is produced by the contact of two substances of a different nature; this difference would be variable, on the contrary, with the cause which produces it, if it were due to a friction or to a pressure.



Ce premier cas est le seul qui puisse avoir lieu lorsque l’action électromotrice se développe entre les diverses parties d’un même corps non conducteur ; la tourmaline en offre un exemple quand elle change de température.

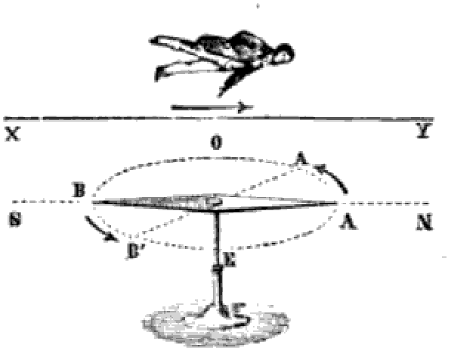
This first case is the only one which can take place when the electromotive action develops between the different parts of the same non-conductive body; tourmaline offers an example of this when it changes temperature.

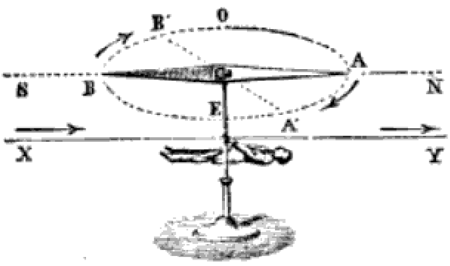


Dans le second cas, il n'y a plus de tension électrique, les corps légers ne sont plus sensiblement attirés, et l'électromètre ordinaire ne peut plus servir à indiquer ce qui se passe dans le corps; cependant l'action électromotrice continue d'agir; car si de l'eau, par exemple, un acide, un alcali ou une dissolution saline font partie du circuit, ces corps sont décomposés, surtout quand l'action électromotrice est constante, comme on le sait depuis long-temps; et en outre, ainsi que M. OErsted

In the second case, there is no longer any electric tension, light bodies are no longer sensibly attracted, and the ordinary electrometer can no longer be used to indicate what is happening in the body; however the electromotive action continues to act; for if water, for example, an acid, an alkali or a saline solution form part of the circuit, these bodies are decomposed, especially when the electromotive action has since been constant, as we have long known.

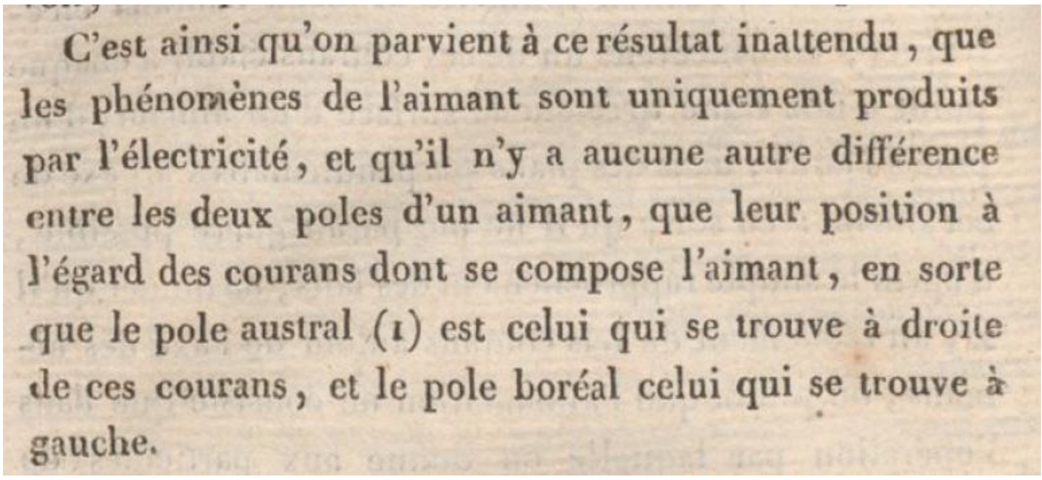
According to part III of Deschanel's Elementary *Treaties on natural philosophy*, , Ampere's law could be visualized as; "an observer". He observes the current passes through his feet and leaving his head from austral (south) to the boreal (north) , then he sees "the deflection of a north - seeking pole" (A) to the left while the deflection of a south - seeking pole (B) to the right.





This is ampere's rule for the "direction in which either pole of a needle is deflected by a current".

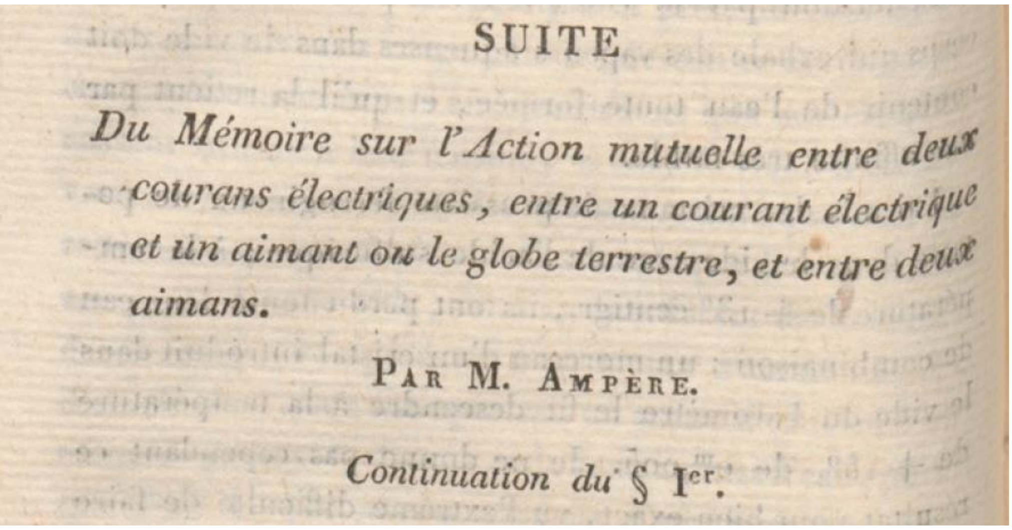
In the end of his memoire "Of the mutual action of two electric currents" which we are considering now he concludes that;



C'est ainsi qu'on parvient à ce résultat inattendu, que les phénomènes de l'aimant sont uniquement produits par l'électricité, et qu'il n'y a aucune autre différence entre les deux poles d'un aimant, que leur position à l'égard des courans dont se compose l'aimant, en sorte que le pole austral (1) est celui qui se trouve à droite de ces courans, et le pole boréal celui qui se trouve à gauche.

This is how we arrive at this unexpected result, that the phenomena of the magnet are solely produced by electricity, and that there is no other difference between the two poles of a magnet, than their position with respect to the currents of which the magnet is composed, so that the *austral pole* is that which is to the right of these currents, and the *boreal pole* that which is to the left.

In continuation of this Memoire he published "Continuation of the Memoir on the Mutual Action between two Electric Currents, between an Electric Current and a Magnet or the Terrestrial Globe, and between two Magnets" on the same journal namely, *Annales de Chimie et de Physique* 1820.



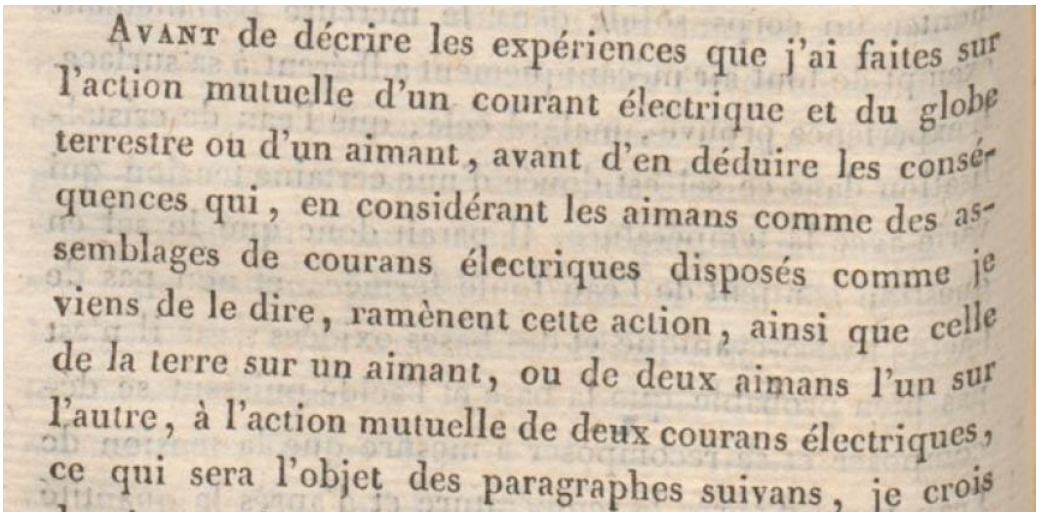
SUITE

Du Mémoire sur l'Action mutuelle entre deux courans électriques, entre un courant électrique et un aimant ou le globe terrestre, et entre deux aimans.

PAR M. AMPÈRE.

Continuation du § I$^{\text{er}}$.

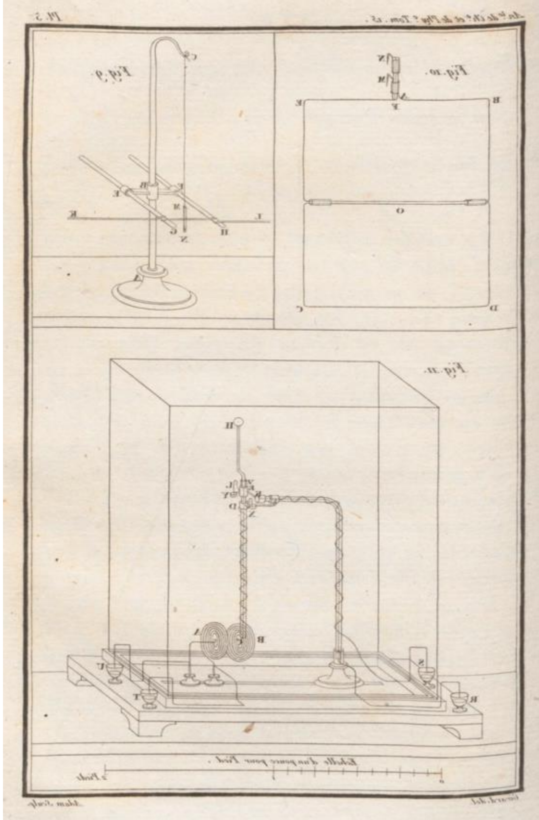
In the beginning he starts to describes his additional experiments and their results to those he did before succinctly as;

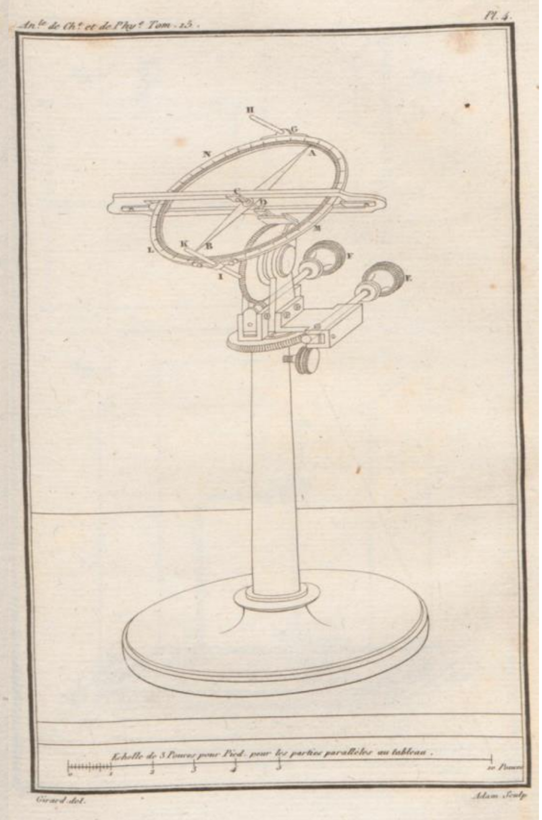


Avant de décrire les expériences que j'ai faites sur l'action mutuelle d'un courant électrique et du globe terrestre ou d'un aimant, avant d'en déduire les conséquences qui, en considérant les aimans comme des assemblages de courans électriques disposés comme je viens de le dire, ramènent cette action, ainsi que celle de la terre sur un aimant, ou de deux aimans l'un sur l'autre, à l'action mutuelle de deux courans électriques, ce qui sera l'objet des paragraphes suivans, je crois

Before describing the experiments I made on the mutual action of an electric current and the terrestrial globe or a magnet, before deducing the consequences which, considering magnets as assemblies of electric currents arranged as I have just said, reduce this action, as well as that of the earth on a magnet, or of two magnets one on the other, to the mutual action of two electric currents, which will be the object of following paragraphs.

Then he continues to describe his first experimental devices schematically as below in plots and detailed explanations.




Ann. de Ch. et de Phys. Tom. 15.
Pl. 4.
H
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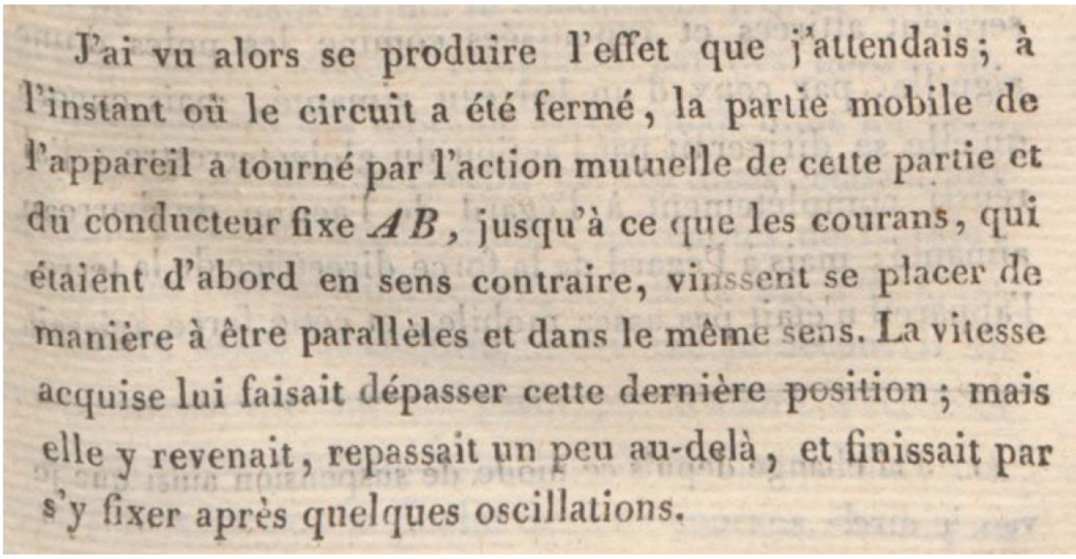
E
Echelle de 3 Pouces pour Pied, pour les parties parallèles au tableau.
Girard del.
Adam Sculp.

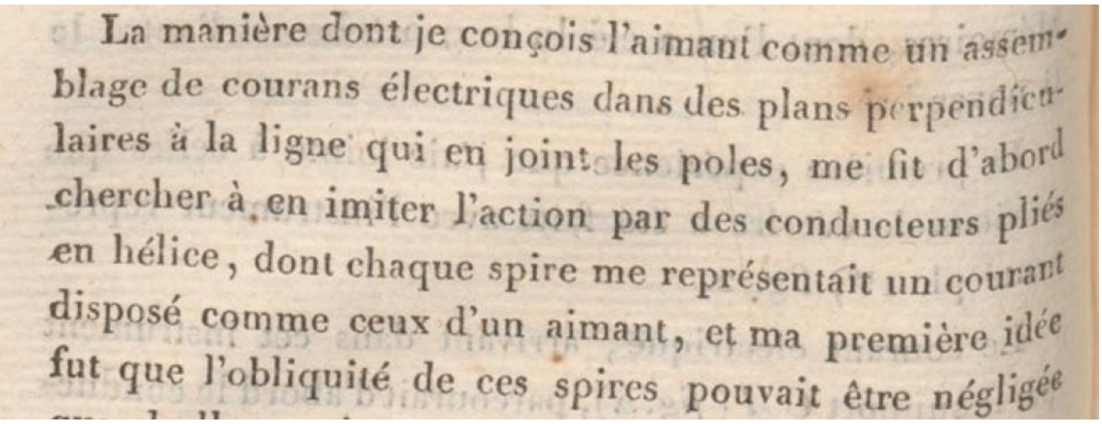
His repeated four experiments with these apparatuses led him to mathematically formulate the magnetic force between two parallel electrical currents as we know as *Ampere's law*. In order to do his experiments, he develops measuring techniques for electricity with a compass-like instrument with a "free moving magnetized needle to measure electrical flow. Today it is known as galvanometer.

He explains his observations as;

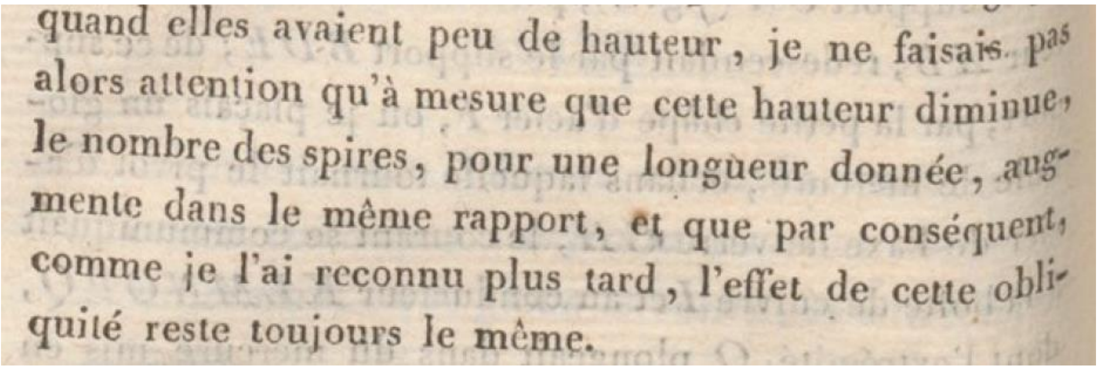


J'ai vu alors se produire l'effet que j'attendais ; à l'instant où le circuit a été fermé, la partie mobile de l'appareil a tourné par l'action mutuelle de cette partie et du conducteur fixe *AB*, jusqu'à ce que les courans, qui étaient d'abord en sens contraire, vinssent se placer de manière à être parallèles et dans le même sens. La vitesse acquise lui faisait dépasser cette dernière position ; mais elle y revenait, repassait un peu au-delà, et finissait par s'y fixer après quelques oscillations.

Then saw the effect that I expected take place; at the instant when the circuit was closed, the mobile part of the apparatus rotated by the mutual action of this part and of the fixed conductor AB (the first plot), until the currents, which were at first in the opposite direction, came to a position so as to be parallel and in the same direction. The acquired speed made him (the fixed conductor AB) exceed this last position; but it returned to it, passed a little beyond it, and ended by settling there after a few oscillations.



La manière dont je conçois l'aimant comme un assemblage de courans électriques dans des plans perpendiculaires à la ligne qui en joint les poles, me fit d'abord chercher à en imiter l'action par des conducteurs pliés en hélice, dont chaque spire me représentait un courant disposé comme ceux d'un aimant, et ma première idée fut que l'obliquité de ces spires pouvait être négligée



quand elles avaient peu de hauteur, je ne faisais pas alors attention qu'à mesure que cette hauteur diminue, le nombre des spires, pour une longueur donnée, augmente dans le même rapport, et que par conséquent, comme je l'ai reconnu plus tard, l'effet de cette obliquité reste toujours le même.

The way I conceive the magnet as an assemblage of electric currents in planes, perpendicular to the line which joins the poles, first made me try to imitate their action by conductors bent in a helix, each turn of which represented to me a current arranged like those of a magnet, and my first idea was that the obliquity of these whorls could be neglected when they had little height, I did not pay attention then that *as this height decreases, number of whorls, for a given length, increases in the same relation*, and that therefore, as I later recognized, the effect of this obliquity always remains the same.

He applied mathematics to generalize these experiments as physical laws. For instance, the so-called Ampere's law proposed that, "the mutual action of two lengths of current-carrying wire is proportional to their lengths and to the intensities of their currents". In applying this law to magnetism, he showed that there exists a harmony between his law and Coulomb's law of electric action.

His works in newborn electrodynamics paved the way for manifestation of the idea of "electrodynamic molecule" later called electron. This elementary particle was shown to be the "component element" of the phenomena of electricity and magnetism. Similar to Newton's predictive formulation of mechanical motion, Ampere provided a simultaneous physical and mathematical explanation of electromagnetic motion as well. He established a physical interpretation of electromagnetic phenomena which was "empirically demonstrable and mathematically predictive".

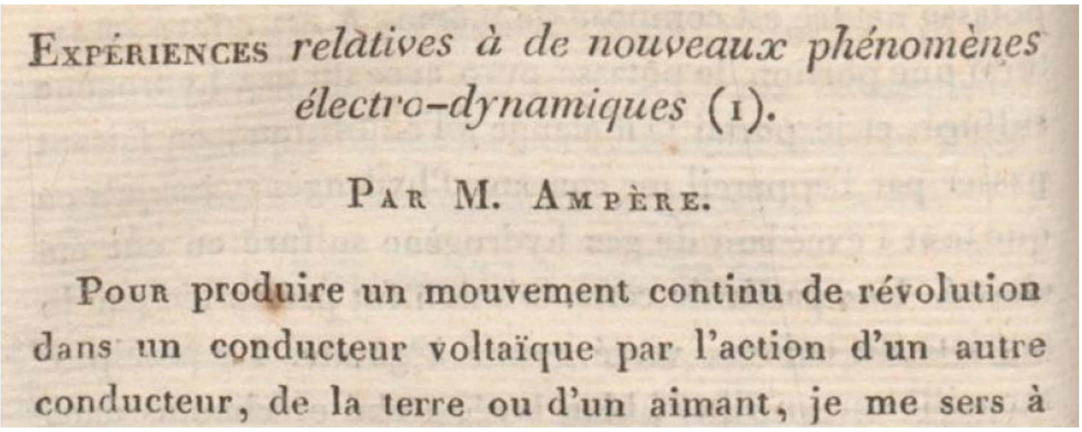
As we considered before the final definition of fundamental law of electricity proposed by Charles Coulomb was;

"*The electric action, whether repulsive or attractive, of the two electrified spheres, and therefore of two electrified molecules, is in the ratio compounded of the densities of the electric fluid of the two electrified molecules and inversely as the square of the distances*".

Coulomb did not show the proportionality of electric force to the product of the charges experimentally or mathematically. He simply proposed that this is true. But Ampere Firstly, supposed an attractive force as positive and a repulsive force as negative. Then he showed that, the electrostatic force between these electrified bodies is proportional to $-\frac{qq'}{r^2}$. This is also hold in magnetic force between two magnetic poles as $-\frac{pp'}{r^2}$.

We mentioned that, it was Oersted who discovered and characterized the existence of an interaction between a current-carrying wire and a magnetic needle. He also proposed two terms of *electromagnetic* and *electromagnetism*. Following Oersted it was Ampere who discovered the existence of the attractions and repulsions between current carrying conductors in 1820.

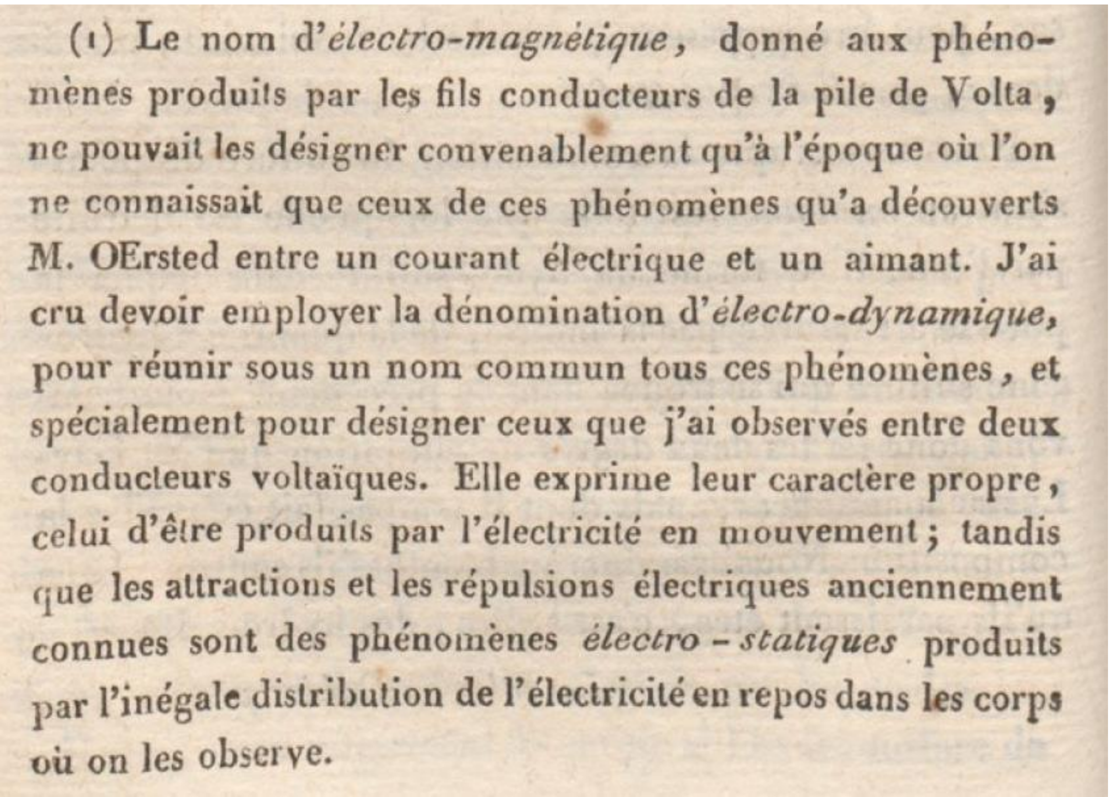
He proposed two terms for attractions and repulsions among electrified bodies. The terms *electrostatic* for electrified bodies at rest relative to one another and *electrodynamic* for current-carrying bodies such as wires were coined by him. In his article *Expérences relàtives à de nouveaux phénomènes électro-dynamiques* published in *Annales de chimie et de physique* 1822 he units these separate terms under a common name namely electrodynamic.



EXPÉRIENCES *relàtives à de nouveaux phénomènes électro-dynamiques* (1).

PAR M. AMPÈRE.

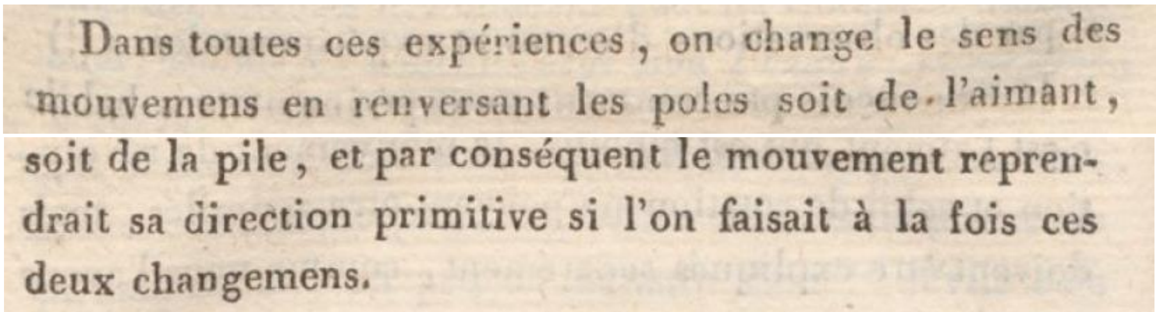
POUR produire un mouvement continu de révolution dans un conducteur voltaïque par l'action d'un autre conducteur, de la terre ou d'un aimant, je me sers à he explains these terms as;



(1) Le nom d'*électro-magnétique*, donné aux phénomènes produits par les fils conducteurs de la pile de Volta, ne pouvait les désigner convenablement qu'à l'époque où l'on ne connaissait que ceux de ces phénomènes qu'a découverts M. OErsted entre un courant électrique et un aimant. J'ai cru devoir employer la dénomination d'*électro-dynamique*, pour réunir sous un nom commun tous ces phénomènes, et spécialement pour désigner ceux que j'ai observés entre deux conducteurs voltaïques. Elle exprime leur caractère propre, celui d'être produits par l'électricité en mouvement; tandis que les attractions et les répulsions électriques anciennement connues sont des phénomènes *électro-statiques* produits par l'inégale distribution de l'électricité en repos dans les corps où on les observe.

The name electro-magnetic, given to the phenomena produced by the conducting wires of the pile of Volta, could only designate them suitably at the time when we knew only those of these phenomena which Mr. Oersted discovered between an electric current and a magnet. I thought I should use the term electro-dynamics to unite all these phenomena under a common name, and especially to designate those which I have observed between two voltaic conductors. It expresses their own character, that of being produced by electricity in motion; while the electrical attractions and repulsions formerly known are electrostatic phenomena produced by the unequal distribution of electricity at rest in the bodies where they are observed.

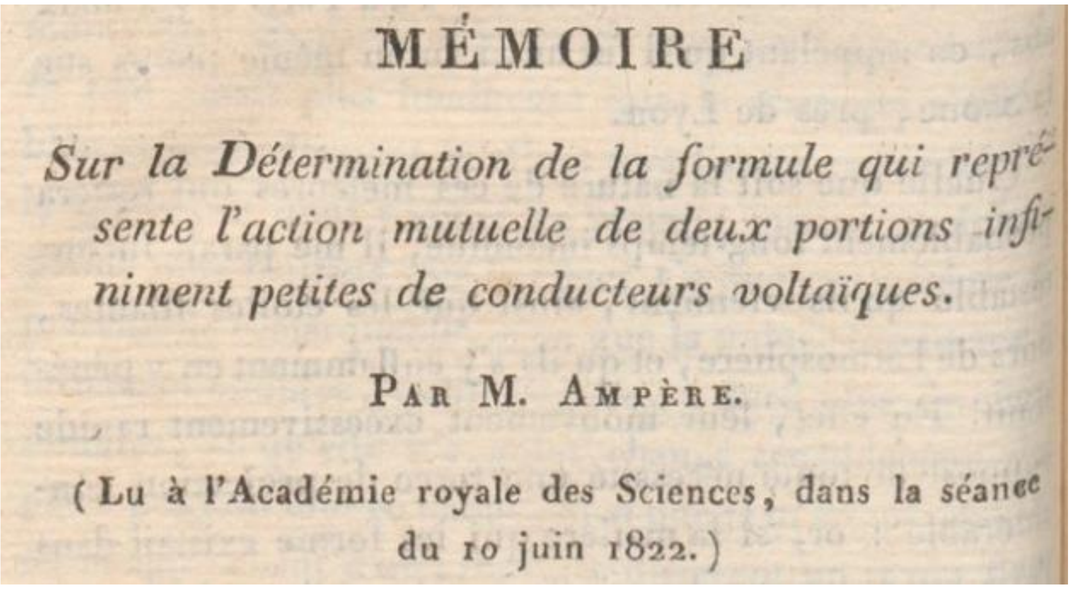
In the end he states that



Dans toutes ces expériences, on change le sens des mouvemens en renversant les poles soit de l'aimant, soit de la pile, et par conséquent le mouvement reprendrait sa direction primitive si l'on faisait à la fois ces deux changemens.

In all these experiments, the direction of the movements is changed by reversing the poles either of the magnet or of the battery, and consequently the movement would resume its original direction if these two changes were made at the same time.

In 1822 Ampere found his final expression for the force acting between two current elements separated by a distance *r*. He published his results in "On the determination of the formula which represents the mutual action of two infinitely small portions of voltaic conductors".



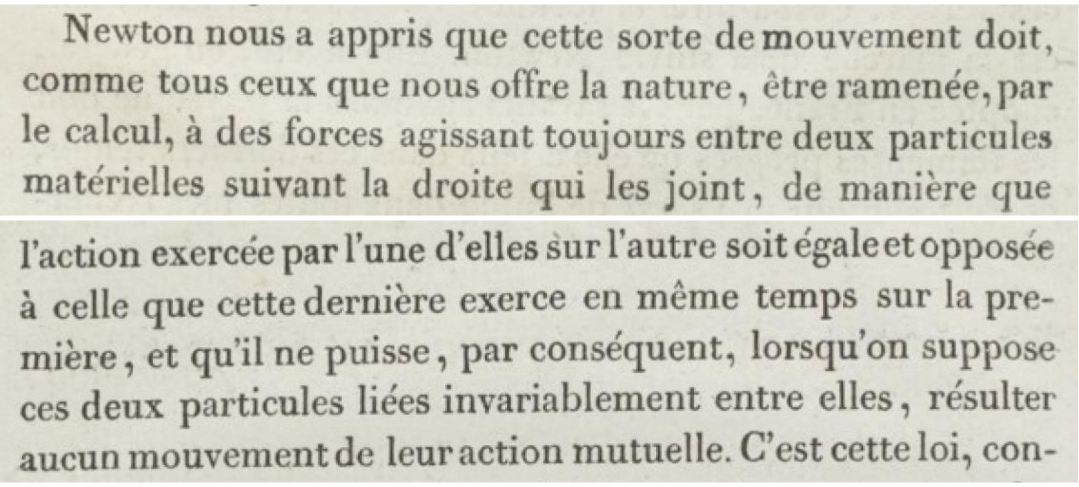
MÉMOIRE

Sur la Détermination de la formule qui représente l'action mutuelle de deux portions infiniment petites de conducteurs voltaïques.

PAR M. AMPÈRE.

(Lu à l'Académie royale des Sciences, dans la séance du 10 juin 1822.)

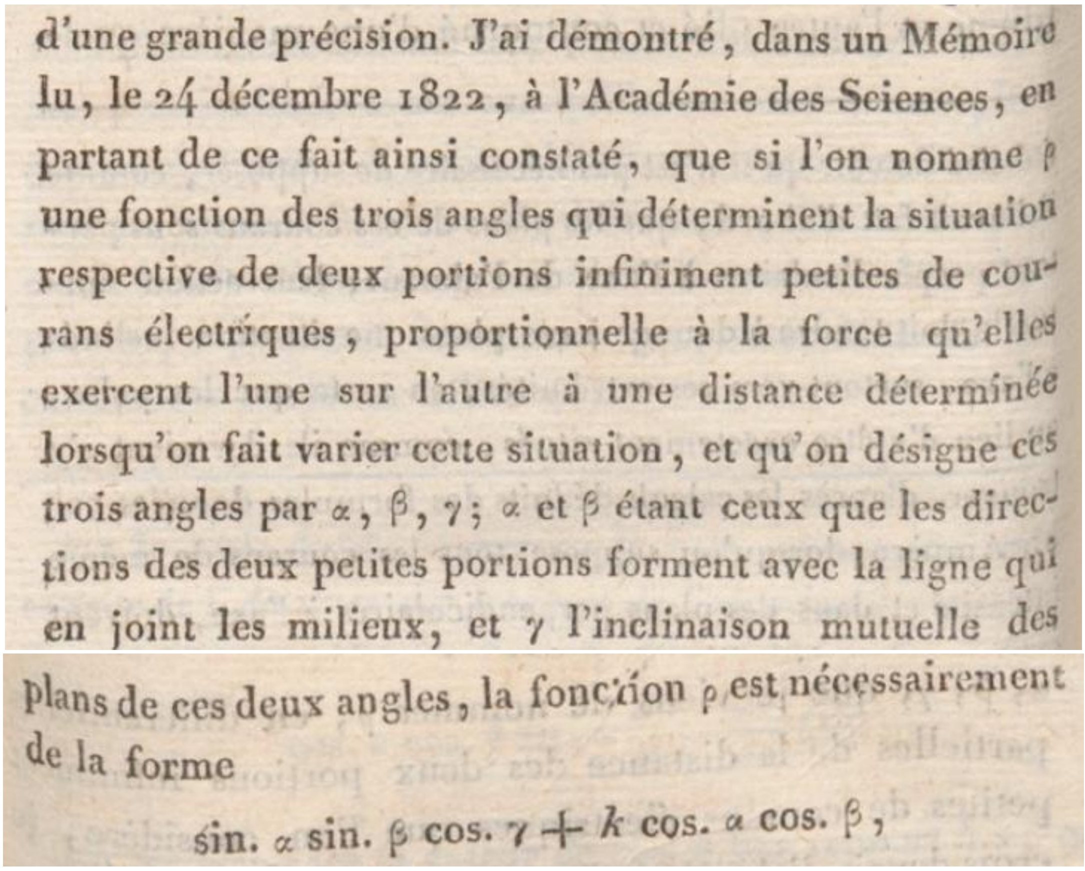
This formulation was based on previous efforts by him and others from Gilbert to Oersted. It is very similar to Newton formulation of Motion and gravity. He himself mention this fact in his "Theory of the Electrodynamic Phenomena, Uniquely Deduced from Experience" published in 1826.



Newton nous a appris que cette sorte de mouvement doit, comme tous ceux que nous offre la nature, être ramenée, par le calcul, à des forces agissant toujours entre deux particules matérielles suivant la droite qui les joint, de manière que l'action exercée par l'une d'elles sur l'autre soit égale et opposée à celle que cette dernière exerce en même temps sur la première, et qu'il ne puisse, par conséquent, lorsqu'on suppose ces deux particules liées invariablement entre elles, résulter aucun mouvement de leur action mutuelle. C'est cette loi, con-

Newton taught us that this kind of movement must, like all those that nature offers us, be reduced, by calculation, to forces always acting between two material particles following the straight line which joins them, so that the action exerted by one of them on the other is equal and opposite to that which the latter exerts at the same time on the first, and that it cannot, consequently, when one supposes these two particles bound invariably between them, result no movement of their mutual action.

We consider his original paper for this formulation published in 1822.



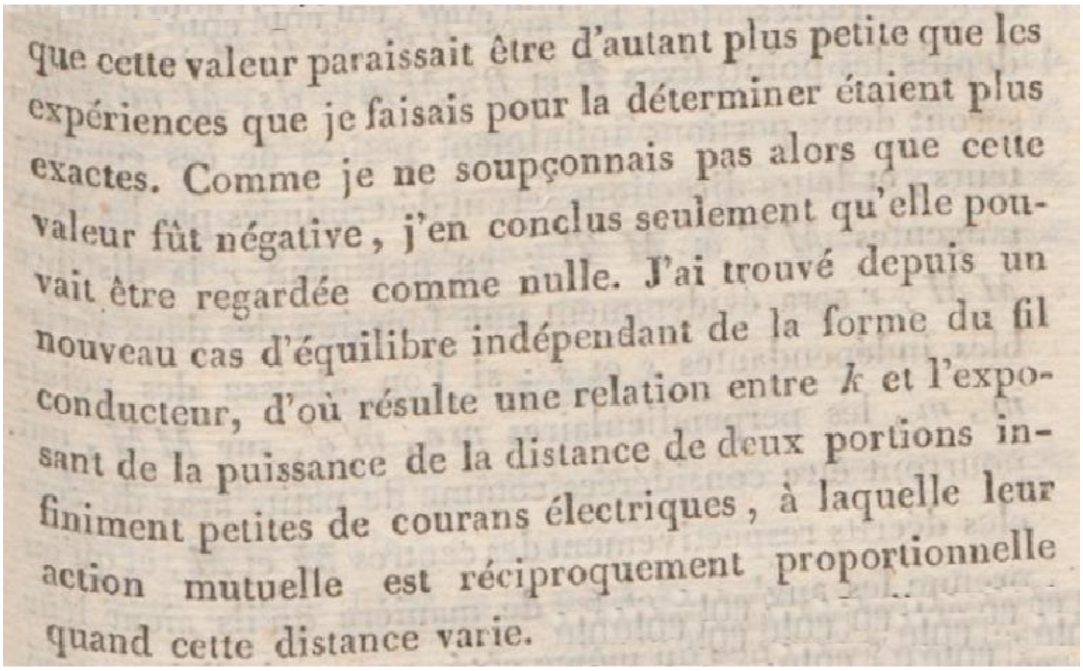
d'une grande précision. J'ai démontré, dans un Mémoire lu, le 24 décembre 1822, à l'Académie des Sciences, en partant de ce fait ainsi constaté, que si l'on nomme ρ une fonction des trois angles qui déterminent la situation respective de deux portions infiniment petites de courans électriques, proportionnelle à la force qu'elles exercent l'une sur l'autre à une distance déterminée lorsqu'on fait varier cette situation, et qu'on désigne ces trois angles par α, β, γ; α et β étant ceux que les directions des deux petites portions forment avec la ligne qui en joint les milieux, et γ l'inclinaison mutuelle des plans de ces deux angles, la fonction ρ est nécessairement de la forme

$$\sin.\,\alpha \sin.\,\beta \cos.\,\gamma + k \cos.\,\alpha \cos.\,\beta,$$

I demonstrated, in a memoir read on December 24, 1822, at the Academy of Sciences, starting from this fact thus established, that if we call ρ a function of the three angles which determine the respective situation of two portions infinitely small electric currents, proportional to the force they exert on each other at a determined distance when this situation is varied, and these three angles are designated by α, β, γ; α and β being those which the directions of the two small portions form with the line which joins their midpoints, and Gama the mutual inclination of the planes of these two angles, the function ρ is necessarily of the form

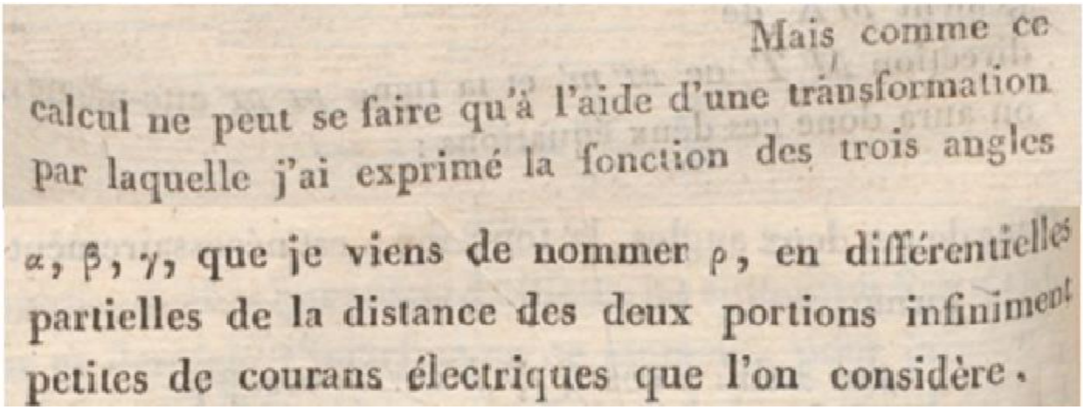
$$sin\,\alpha \sin\beta \;\; cos\gamma + k\cos\alpha\cos\beta$$

Here k is a constant coefficient denoted by him as, $k = \frac{n}{m} = -\frac{1}{2}$.



que cette valeur paraissait être d'autant plus petite que les expériences que je faisais pour la déterminer étaient plus exactes. Comme je ne soupçonnais pas alors que cette valeur fût négative, j'en conclus seulement qu'elle pouvait être regardée comme nulle. J'ai trouvé depuis un nouveau cas d'équilibre indépendant de la forme du fil conducteur, d'où résulte une relation entre k et l'exposant de la puissance de la distance de deux portions infiniment petites de courans électriques, à laquelle leur action mutuelle est réciproquement proportionnelle quand cette distance varie.

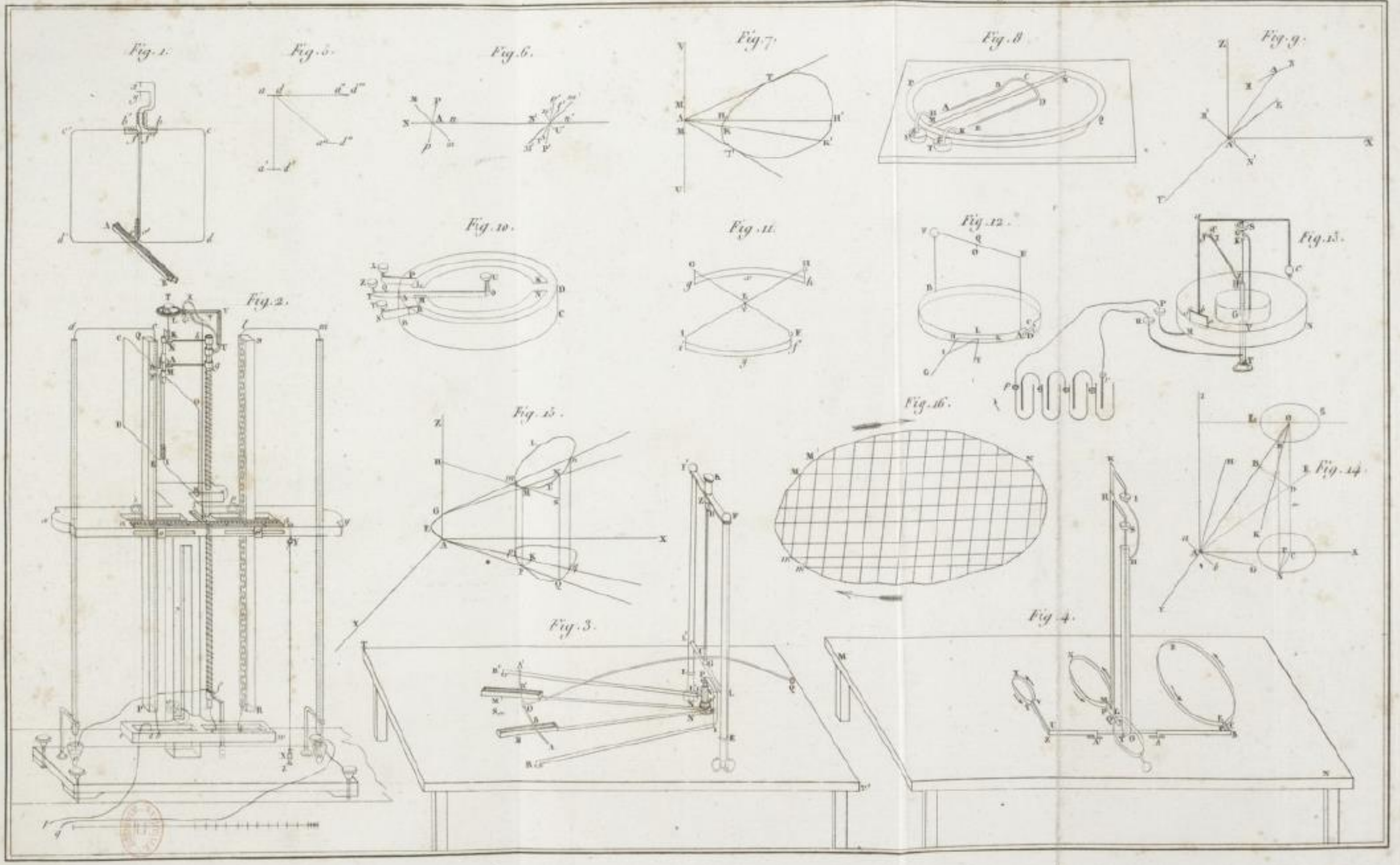
this value appeared to be all the smaller than the experiments which I made for more accurate determination. As I did not suspect then that this value was negative, I only concluded that it could be regarded as zero. Thus, I have found a new case of equilibrium independent of the shape of the conducting wire, which the results showed a relation between $\boldsymbol{k}$ and the exponent of the power of the distance of two infinitely small portions of electric currents, at which their mutual action is reciprocally proportional when this distance varies.



Mais comme ce calcul ne peut se faire qu'à l'aide d'une transformation par laquelle j'ai exprimé la fonction des trois angles α, β, γ, que je viens de nommer ρ, en différentielles partielles de la distance des deux portions infiniment petites de courans électriques que l'on considère.

But as this calculation can only be done with the aid of a transformation by which I expressed the function of three angles $\alpha, \beta\ \gamma$, which I have just named ρ, in partial differentials of the distance of the two infinitely small portions of electric currents which are considered.

Théorie des Phénomènes Électro-dynamiques. Pl. 1.

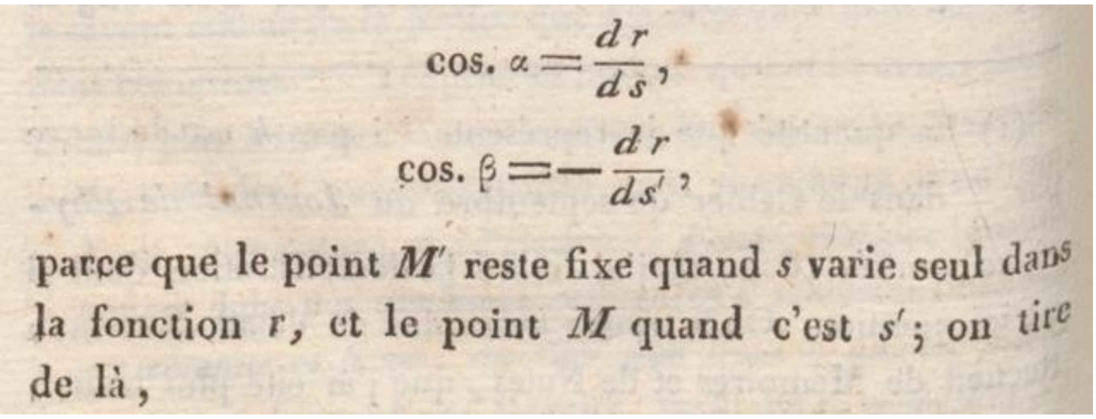


Then he considers this transformation to get the value of constant k for almost ten pages. we consider these pages with some eliminations of trivial parts.

Based on the figure 14 of the plot we have brought here he continues. If BM and B'M' be two lines representing conducting wires with S and S' represent their arcs counted from the fixed points B and B' then;

$$Mm = ds\ , M'm' = ds'$$

Would be two infinitely small portions of conducting wires in the directions of two tangents MT and $M'\ T'$. If (r) be the distance between M and M' then r is function of two independent variables s and s'. Now according to existing perpendicularity of lower points me $m'e'$, and changes in angles and arcs we have two equations.



$$\cos.\ \alpha = \frac{dr}{ds},$$

$$\cos.\ \beta = -\frac{dr}{ds'},$$

parce que le point M' reste fixe quand s varie seul dans la fonction r, et le point M quand c'est s'; on tire de là,

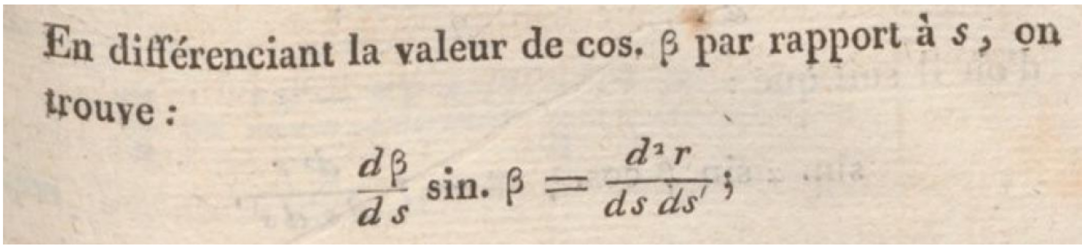
because the point M' remains fixed when s varies alone in the function r, and the point M when it is s'; we pull from there,

$$cos\alpha\ cos\beta = -\frac{dr}{ds} \cdot \frac{dr}{ds'}$$

While for opposite directions we get,

$$cos\alpha\ cos\beta = \frac{dr}{ds} \cdot \frac{dr}{ds'}$$

Now,



En différenciant la valeur de cos. β par rapport à s, on trouve :

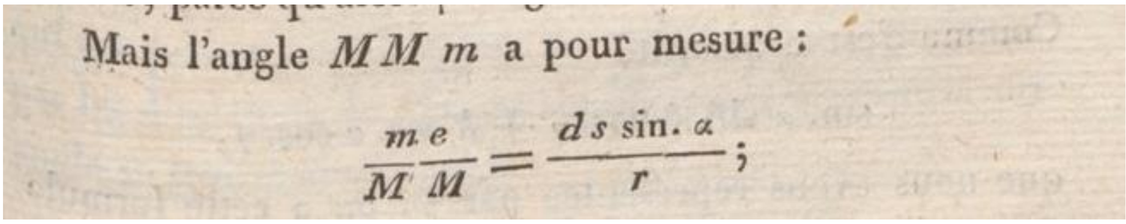
$$\frac{d\beta}{ds}\sin.\beta = \frac{d^2 r}{ds\,ds'};$$

Differentiating the value of cos, p with respect to s, we find;

$$\frac{d\beta}{ds}sin\beta = \frac{d^2r}{dsds'}$$

Where the angle β decreases and increase infinitesimally with s $(s + ds)$ as long as the angle γ such that,

$$d\beta = -MM' = m\,cos\,\gamma$$



Mais l'angle $MM\,m$ a pour mesure :

$$\frac{m\,e}{M\,M} = \frac{ds\,\sin.\alpha}{r};$$

But the angle MM m has the measure:

$$\frac{m}{M}\frac{e}{M} = \frac{ds\,sin\alpha}{r}$$

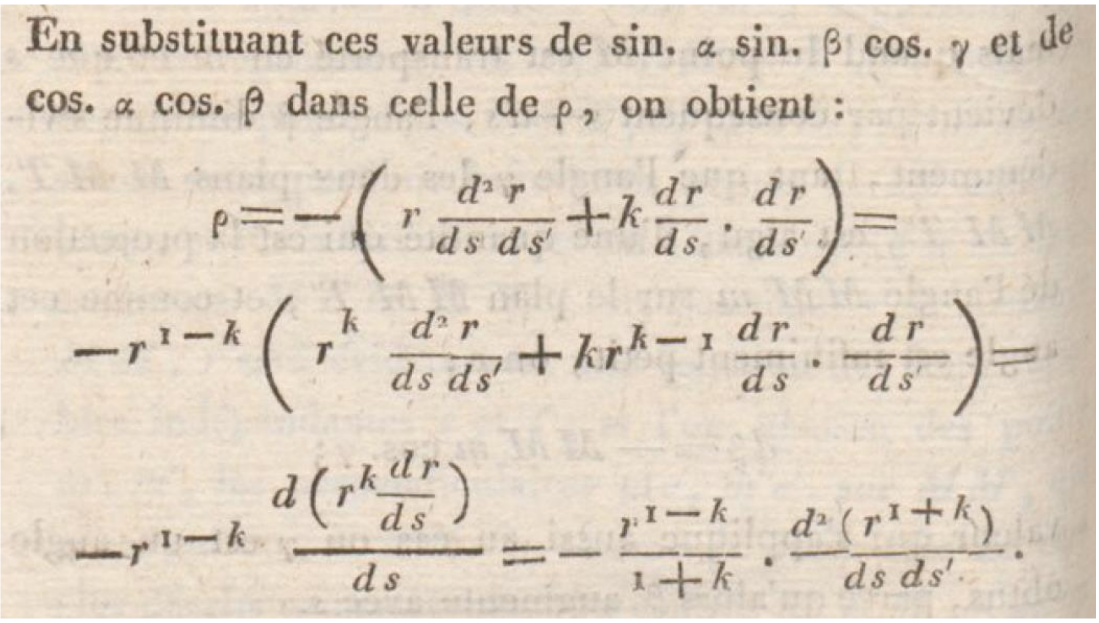
Therefore,

$$\frac{d\beta}{ds} = -\frac{sin\alpha\,cos\gamma}{\gamma}$$

From which it follows that,

$$sin\,\alpha\sin\beta\ \ cos\,\gamma = -r\frac{d^2r}{dsds'}$$

With substituting the value of $sin\,\alpha\sin\beta\ \ cos\,\gamma$ and $sin\,\alpha\sin\beta$ in the function ρ we get,



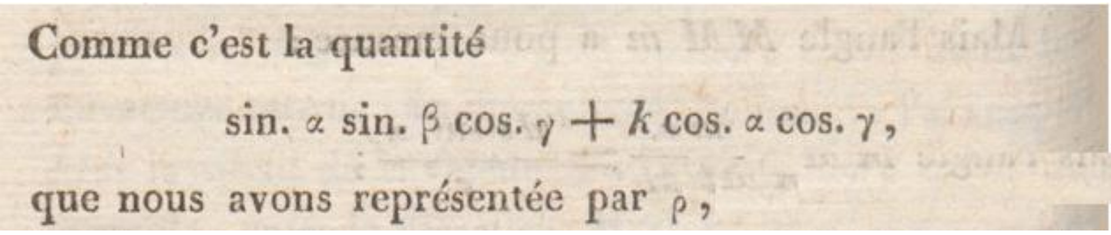
En substituant ces valeurs de sin. α sin. β cos. γ et de cos. α cos. β dans celle de ρ, on obtient :

$$\rho = -\left(r\frac{d^2 r}{ds\,ds'} + k\frac{dr}{ds}\cdot\frac{dr}{ds'}\right) =$$

$$-r^{1-k}\left(r^k\frac{d^2 r}{ds\,ds'} + kr^{k-1}\frac{dr}{ds}\cdot\frac{dr}{ds'}\right) =$$

$$-r^{1-k}\frac{d\left(r^k\frac{dr}{ds}\right)}{ds} = -\frac{r^{1-k}}{1+k}\cdot\frac{d^2(r^{1+k})}{ds\,ds'}.$$

And then,



Comme c'est la quantité

$$\text{sin. }\alpha\text{ sin. }\beta\text{ cos. }\gamma + k\text{ cos. }\alpha\text{ cos. }\gamma,$$

que nous avons représentée par ρ,

As in the quantities,

$$sin\ \alpha \sin\beta\ \ cos\gamma + k\cos\alpha\cos\beta$$

that we have represented by ρ.

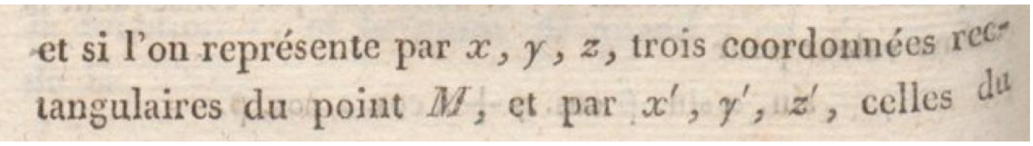
According to analytic trigonometry, we get,

$$in\ \alpha\sin\beta\ \ cos\gamma + k\cos\alpha\cos\beta = -\frac{r^{1-k}}{1+k}\,\frac{d^2(r^{1+1})}{ds\,ds'}$$

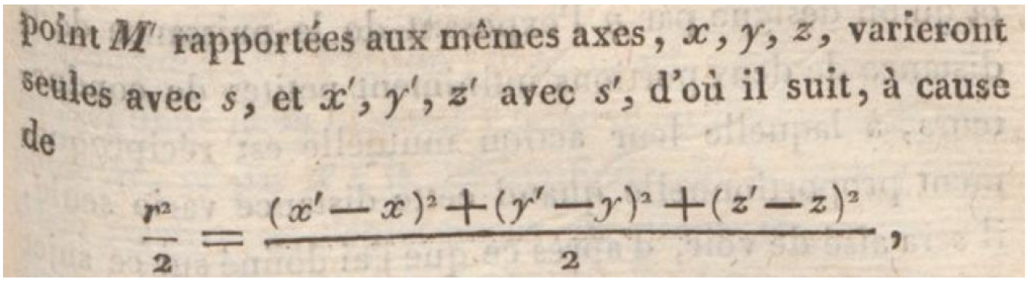
If we suppose $K = 1$ it becomes:

$$sin\ \alpha\sin\beta\ \ cos\gamma + \cos\alpha\cos\beta = -\frac{d(r/2)}{ds\,ds'}$$

Now,



et si l'on représente par x, y, z, trois coordonnées rectangulaires du point M, et par x', y', z', celles du



point M' rapportées aux mêmes axes, x, y, z, varieront seules avec s, et x', y', z' avec s', d'où il suit, à cause de

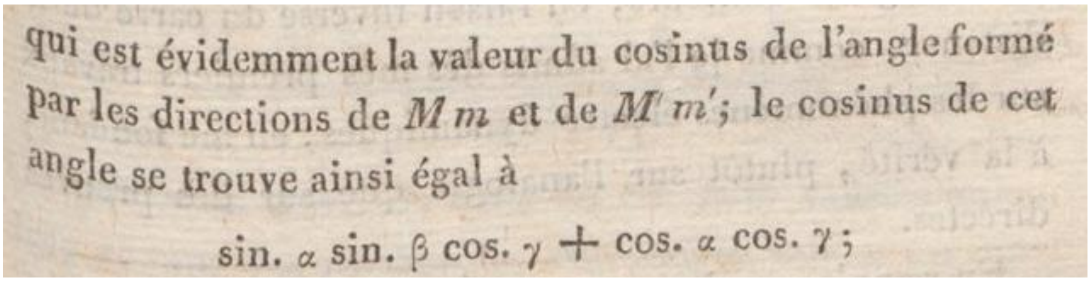
$$\frac{r^2}{2} = \frac{(x'-x)^2 + (y'-y)^2 + (z'-z)^2}{2},$$

And if we represent by x, y, z, three coordinates rectangular lines of point M, and by x', y', z', those of point M' referred to the same axes, x, y, z, will vary only with s, and x', y', z' with s', from which it follows, because

$$\frac{r^2}{2} = \frac{(x'-x)^r + (y'-y)^r + (z'-z)^r}{2}$$

With some differentiating we get,

$$sin\,\alpha \sin\beta\ \ cos\gamma + \cos\alpha\cos\beta = \frac{dx}{ds}\frac{dx'}{ds'} + \frac{dy}{ds}\frac{dxy'}{ds'} + \frac{dz}{ds}\frac{dz'}{ds'}$$



qui est évidemment la valeur du cosinus de l'angle formé par les directions de Mm et de $M'm'$; le cosinus de cet angle se trouve ainsi égal à

$$\sin.\ \alpha\ \sin.\ \beta\ \cos.\ \gamma + \cos.\ \alpha\ \cos.\ \gamma\,;$$

which is obviously the value of the cosine of the angle formed by the directions of Mm and $M'm'$; the cosine of this angle is thus found equal to

$$sin\,\alpha \sin\beta\ \ cos\gamma + \cos\alpha\cos\gamma$$

If we name i and i' the actions exerted at the distance 1 in the situation or

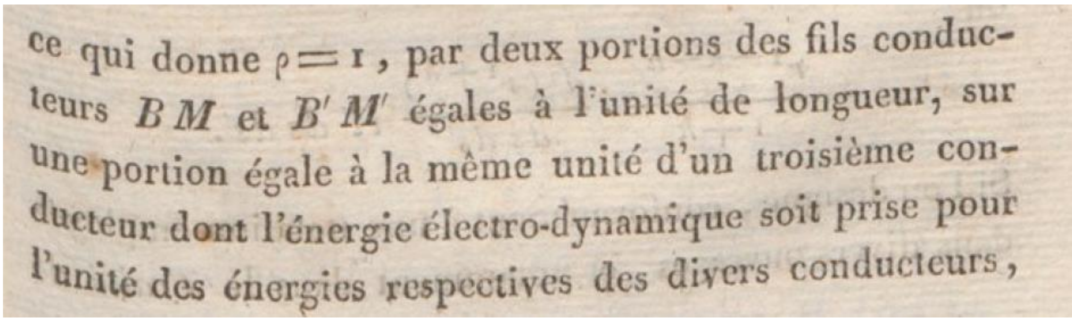
$$\alpha = \beta = \frac{\pi}{2}$$

Or

$$\gamma = 0$$

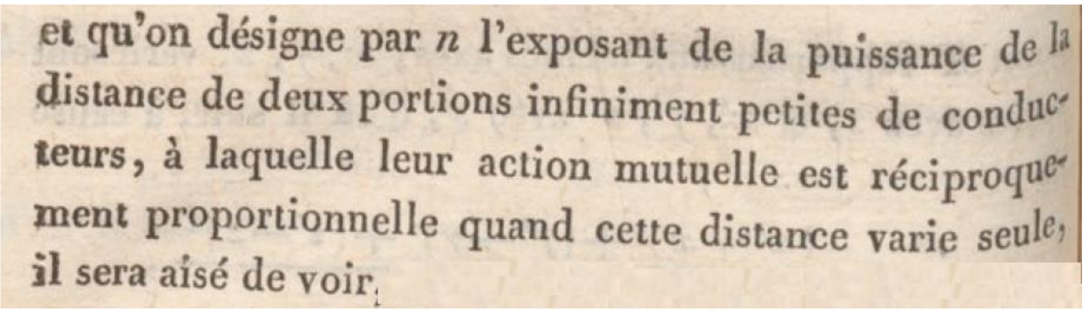
Which gives $\rho = 1$.

It means that,



ce qui donne $\rho = 1$, par deux portions des fils conducteurs *BM* et *B'M'* égales à l'unité de longueur, sur une portion égale à la même unité d'un troisième conducteur dont l'énergie électro-dynamique soit prise pour l'unité des énergies respectives des divers conducteurs,

by two portions of the conductive wires *BM* and *B'M'* equal to the unit of length, on a portion equal to the same unit of a third conductor whose electro-dynamic energy is taken as unit of the respective energies of the various conductors.



et qu'on désigne par *n* l'exposant de la puissance de la distance de deux portions infiniment petites de conducteurs, à laquelle leur action mutuelle est réciproquement proportionnelle quand cette distance varie seule, il sera aisé de voir,

and that we designate by n the exponent of the power of the distance of two infinitely small portions of conductors, to which their mutual action is reciprocally proportional when this distance varies alone.

following, that the intensities of action of the two small portions of conductors (g and h) thus, their lengths are ds and ds', by ids and $i'ds'$, and their mutual action will be,

$$\frac{\rho i i' ds\, ds'}{r^n}$$

the exponent n being equal to *2* if this action is, all other things being equal, in inverse proportion to the square of the distance. By replacing in this expression, the function p by its values found above, it becomes;

$$-r^{1-k-n}\frac{d(r^k\frac{dr}{ds'})}{ds}\, ii'ds\, ds'$$

Or

$$-\frac{r^{1-k-n}}{1+k}\,\frac{d^2(r^{1+k})}{ds\, ds'}\, ii'ds\, ds'$$

Using "*Traité de Mécanique*" by S. D. Poisson (1781 –1840), and the displacement of r with respect to the points MM' he expresses,

$$\frac{dr}{ds}ds \quad and \quad \frac{dr}{ds'}ds'$$

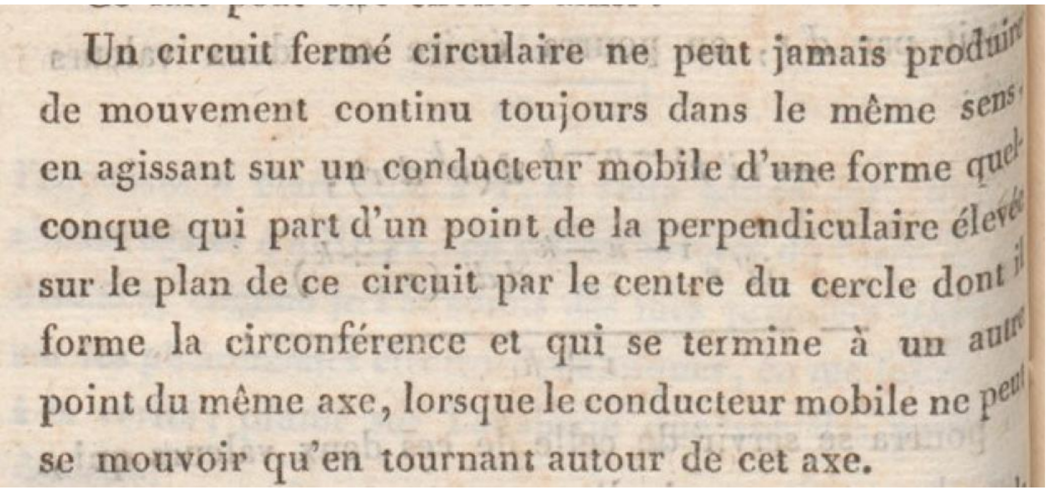
Using $d'r$ notation we can write these two as,

$$-ii'r^{1-n-k}d(r^k d'r)$$

And

$$-\frac{ii'r^{1-n-k}dd'(r^{1+k})}{1+k}$$

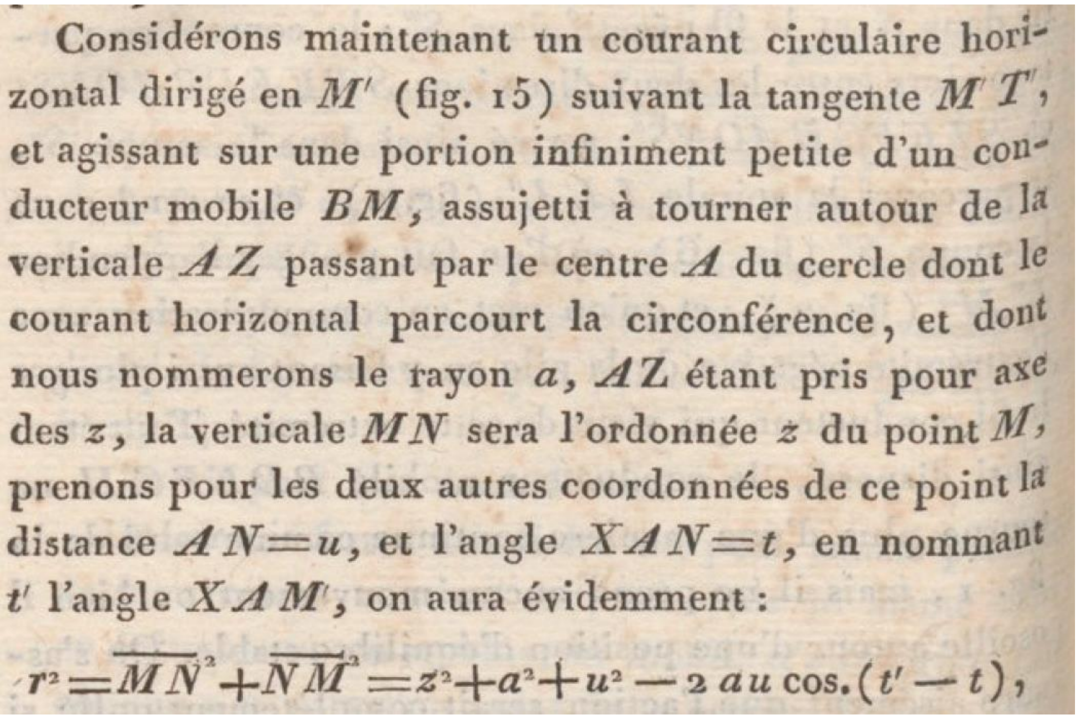
Here r is a function of the six coordinates of the two points M and M' with d and d' as their partial differentials respectively.



Un circuit fermé circulaire ne peut jamais produire de mouvement continu toujours dans le même sens, en agissant sur un conducteur mobile d'une forme quelconque qui part d'un point de la perpendiculaire élevée sur le plan de ce circuit par le centre du cercle dont il forme la circonférence et qui se termine à un autre point du même axe, lorsque le conducteur mobile ne peut se mouvoir qu'en tournant autour de cet axe.

A circular closed circuit can never produce continuous motion always in the same direction by acting on a movable conductor of any shape whatsoever which starts from a point on the perpendicular to the plane of this circuit by the center of the circle of which it is forms the circumference and which ends at another point on the same axis, when the moving conductor can only move by turning around this axis.

Then he describes some experiment on this situation and rlated devices and express that,



Considérons maintenant un courant circulaire horizontal dirigé en M' (fig. 15) suivant la tangente $M'T'$, et agissant sur une portion infiniment petite d'un conducteur mobile BM, assujetti à tourner autour de la verticale AZ passant par le centre A du cercle dont le courant horizontal parcourt la circonférence, et dont nous nommerons le rayon a, AZ étant pris pour axe des z, la verticale MN sera l'ordonnée z du point M, prenons pour les deux autres coordonnées de ce point la distance $AN=u$, et l'angle $XAN=t$, en nommant t' l'angle XAM', on aura évidemment :

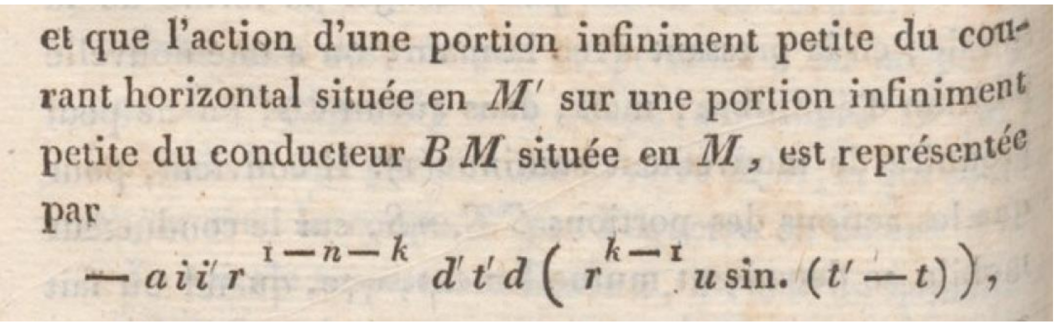
$$r^2=\overline{MN}^2+\overline{NM'}^2=z^2+a^2+u^2-2\,au\cos.(t'-t),$$

Now consider a horizontal circular current directed at M' (fig. 15) along the tangent $M'M''$, and acting on an infinitely small portion of a moving conductor *BM*, subject to rotating around the vertical *AZ* passing through the center A of the circle whose horizontal current travels the circumference, and whose radius we will name a, *AZ* being taken as the z-axis, the vertical *MN* will be the ordinate z of the point M, let us take for the two other coordinates of this point the distance $AN = u$, and the angle $XAN = t$, by naming t' the angle XAM', we will obviously have:

$$r^2 = MN^2 + NM^2 = z^2 + a^2 + u^2 - 2au\cos(t' - t)$$

The value of t' varies alone when the point M' moves, so that

$$d'r = \frac{a\,u\,d't'\sin(t' - t)}{r}$$

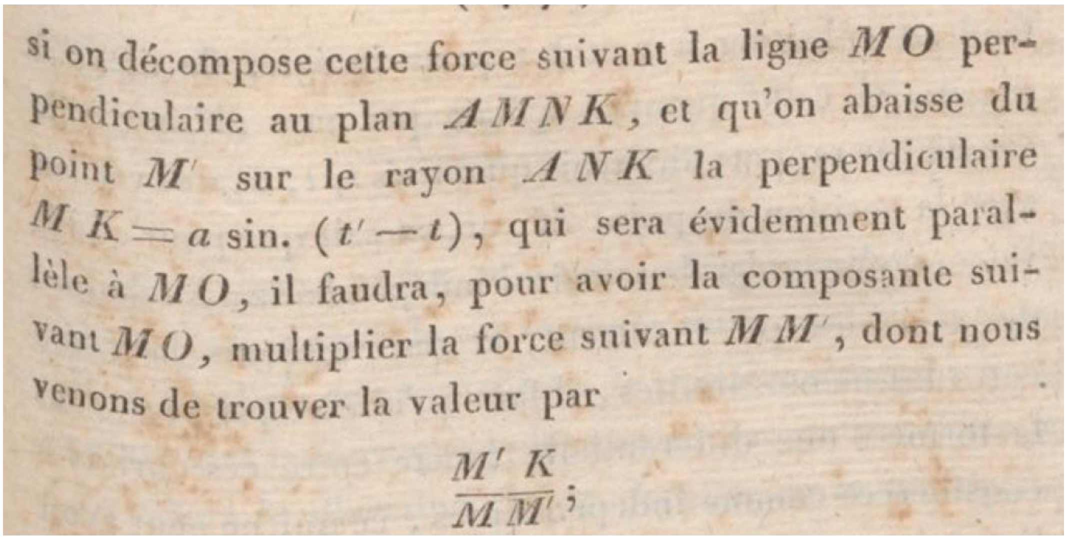


et que l'action d'une portion infiniment petite du courant horizontal située en M' sur une portion infiniment petite du conducteur $B\,M$ située en M, est représentée par

$$-aii'r^{1-n-k}d't'd\left(r^{k-1}u\sin.(t'-t)\right),$$

and that the action of an infinitely small portion of the horizontal current located at M' on an infinitely small portion of the conductor BM located at M, is represented by

$$-aii'r^{1-n-k}d't'd(r^{k-1}u\sin(t'-t))$$



si on décompose cette force suivant la ligne $M\,O$ perpendiculaire au plan $AMNK$, et qu'on abaisse du point M' sur le rayon ANK la perpendiculaire $M\,K = a\sin.(t'-t)$, qui sera évidemment parallèle à $M\,O$, il faudra, pour avoir la composante suivant $M\,O$, multiplier la force suivant $M\,M'$, dont nous venons de trouver la valeur par

$$\frac{M'\,K}{M\,M'};$$

if we decompose this force along the line MO perpendicular to the plane AMNK, and if we lower from the point M on the radius ANK the perpendicular $M\,K = a\,sin.(t'-t)$, which will obviously be parallel to MO, it will be necessary, to have the component according to MO, to multiply the force according to MM', of which we have just found the value by:

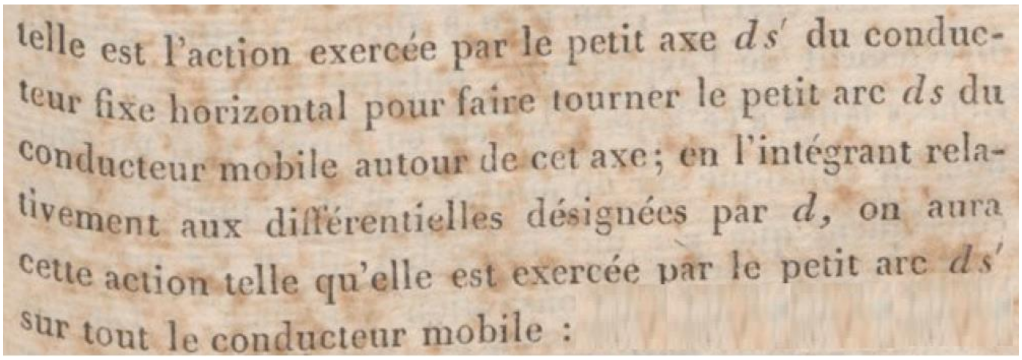
$$M'K/MM'$$

With this we will have;

$$-a^2ii'd't'r^{-n-k}\sin(t'-t)d\{r^{k-1}usin(t'-t)\}$$

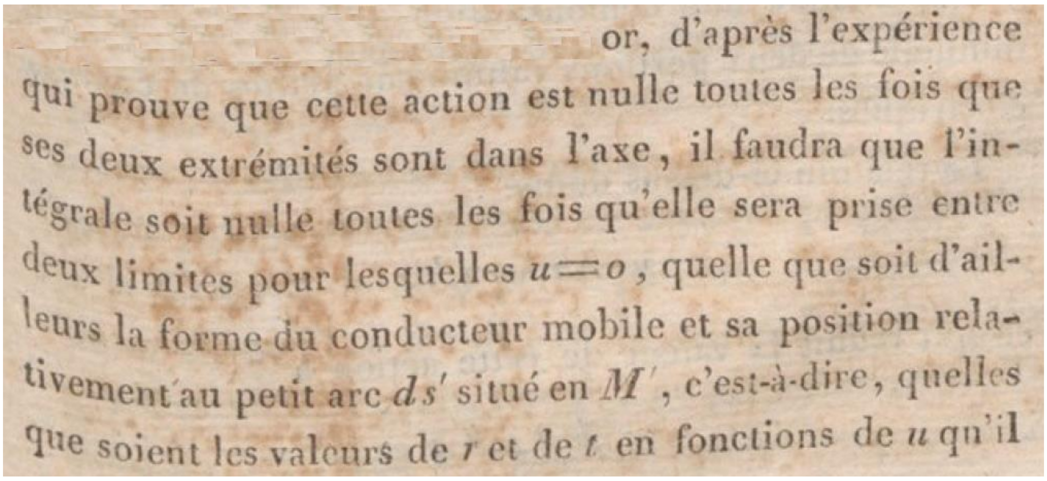
by Multiplying this quantity by the distance MQ=u from the point M to the axis AZ, we will have, for the moment of rotation

$$-a^2ii'dt'^{r^{-n-k}}\mathrm{u}\sin(t'-t)d\{r^{k-1}u\,sin(t'-t)\}$$



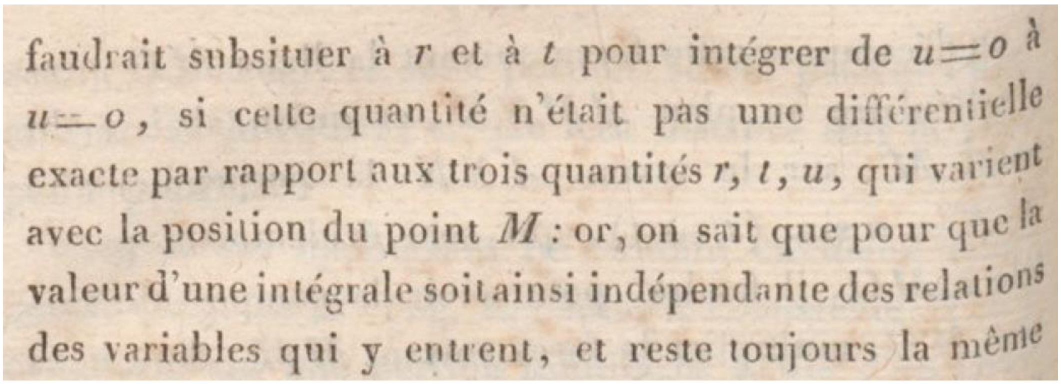
telle est l'action exercée par le petit axe ds' du conducteur fixe horizontal pour faire tourner le petit arc ds du conducteur mobile autour de cet axe ; en l'intégrant relativement aux différentielles désignées par d, on aura cette action telle qu'elle est exercée par le petit arc ds' sur tout le conducteur mobile :

such is the action exerted by the minor axis ds' of the fixed horizontal conductor to cause the minor arc ds of the mobile inductor to rotate around this axis; by integrating it relatively to the differentials designated by d, we will have this action such as it is exerted by the small arc *ds'* on the whole moving conductor:

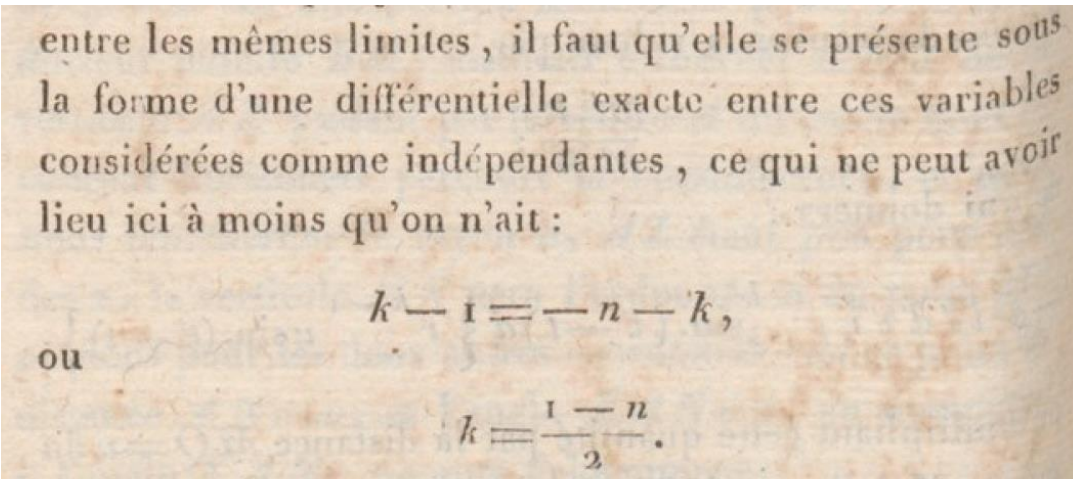


or, d'après l'expérience qui prouve que cette action est nulle toutes les fois que ses deux extrémités sont dans l'axe, il faudra que l'intégrale soit nulle toutes les fois qu'elle sera prise entre deux limites pour lesquelles $u=o$, quelle que soit d'ailleurs la forme du conducteur mobile et sa position relativement au petit arc ds' situé en M', c'est-à-dire, quelles que soient les valeurs de r et de t en fonctions de u qu'il

now, according to the experiment which proves that this action is zero every times that its two extremities are in axis, it will be necessary that the integral be zero all the times that it will be taken between them limits for which $u = o$, whatever the shape of the mobile conductor and its position relatively to the small arc *ds'*, located at M', that is to say, whatever the values of *r* and *t* as functions of *u* that



faudrait subsituer à r et à t pour intégrer de $u=o$ à $u=o$, si cette quantité n'était pas une différentielle exacte par rapport aux trois quantités r, t, u, qui varient avec la position du point M : or, on sait que pour que la valeur d'une intégrale soit ainsi indépendante des relations des variables qui y entrent, et reste toujours la même



entre les mêmes limites, il faut qu'elle se présente sous la forme d'une différentielle exacte entre ces variables considérées comme indépendantes, ce qui ne peut avoir lieu ici à moins qu'on n'ait :

$$k-1=-n-k,$$

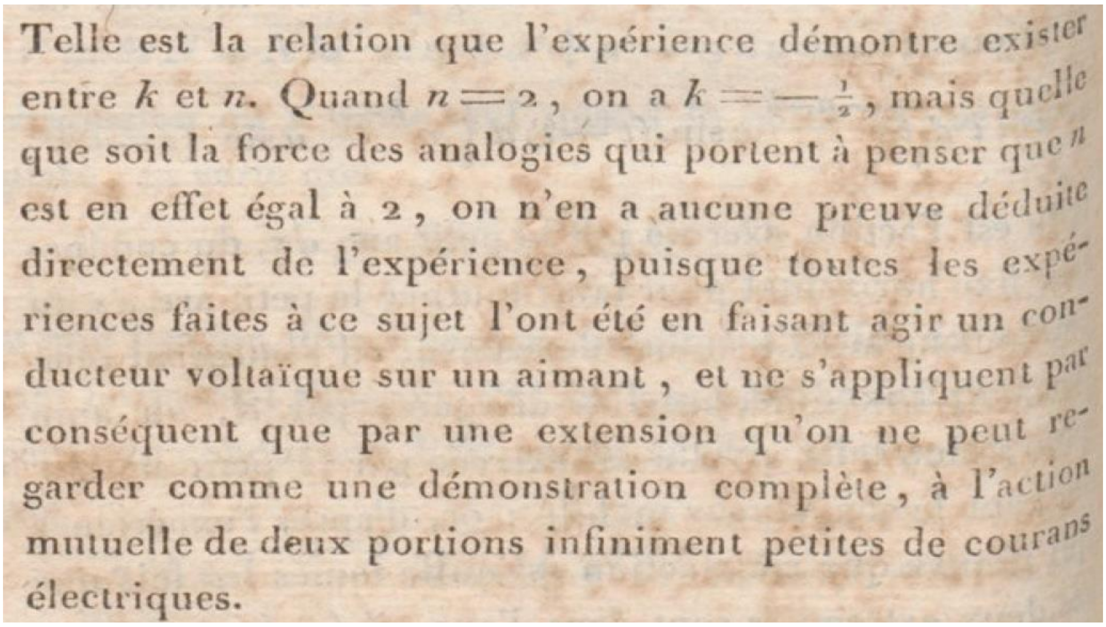
ou

$$k=\frac{1-n}{2}.$$

should be substituted for r and t to integrate from $u=o$ at $u'=o$, if this quantity were not an exact differential with respect to the three quantities r, t, u, which vary with the position of the point M: however, we know that for the value of an integral to be independent of the relations of the variables which enter into it, and always remains the same between the same limits, it must be presented in the form of an exact differential between these variables considered as independent, which cannot take place here unless one has:

$$K-1=-n-k \quad where \quad k=1-n/2$$

This is the value of constant *k* which Ampère deduced from his experiments and their corresponding calculations. He describes it as,



Telle est la relation que l'expérience démontre exister entre k et n. Quand $n=2$, on a $k=-\frac{1}{2}$, mais quelle que soit la force des analogies qui portent à penser que n est en effet égal à 2, on n'en a aucune preuve déduite directement de l'expérience, puisque toutes les expériences faites à ce sujet l'ont été en faisant agir un conducteur voltaïque sur un aimant, et ne s'appliquent par conséquent que par une extension qu'on ne peut regarder comme une démonstration complète, à l'action mutuelle de deux portions infiniment petites de courans électriques.

Such is the relation that experience demonstrates to exist between k and n. When $n = 2$, we have $k = -1/2$, but whatever the strength of the analogies which lead us to think that it is indeed equal to 2, we have no proof of it deduced directly from experience, since all the experiments made on this subject have been made by causing a voltaic conductor to act on a magnet, and consequently apply only by an extension which cannot be regarded as a complete demonstration, to the mutual action of two portions infinitely small electrical currents.

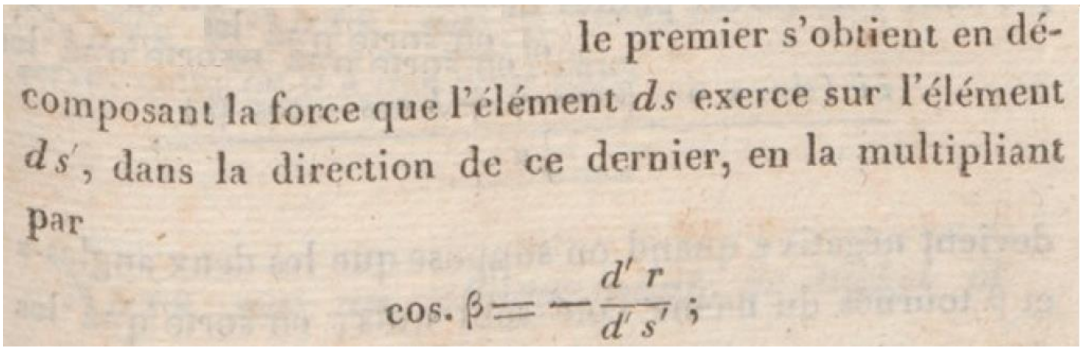
Now based on above relation we get, $n = 1 - 2k$ and could reduce the value of this action to,

$$-ii'r^k d(r^k d'r)$$

Or at

$$\frac{-ii'r^k dd'(r^{1+k})}{1+k}$$

Here Ampere put on two"remarkable rsults" from the process above as,



le premier s'obtient en décomposant la force que l'élément ds exerce sur l'élément ds', dans la direction de ce dernier, en la multipliant par

$$\cos.\ \beta = -\frac{d'r}{d's'};$$

the first is obtained by breaking down the force that the element *ds* exerts on the element *ds'*, in the direction of the latter, with multiplying it by

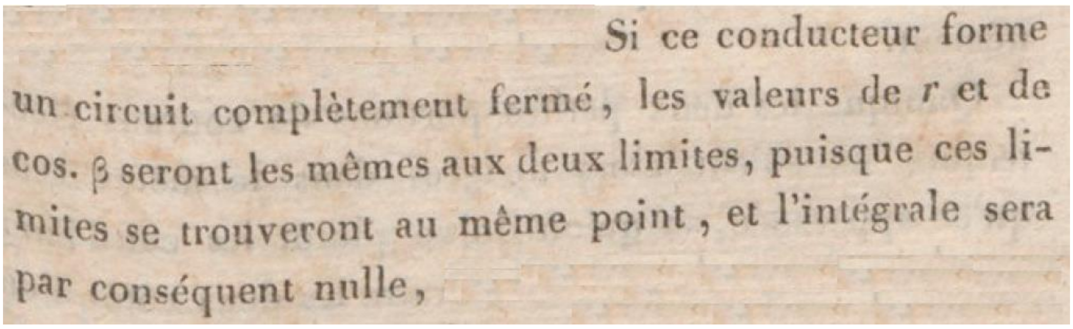
$$cos\beta = -\frac{d'r}{d's'}$$

This gives us

$$\frac{ii'}{ds} r^k d'r d(r^k d'r)$$

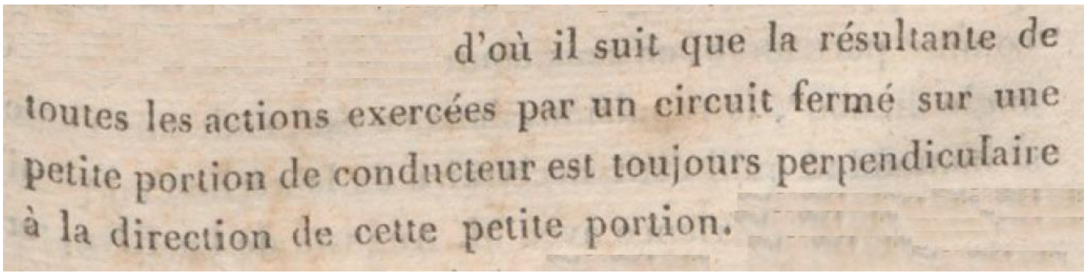
With integrating taken between the limits marked by the two ends of the conductor BM with respect to d we have,

$$\frac{ii'(r^k d'r)^2}{2\,d\,s'} + c = 1/2ii'^{r^{2k}} d's'\, cos^2\beta + c$$



Si ce conducteur forme un circuit complètement fermé, les valeurs de r et de cos. β seront les mêmes aux deux limites, puisque ces limites se trouveront au même point, et l'intégrale sera par conséquent nulle,

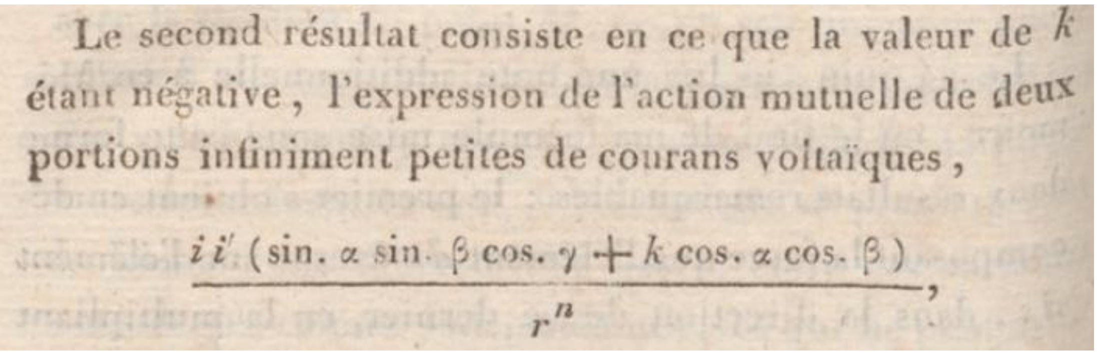
If this conductor forms a completely closed circuit, the values of r and $cos\beta$ will be the same at the two limits, since these limits will be at the same point, and the integral will therefore be zero,



d'où il suit que la résultante de toutes les actions exercées par un circuit fermé sur une petite portion de conducteur est toujours perpendiculaire à la direction de cette petite portion.

whence it follows that the resultant of all the actions exerted by a closed circuit on a small portion of conductor is always perpendicular to the direction of this small portion.

He notices that, this is the same for "any assemblage of closed circuits, and consequently with a magnet".



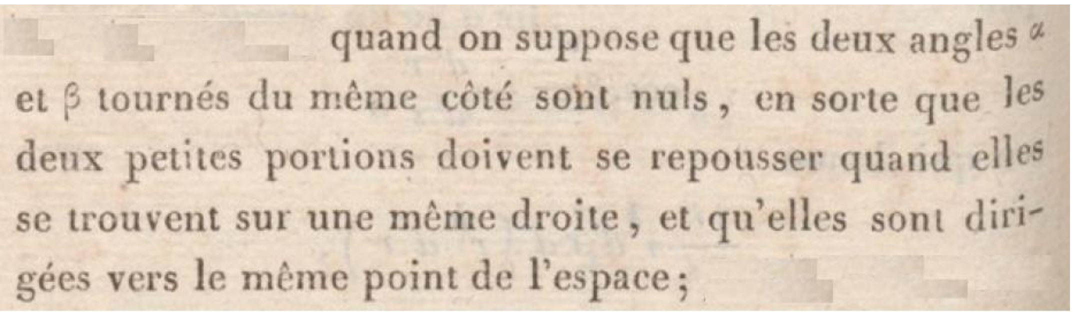
Le second résultat consiste en ce que la valeur de k étant négative, l'expression de l'action mutuelle de deux portions infiniment petites de courans voltaïques,

$$\frac{i\,i'\,(\sin.\,\alpha\,\sin.\,\beta\,\cos.\,\gamma + k\,\cos.\,\alpha\,\cos.\,\beta)}{r^n},$$

The second result consists the negative value of k such that the expression of the mutual action of two infinitely small portions of voltaic currents would be,

$$\frac{ii'(sin\,\alpha \sin\beta\;\; cos\gamma + k\cos\alpha\cos\beta)}{r^n}$$

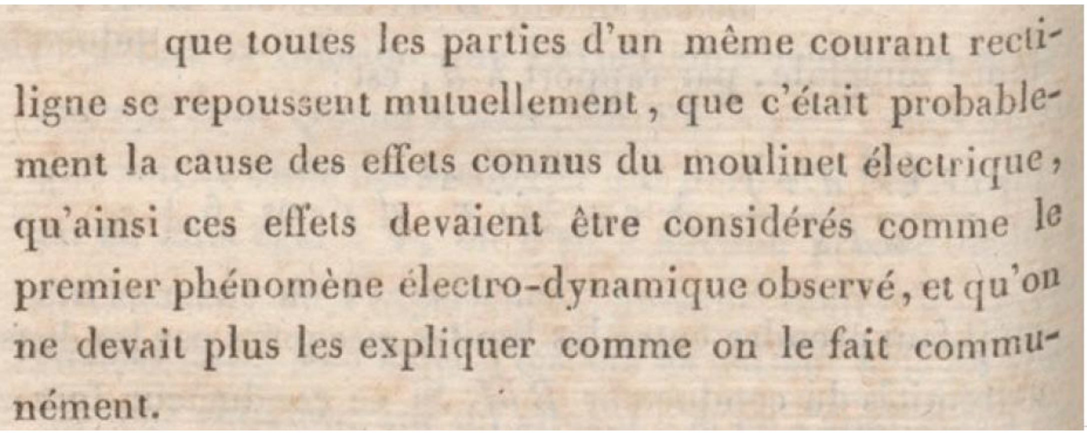
The value of this expression would also be negative when,



quand on suppose que les deux angles α et β tournés du même côté sont nuls, en sorte que les deux petites portions doivent se repousser quand elles se trouvent sur une même droite, et qu'elles sont dirigées vers le même point de l'espace;

it is assumed that the two angles α and β turned to the same side are zero, so that the two small portions must repel each other when they are on the same straight line, and that are directed towards the same point of space;

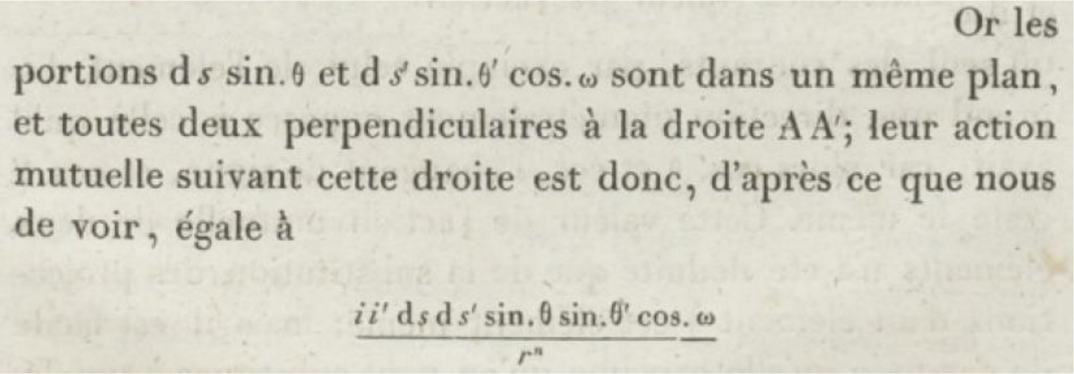
Ampere concludes that,



que toutes les parties d'un même courant rectiligne se repoussent mutuellement, que c'était probablement la cause des effets connus du moulinet électrique, qu'ainsi ces effets devaient être considérés comme le premier phénomène électro-dynamique observé, et qu'on ne devait plus les expliquer comme on le fait communément.

all the parts of the same rectilinear current mutually repel each other, that this was probably the cause of the known effects of the electric mill, that thus these effects should be considered as the first electro-dynamic phenomenon observed, and that they should no longer be explained as is commonly done.

In the *théorie mathématique des phénomènes électrodynamiques* this force appeared as below with replacing $\alpha, \beta\ and\ \gamma$ with $\theta, \theta'\ and\ \omega$ respectively. However, in practice they are the same angle with different name i.e. angle of revolution.



Or les portions $ds \sin.\theta$ et $ds' \sin.\theta' \cos.\omega$ sont dans un même plan, et toutes deux perpendiculaires à la droite AA'; leur action mutuelle suivant cette droite est donc, d'après ce que nous de voir, égale à

$$\frac{ii'\,ds\,ds' \sin.\theta \sin.\theta' \cos.\omega}{r^n}$$

Or portions $ds\ sin.\theta$ and $ds'\ sin.\theta'\ cos\ \omega$ are in the same plane, and both perpendicular to line AA'; their mutual action along this line is therefore, from what we can see, equal to

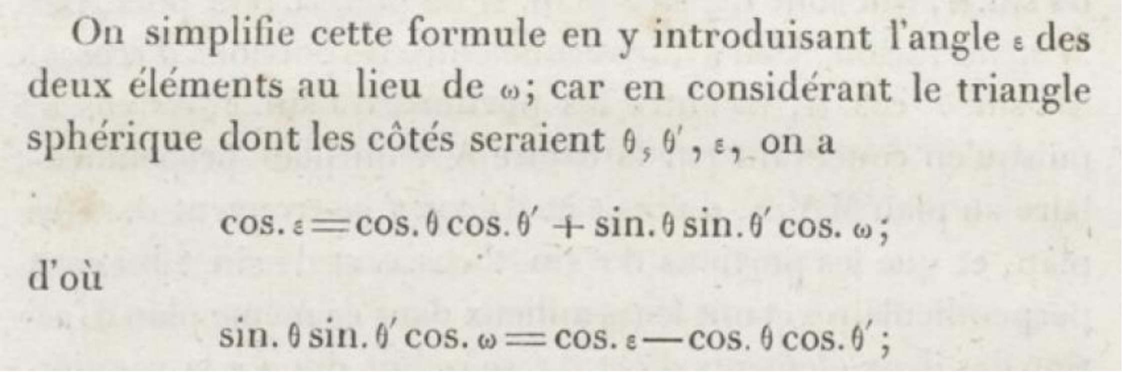
$$\frac{ii'ds\ ds' sin\ \theta\ sin\theta'\ cos\ \omega}{r^n}$$

For two portions of cosine we have,

$$\frac{ii'k\ ds\ ds' cos\ \theta\ cos\theta'}{r^n}$$

Therefore, with the action of the two elements *ds, ds'* on each other we get,

$$\frac{ii'dsds'}{r^n}(\sin\theta\sin\theta'\cos\omega + k\cos\theta\cos\theta')$$



On simplifie cette formule en y introduisant l'angle ε des deux éléments au lieu de ω; car en considérant le triangle sphérique dont les côtés seraient θ, θ', ε, on a

$$\cos.\varepsilon = \cos.\theta \cos.\theta' + \sin.\theta \sin.\theta' \cos.\omega;$$

d'où

$$\sin.\theta \sin.\theta' \cos.\omega = \cos.\varepsilon - \cos.\theta \cos.\theta';$$

We simplify this formula by introducing the angle ε of the two elements instead of ω; because by considering the spherical triangle whose sides would be $\theta, \theta', \varepsilon$ we have

$$\cos\varepsilon = cos\ \theta, cos\theta' + sin\ \theta\ sin\theta'\ cos\ \omega$$

This follows that since $k - 1 = h = 3/2$ then,

$$\frac{ii'dsds'}{r^n}(\cos\varepsilon + h\cos\theta\cos\theta')\ .$$

Now we leave his works in early 19 century and come now to express Ampere's force in vector notation using SI units.

Using Newton's law of universal gravitation of Faraday's strives, Charles Coulomb express his law of mutual interaction between to charges. Using vector notations, it would be,

$$\pm\vec{F}_Q = \frac{qq'}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$$

Here $\pm\vec{F}_Q$ is the (electric) force per Coulomb (C) exerts on electric charges q and q' respectively in the direction of unit vector $\hat{r}$. The electric constant $\varepsilon_0 = 8.85 \times 10^{-12} A^2 s^2 N^{-1} m^{-2}$ is called vacuum permittivity or permittivity of free space.

For magnetic force similarly we have,

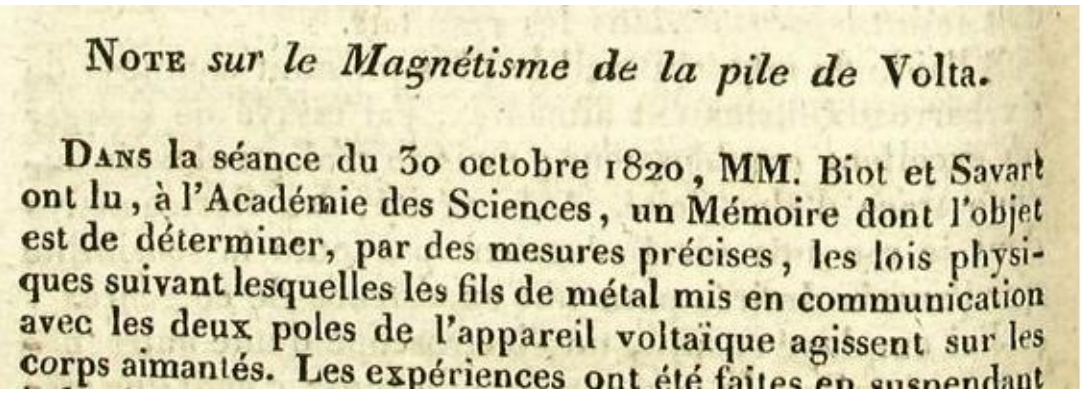
$$\pm\vec{F}_P = \frac{\mu_0\, pp'}{4\pi}\frac{\hat{r}}{r^2}$$

Here $\pm\vec{F}_P$ is the (magnetic) force per ampere-meters (Am) exerts on magnetic poles p and p' respectively in the direction of unit vector $\hat{r}$. The magnetic constant $\mu_0 = 4\pi \times 10^{-7} kg\mathrm{Am}^2 s^{-2}$ is called vacuum permeability or permeability of free space.

As we mentioned before ε_0 and μ_0 are constant of proportionality for electric charges and magnetic poles permittivity in free space respectively. In Ampere's force law, k is also a constant of proportionality i.e. $k = n/m$. It is a law in magnetostatics describing the force of attraction or repulsion between two current-carrying wires which generates a magnetic field.

Biot–Savart law describes the magnetic field generated by a constant electric current. Two French physicist Jean-Baptiste Biot (1774 –1862) and Félix Savart (1791-1841), discovered this relationship in 1820. They published their results "Note on the

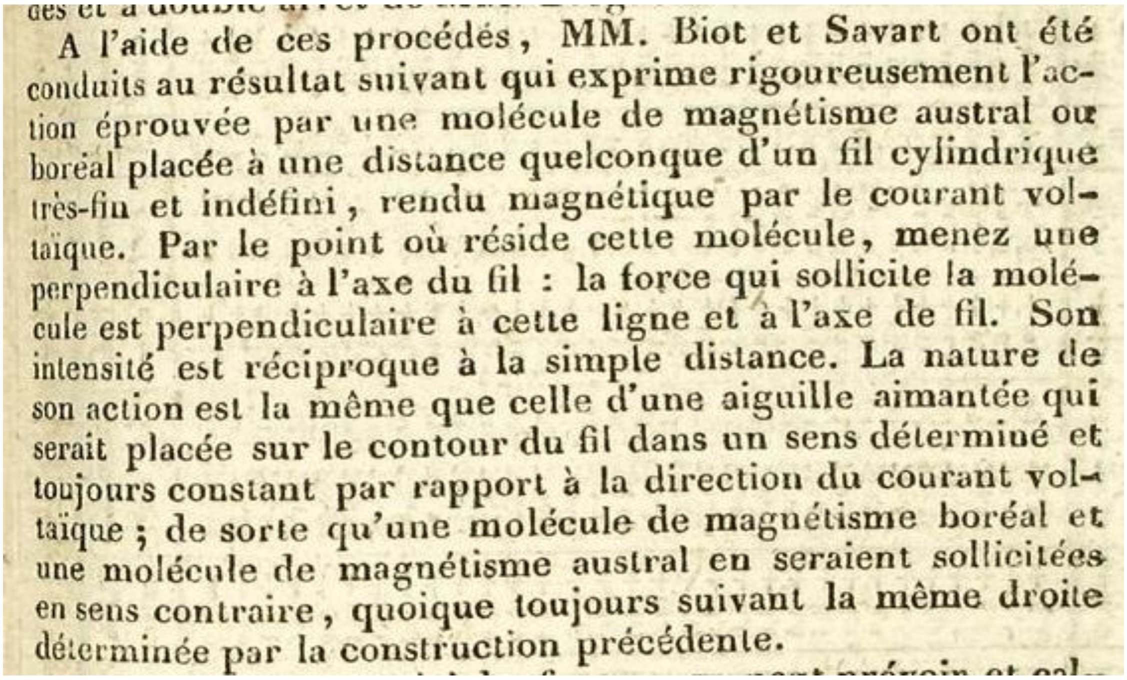
Magnetism of the Volta pile." in *annales de chimie et de physique, 1820 tome 15.*



NOTE *sur le Magnétisme de la pile de* Volta.

DANS la séance du 30 octobre 1820, MM. Biot et Savart ont lu, à l'Académie des Sciences, un Mémoire dont l'objet est de déterminer, par des mesures précises, les lois physiques suivant lesquelles les fils de métal mis en communication avec les deux poles de l'appareil voltaïque agissent sur les corps aimantés. Les expériences ont été faites en suspendant

"In the meeting of October 30, 1820, MM. Biot and Savart read, at the Academy of Sciences, a Memory whose object is to determine, by precise measurements, the physical laws according to which the metal wires placed in communication with the two poles of the voltaic apparatus act on magnetic bodies".



A l'aide de ces procédés, MM. Biot et Savart ont été conduits au résultat suivant qui exprime rigoureusement l'action éprouvée par une molécule de magnétisme austral ou boréal placée à une distance quelconque d'un fil cylindrique très-fin et indéfini, rendu magnétique par le courant voltaïque. Par le point où réside cette molécule, menez une perpendiculaire à l'axe du fil : la force qui sollicite la molécule est perpendiculaire à cette ligne et à l'axe de fil. Son intensité est réciproque à la simple distance. La nature de son action est la même que celle d'une aiguille aimantée qui serait placée sur le contour du fil dans un sens déterminé et toujours constant par rapport à la direction du courant voltaïque ; de sorte qu'une molécule de magnétisme boréal et une molécule de magnétisme austral en seraient sollicitées en sens contraire, quoique toujours suivant la même droite déterminée par la construction précédente.

"By means of these methods, MM. Biot and Savart were led to the following result which rigorously expresses the action experienced by a molecule of austral or boreal magnetism placed at any distance from a very fine and indefinite cylindrical wire, rendered magnetic by the voltaic current. Through the point where this molecule resides, lead a perpendicular to the axis of the wire: the force which solicits the molecule is perpendicular to this line and to the axis of thread. Its intensity is reciprocal to simple distance. The nature of its action is the same as that of a magnetic needle which would be placed on the contour of the thread in a determined direction and always constant with respect to the direction of the voltaic current; so that a molecule of boreal magnetism and a molecule of southern magnetism would be solicited in the direction contrary, although always following the same straight line determined by the construction former".

If B stands for magnetic field generated by a current I at distance r then,

$$B(r) = \frac{\mu_0}{4\pi} \int_C \frac{Idl \times \hat{r}}{|r'|^2}$$

Here C is the path of electric flow and l is a point on path thus, dl is a vector along the path. This is the mathematical definition of Biot–Savart law in 2-D space.

According to "26th CGPM Resolutions" (2020), Ampere's force law is defined as,

$$F_m = 2k_A \frac{II'}{r}$$

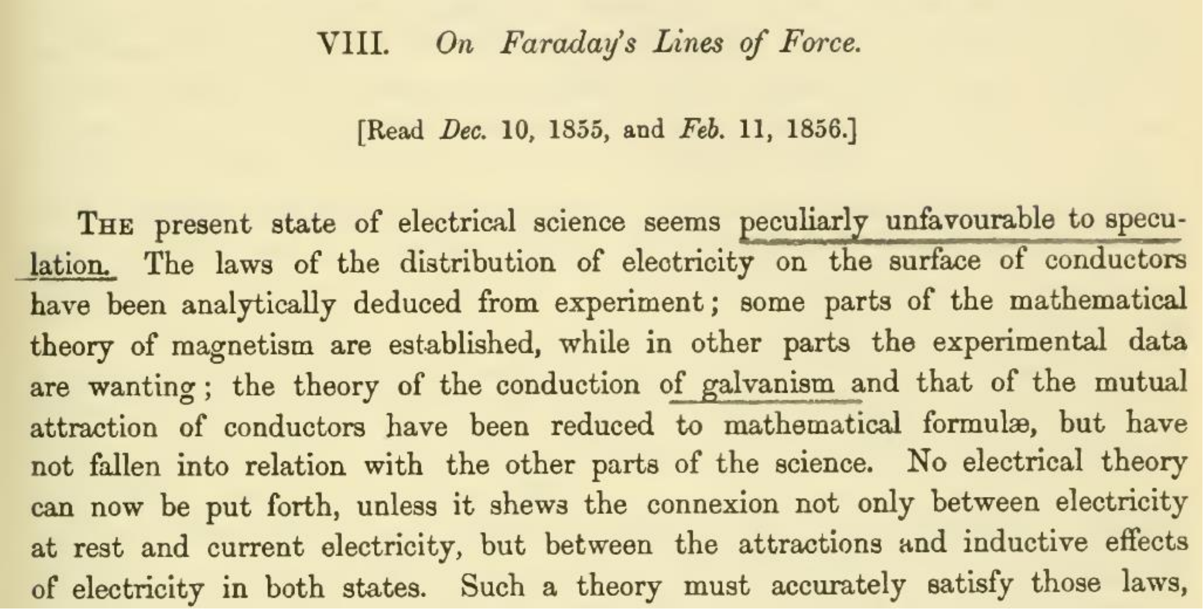
This is the definition of magnetic force (F_m) per unit length (l) between two straight parallel conductors (II') and k_A is a magnetic force constant.

$$k_A = \frac{\mu_0}{4\pi}$$

The story of electron (cont. 2)

James Clerk Maxwell (1831-1879), formulated characteristics of coupled electric and magnetic fields. His equations are mathematical definitions of electromagnetic or as Ampere called "electrodynamic" phenomena. According to his four equations, "the electromagnetic field propagates in vacuum as an undamped wave" with constant velocity $c = 3 \times 10^8 m/s$. This constant (coefficient of proportionality) also evaluated by him. This constant will be considered later. Now we are going to weigh an "electron".

Maxwell started to study electromagnetism with his paper "On Faraday's lines of force" in 1855 published in *Transactions of the Cambridge Philosophical Society*.



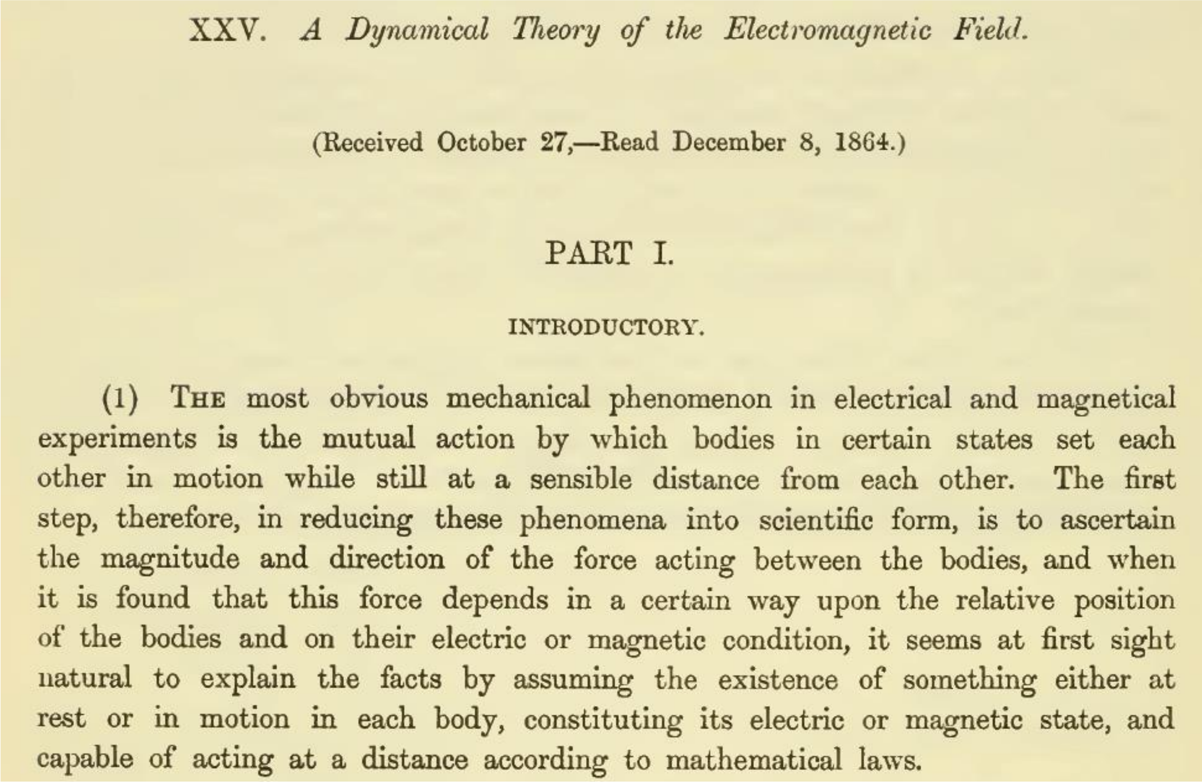
VIII. *On Faraday's Lines of Force.*

[Read *Dec.* 10, 1855, and *Feb.* 11, 1856.]

THE present state of electrical science seems peculiarly unfavourable to speculation. The laws of the distribution of electricity on the surface of conductors have been analytically deduced from experiment; some parts of the mathematical theory of magnetism are established, while in other parts the experimental data are wanting; the theory of the conduction of galvanism and that of the mutual attraction of conductors have been reduced to mathematical formulæ, but have not fallen into relation with the other parts of the science. No electrical theory can now be put forth, unless it shews the connexion not only between electricity at rest and current electricity, but between the attractions and inductive effects of electricity in both states. Such a theory must accurately satisfy those laws,

In this paper he tried to show and partly fill the gaps between the electro and magnetic phenomena of nature and their mathematical definitions.

In 1864 he published "A Dynamical Theory of the Electromagnetic Field" in *Royal Society Transactions*. Here he introduces his problem to investigate as mutual interaction between electro or magnetic bodies while they are in a "sensible distance". He assumed "the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, and capable of acting at a distance". However, he mentions that the theory of existing of such attracting or repulsing particles is mostly developed by M. W. Weber.



XXV. *A Dynamical Theory of the Electromagnetic Field.*

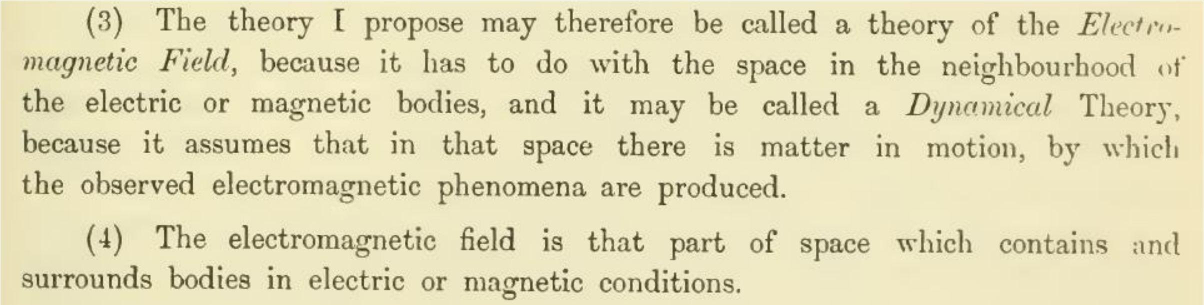
(Received October 27,—Read December 8, 1864.)

PART I.

INTRODUCTORY.

(1) THE most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric or magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, and capable of acting at a distance according to mathematical laws.

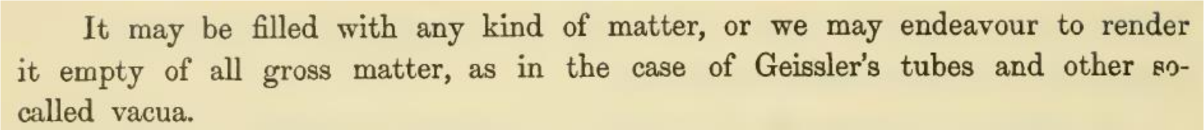
But he preferred to explain these phenomena in another way. He supposed them to be produced by "actions which go on in the surrounding medium as well as in the excited bodies". Then he tried to cinematically explain this action i.e. without the existence of acting forces. He writes;



(3) The theory I propose may therefore be called a theory of the *Electromagnetic Field*, because it has to do with the space in the neighbourhood of the electric or magnetic bodies, and it may be called a *Dynamical* Theory, because it assumes that in that space there is matter in motion, by which the observed electromagnetic phenomena are produced.

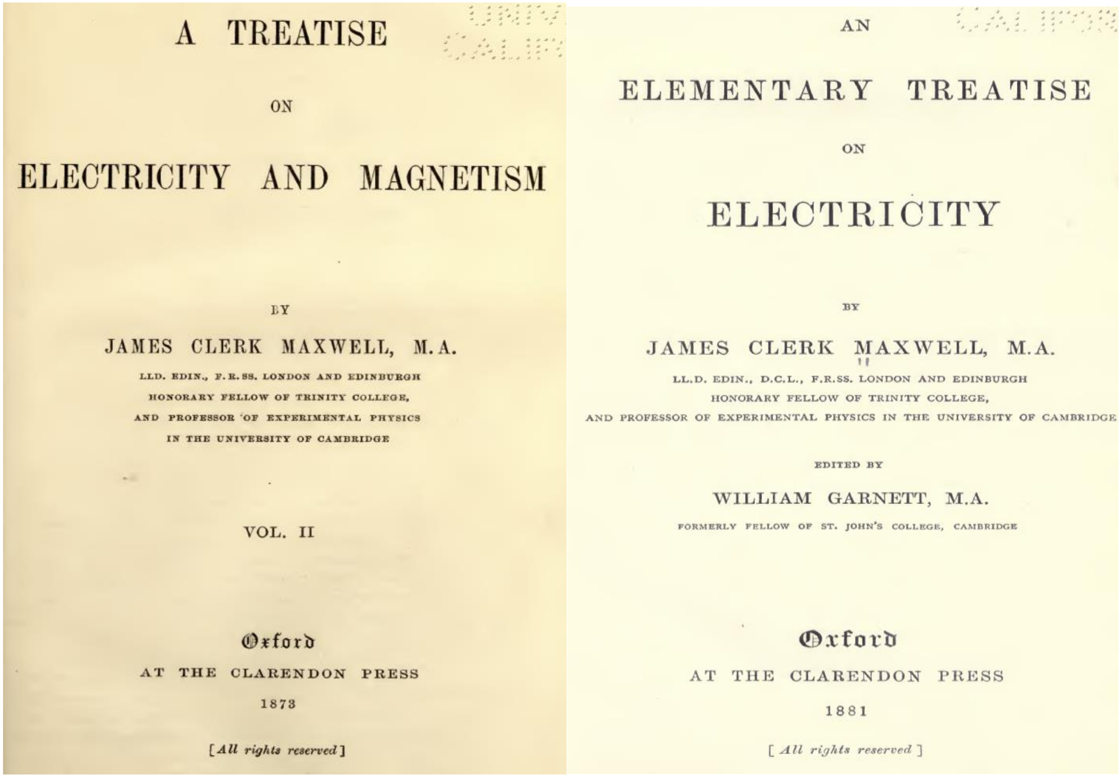
(4) The electromagnetic field is that part of space which contains and surrounds bodies in electric or magnetic conditions.

For his it doesn't matter this space is made of what.



It may be filled with any kind of matter, or we may endeavour to render it empty of all gross matter, as in the case of Geissler's tubes and other so-called vacua.

In 1873 he published "a treatise on electricity and magnetism" in two volume and in 1881 "an elementary treatise on electricity".



A TREATISE
ON
ELECTRICITY AND MAGNETISM

BY
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VOL. II

Oxford
AT THE CLARENDON PRESS
1873

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Oxford
AT THE CLARENDON PRESS
1881

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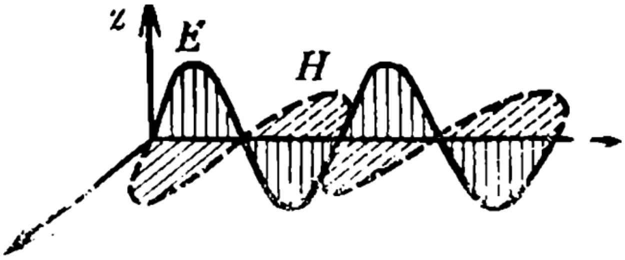
When Maxwell was trying his equations on some specific problems, he found the value of constant c. He also found that this is equal to the speed of light. Thus, he concluded that, light is an electromagnetic wave. This wave is composed of electro (E) and magnetic (H) waves moving perpendicular to each other.



Maxwell proposed his equations in 1864 in the paper "A Dynamical Theory of the Electromagnetic Field". They were mathematical definitions of previous experiments and theoretical results. However, people like Coulomb and Ampere also tried to explain the phenomena of electromagnetism by mathematical equations but it was Maxwell who proposed the most profound and general mathematical definitions. We will face with the related constant *C* in the next part.

However, later **Heinrich Hertz** (1857-1894) in his fanous book, *Untersuchungen Ueber Die Ausbreitung Der Elektrischen Kraft* "Investigations on the Propagation of Electrical Energy", published in 1892, he confirmed Maxwell's equations based on some experiments.

UNTERSUCHUNGEN
UEBER DIE
AUSBREITUNG
DER ELEKTRISCHEN KRAFT
VON
DR. HEINRICH HERTZ
PROFESSOR DER PHYSIK AN DER UNIVERSITÄT BONN

MIT 40 FIGUREN IM TEXT

LEIPZIG
JOHANN AMBROSIUS BARTH
1892

Working on electromagnetic induction, he showed that these equations are representing a phenomenon similar to "action at the distance" in gravitation. Then in the autumn of 1886 something interesting happened. He noticed that discharging a Leyden jar into a kind of coil called "Riess spiral" produced a spark in the other coil. **Peter Theophil Riess** (1804–1883) spiral were a pair of coiled conductors with metal balls at their ends.

In the seventh article of *Untersuchungen Ueber Die Ausbreitung Der Elektrischen Kraft* namely "on the speed of propagation of the electrodynamic effects", he explains the experiment and its results. He uses an apparatus brought below. It was consisted of primary (AA') and secondary (BC) conductors along with copper wires and spark gap connected to an inductor (J).

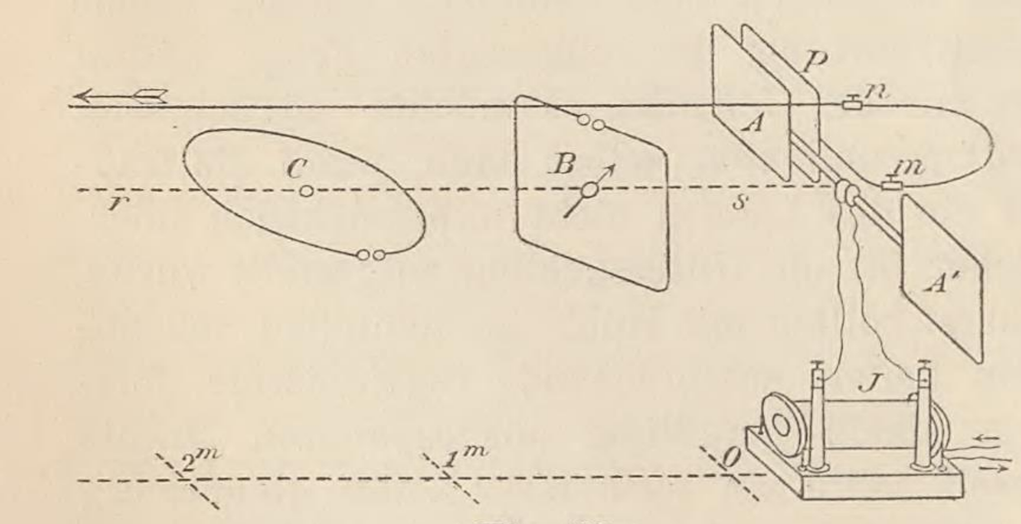
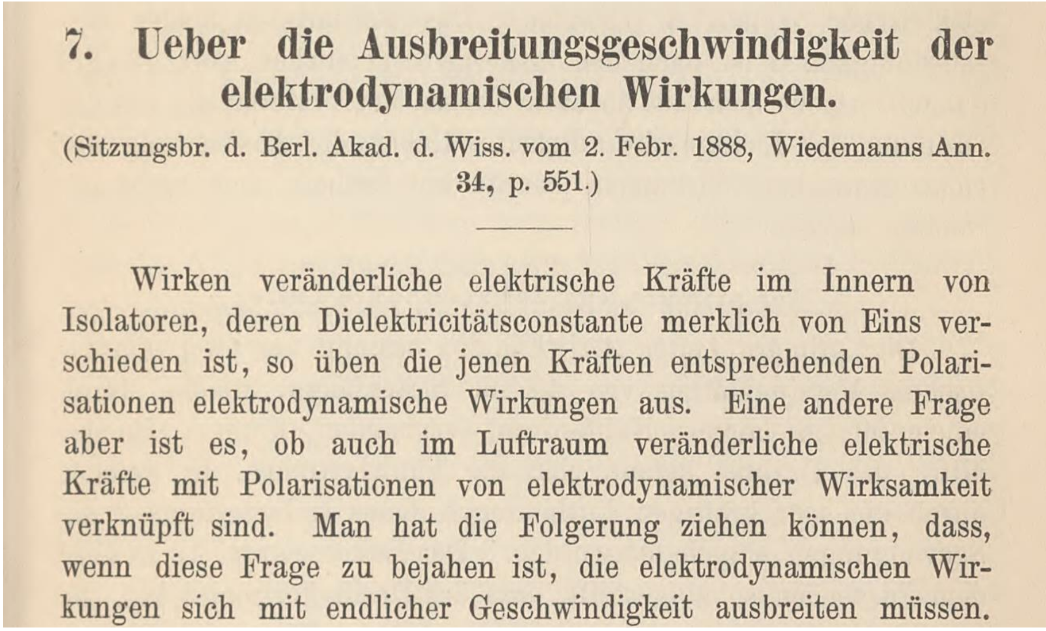


Fig. 25.

At the beginning of this paper he writes,



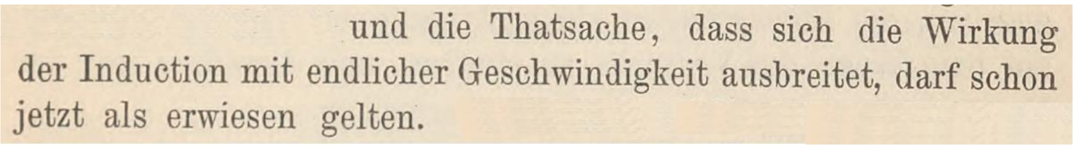
7. Ueber die Ausbreitungsgeschwindigkeit der elektrodynamischen Wirkungen.

(Sitzungsbr. d. Berl. Akad. d. Wiss. vom 2. Febr. 1888, Wiedemanns Ann. 34, p. 551.)

Wirken veränderliche elektrische Kräfte im Innern von Isolatoren, deren Dielektricitätsconstante merklich von Eins verschieden ist, so üben die jenen Kräften entsprechenden Polarisationen elektrodynamische Wirkungen aus. Eine andere Frage aber ist es, ob auch im Luftraum veränderliche elektrische Kräfte mit Polarisationen von elektrodynamischer Wirksamkeit verknüpft sind. Man hat die Folgerung ziehen können, dass, wenn diese Frage zu bejahen ist, die elektrodynamischen Wirkungen sich mit endlicher Geschwindigkeit ausbreiten müssen.

If variable electrical forces act inside insulators whose dielectric constant is noticeably different from one, the polarizations corresponding to those forces exert electrodynamic effects. Another question, however, is whether variable electrical forces are also linked to polarizations of electrodynamic effectiveness in the air space. One has to draw the following conclusion that if this question is to be answered in the affirmative, the electrodynamic effects must propagate with finite speed.

The last point here is also his conclusion,

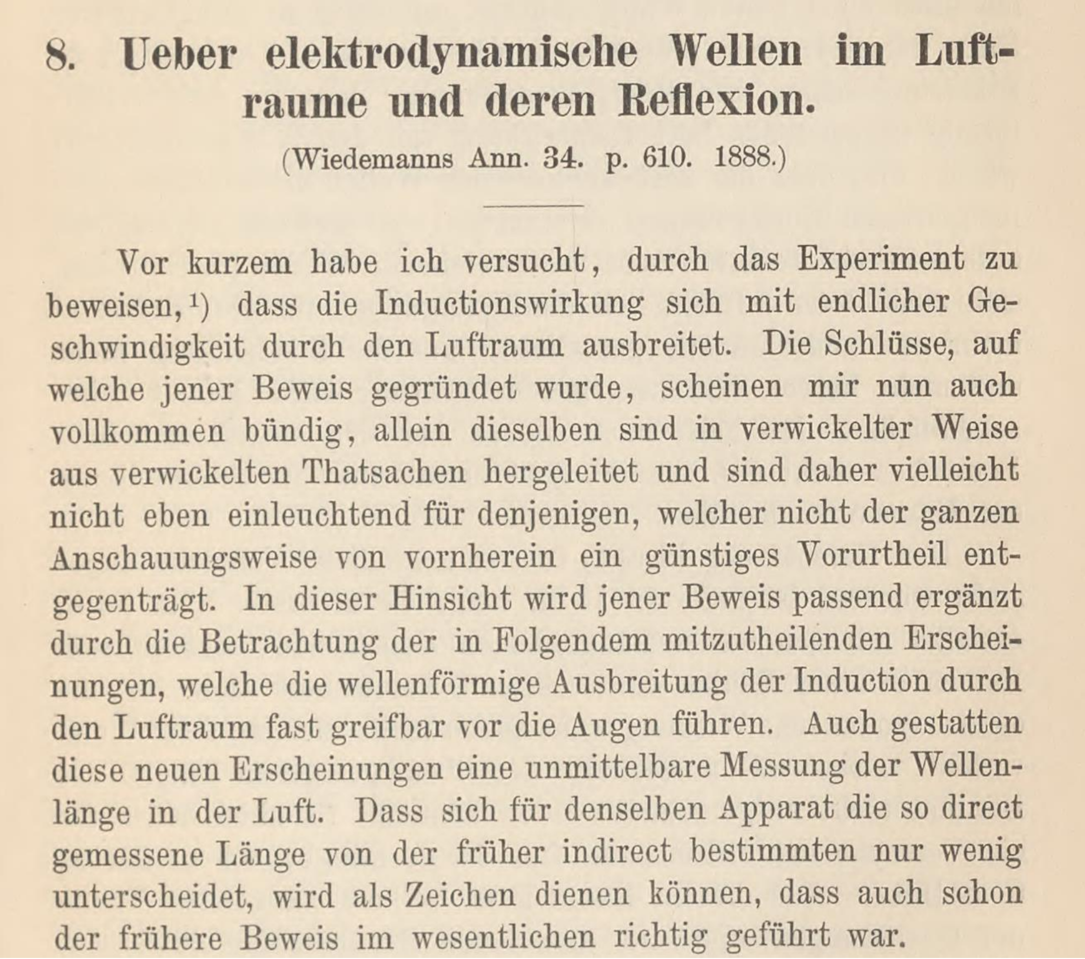


und die Thatsache, dass sich die Wirkung der Induction mit endlicher Geschwindigkeit ausbreitet, darf schon jetzt als erwiesen gelten.

and the fact that the action of induction propagates with finite speed may now be taken for granted.

In fact, he produced and measured the wavelength and velocity electromagnetic waves. His experiment showed that, the nature of the reflection and refraction in EM waves was the same as those of light. Thus, light was just a phenomenon of electromagnetic radiation obeying the Maxwell equations as well.

Proceeding these results, he published *Ueber elektrodynamische Wellen im Luftraume und deren Reflexion* "On electrodynamic waves in the air and their reflection" in 188.



8. Ueber elektrodynamische Wellen im Luftraume und deren Reflexion.

(Wiedemanns Ann. 34. p. 610. 1888.)

Vor kurzem habe ich versucht, durch das Experiment zu beweisen,¹) dass die Inductionswirkung sich mit endlicher Geschwindigkeit durch den Luftraum ausbreitet. Die Schlüsse, auf welche jener Beweis gegründet wurde, scheinen mir nun auch vollkommen bündig, allein dieselben sind in verwickelter Weise aus verwickelten Thatsachen hergeleitet und sind daher vielleicht nicht eben einleuchtend für denjenigen, welcher nicht der ganzen Anschauungsweise von vornherein ein günstiges Vorurtheil entgegenträgt. In dieser Hinsicht wird jener Beweis passend ergänzt durch die Betrachtung der in Folgendem mitzutheilenden Erscheinungen, welche die wellenförmige Ausbreitung der Induction durch den Luftraum fast greifbar vor die Augen führen. Auch gestatten diese neuen Erscheinungen eine unmittelbare Messung der Wellenlänge in der Luft. Dass sich für denselben Apparat die so direct gemessene Länge von der früher indirect bestimmten nur wenig unterscheidet, wird als Zeichen dienen können, dass auch schon der frühere Beweis im wesentlichen richtig geführt war.

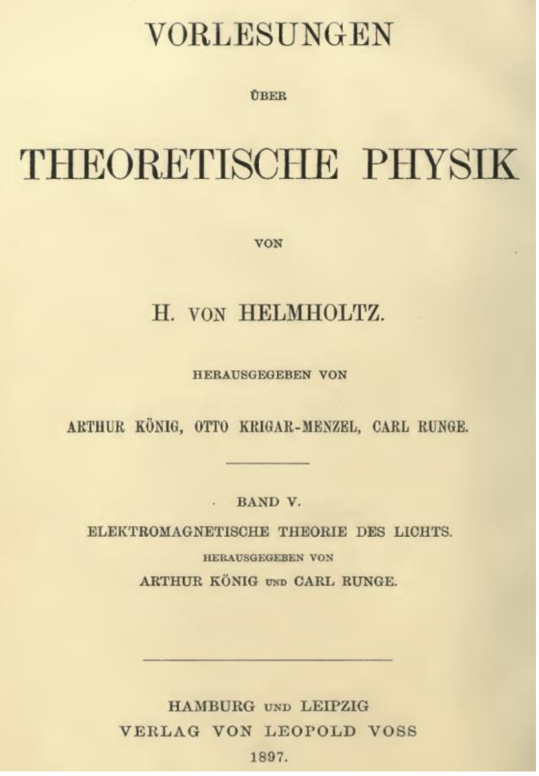
A short time ago I tried to prove by experiment that the induction effect propagates through air space with a finite velocity. The conclusions on which that proof was founded now seem to me perfectly conclusive, but they are intricately derived from intricate facts, and are therefore perhaps not exactly plausible for those who do not have a favorable prejudice against the whole way of looking at things from the outset. In this respect, that proof is suitably supplemented by the consideration of the phenomena to be communicated in the following, which almost tangibly demonstrate the wave-like propagation of the induction through the air space. Also allow this new phenomenon a direct measurement of the wavelength in the air. The fact that for the same apparatus the direct measurement of the length differs only slightly from that previously indirect one which can serve as a sign that the earlier movement is essentially correct.

what is the nature of electricity? A sort of fluid current or a current of particles!? What is the nature of so-called electric charge? Seemingly they were interacting together similar to moon and earth. But we know that, electrical forces are attractive and repulsive while gravitation is just an attractive force. Is there a material curriеries electricity?

As we mentioned earlier it was in 1873 which the existence of atom confirmed by the first measurements of Avogadro's number. This made physicist to went back to their drawing board. The question was, is there a material which curriеries electricity?

IN 1881 **Hermann Helmholtz** (1821 – 1894) stated that, "If we have to assume the existence of atoms of chemical elements, we cannot but conclude that both positive and negative electricity are themselves subdivided into elementary portions which behave as atoms of electricity".

Helmholtz was approximately a polymath. His works are extended from philosophy to mathematics, physics and even physiology. In physics his works are mostly belong to hydrodynamics, optics an electromagnetism as well. Between 1897-1907 his lectures in theoretical physics published in 6 volumes. They were in electromagnetism and light, mathematical principle of caustic and dynamics, thermodynamics and electrodynamics. In 1897 *Elektromagnetische Therie des Lichts* "electromagnetism theory of light" issued. He followed Hertz works in electrodynamics.

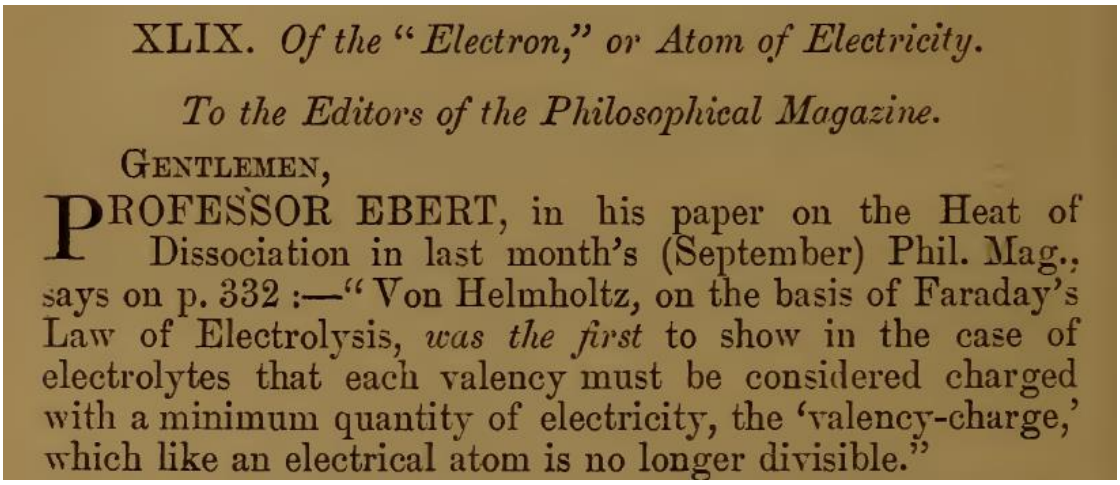


VORLESUNGEN
ÜBER
THEORETISCHE PHYSIK
VON
H. VON HELMHOLTZ.
HERAUSGEGEBEN VON
ARTHUR KÖNIG, OTTO KRIGAR-MENZEL, CARL RUNGE.

BAND V.
ELEKTROMAGNETISCHE THEORIE DES LICHTS.
HERAUSGEGEBEN VON
ARTHUR KÖNIG UND CARL RUNGE.

HAMBURG UND LEIPZIG
VERLAG VON LEOPOLD VOSS
1897.

An Irish physicist **G.J. Stoney** (1826-1911) in his theoretical effort evaluated this "elementary portion of electricity" in his paper "Of the Electron, or Atom of Electricity" published in *The London, Edinburgh and Dublin philosophical magazine and journal of science 1894.*



XLIX. *Of the "Electron," or Atom of Electricity.*

To the Editors of the Philosophical Magazine.

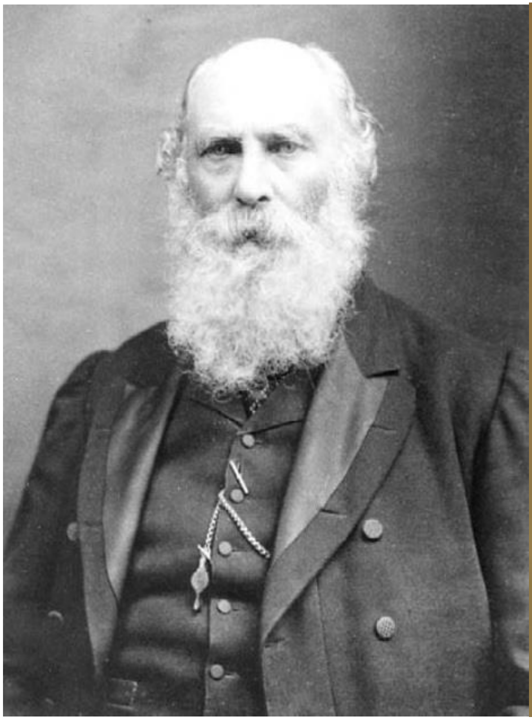
GENTLEMEN,

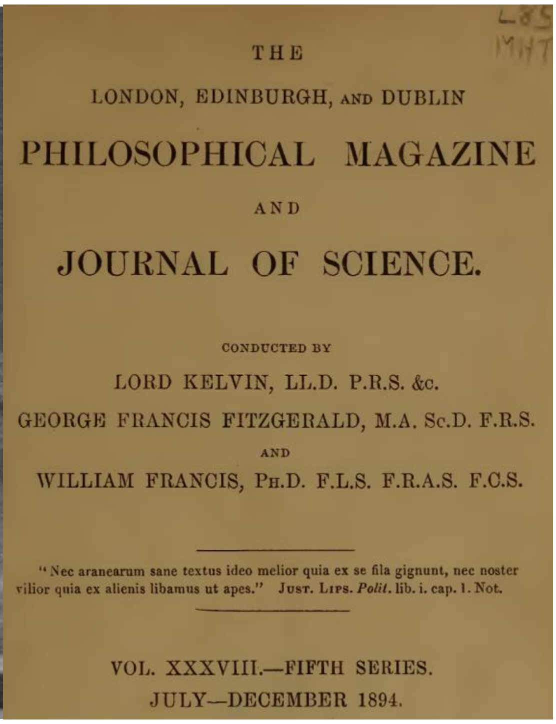
PROFESSOR EBERT, in his paper on the Heat of Dissociation in last month's (September) Phil. Mag., says on p. 332 :—"Von Helmholtz, on the basis of Faraday's Law of Electrolysis, *was the first* to show in the case of electrolytes that each valency must be considered charged with a minimum quantity of electricity, the 'valency-charge,' which like an electrical atom is no longer divisible."

He used electrolysis methods introduced by Faraday and the term "valency-charge" to expressed that, "to deposit one mole of a monovalent element, we require 96484.5 C of charge while one mole contains 6×10^{23}atoms.

$$e = \frac{F}{N_A} = 1.6 \times 10^{-19}$$

This is the amount of electricity (e) for "one atom of the substance to appear on the electrode". He himself called this small portion (quantum) of charge, "electron" in 1891. It was a constant for the amount of electron's charge (e).





L85
MHT

THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

CONDUCTED BY
LORD KELVIN, LL.D. P.R.S. &c.
GEORGE FRANCIS FITZGERALD, M.A. Sc.D. F.R.S.
AND
WILLIAM FRANCIS, Ph.D. F.L.S. F.R.A.S. F.C.S.

"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." Just. Lips. *Polit.* lib. i. cap. 1. Not.

VOL. XXXVIII.—FIFTH SERIES.
JULY—DECEMBER 1894.

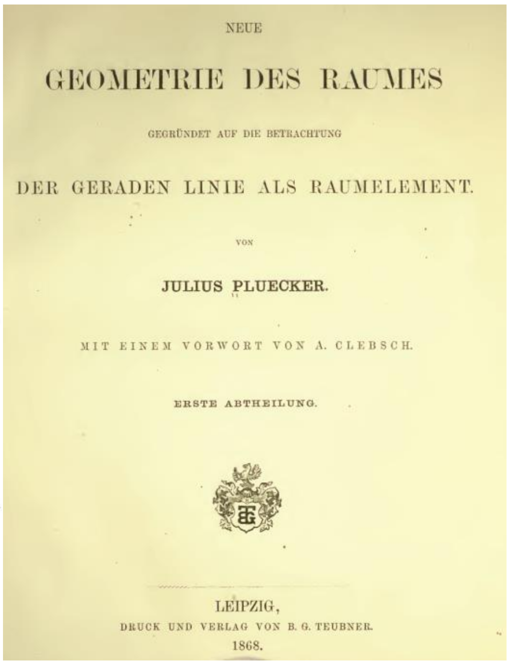
But since in the process of electrolysis a large number of atoms transfer, it seemed that, Stoney results was only an average value. Therefore, the question was still open for the existence of "a definite minimum portion of electricity". Another shortcoming of Stoney's electron was that it had no associated mass or inertia for this amount of electricity.

Remember that it was Michel Faraday who proposed the existence of the electron as an independent physical particle in his studies of current through gases in 1838. In fact, his discovery of cathode rays paved the way for the discovery of electron.

About the half on 19th century a device called "pump" was invented. It enabled scientists to evacuate gas from a glass tube and produce a vacuum.

In the year 1869 a German mathematician and physicist **Julius Plücker** (1801-1868) showed that "electric current passed freely through a glass tube with almost all of the air pumped off from it". He observed that, the glass surface which was placed on the opposite of a cathode (negatively charged electrode) displayed a glow. Therefore, it was seemed that there exist some unknown invisible rays. These rays naturally called cathode rays. His works were mostly at a mathematics and specially geometry such as *Neue Geometrie Des Raumes* New geometry of space.

Ed. Hoffmann
1856
Druck v. J. Haller.
Nach einer Photographie v. Schafgans in Bonn.

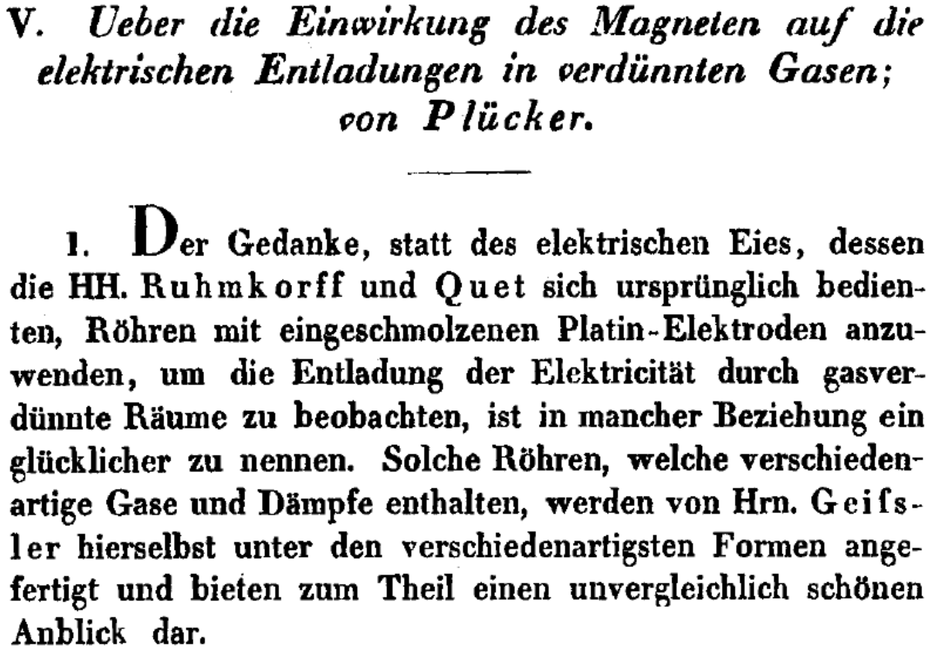


NEUE
GEOMETRIE DES RAUMES
GEGRÜNDET AUF DIE BETRACHTUNG
DER GERADEN LINIE ALS RAUMELEMENT.
VON
JULIUS PLUECKER.
MIT EINEM VORWORT VON A. CLEBSCH.
ERSTE ABTHEILUNG.

LEIPZIG,
DRUCK UND VERLAG VON B. G. TEUBNER.
1868.

In 1857 He published his observations in *Annalen der Physik und chemie, (vol. 179)* With the title of *Ueber die Einwirkung des Magneten auf die elektrischen Entladungen in verdünnten Gasen "On the action of the magnet on the electric discharges in rarefied gases"*. In this paper he studied the rarefied gas spectra observing the deflection of cathode rays by magnetic fields. He starts with comparing *elektrischen Eies* "electric egg" namely

Ruhmkorff induction-coil with *Geissler Tube*. The earlier is an induction coil for obtaining alternating high voltage and the former is a gas-filled-tube with arrangements of electrodes demonstrating electrical glow discharge.

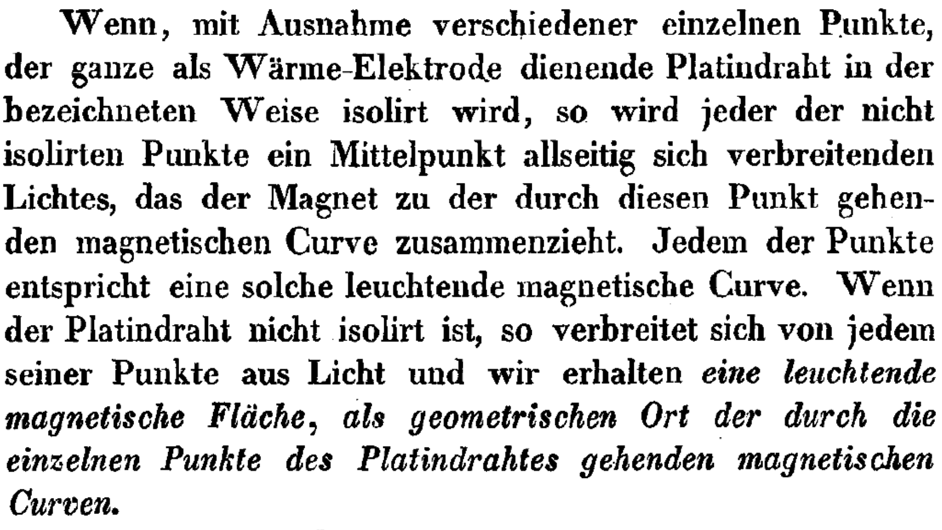


V. *Ueber die Einwirkung des Magneten auf die elektrischen Entladungen in verdünnten Gasen;*
von **Plücker.**

1. Der Gedanke, statt des elektrischen Eies, dessen die HH. Ruhmkorff und Quet sich ursprünglich bedienten, Röhren mit eingeschmolzenen Platin-Elektroden anzuwenden, um die Entladung der Elektricität durch gasverdünnte Räume zu beobachten, ist in mancher Beziehung ein glücklicher zu nennen. Solche Röhren, welche verschiedenartige Gase und Dämpfe enthalten, werden von Hrn. Geiſsler hierselbst unter den verschiedenartigsten Formen angefertigt und bieten zum Theil einen unvergleichlich schönen Anblick dar.

The thought, instead of the electric egg, that the HH. Ruhmkorff and Quet originally used tubes with fused-in platinum electrodes in order to observe the discharge of electricity through gas-thin spaces, is in some respects a fortunate one. Such tubes, which contain various gases and vapors, are manufactured by Mr. Geissler here in the most varied of forms and some of them present an incomparably beautiful sight.

He concludes with

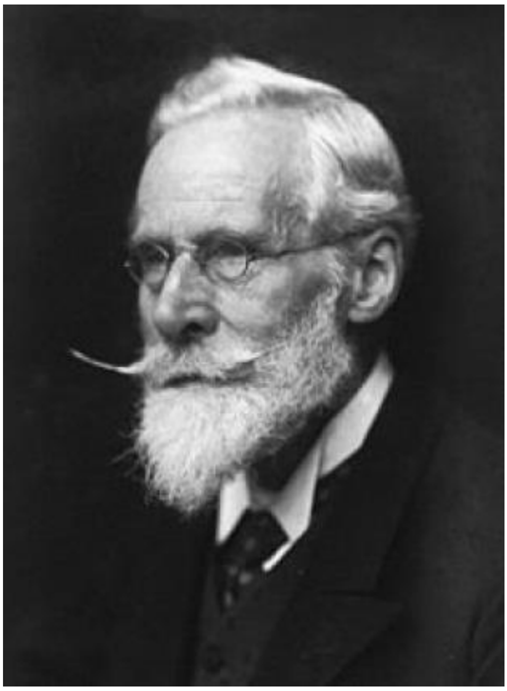


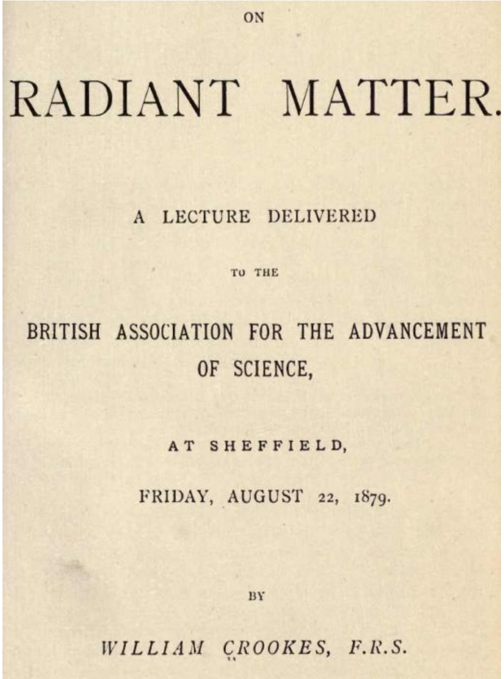
Wenn, mit Ausnahme verschiedener einzelnen Punkte, der ganze als Wärme-Elektrode dienende Platindraht in der bezeichneten Weise isolirt wird, so wird jeder der nicht isolirten Punkte ein Mittelpunkt allseitig sich verbreitenden Lichtes, das der Magnet zu der durch diesen Punkt gehenden magnetischen Curve zusammenzieht. Jedem der Punkte entspricht eine solche leuchtende magnetische Curve. Wenn der Platindraht nicht isolirt ist, so verbreitet sich von jedem seiner Punkte aus Licht und wir erhalten *eine leuchtende magnetische Fläche, als geometrischen Ort der durch die einzelnen Punkte des Platindrahtes gehenden magnetischen Curven.*

Bonn, 27. December 1857.

If, with the exception of various individual points, the entire platinum wire, as a heat electrode, is insulated in the manner described, each of the uninsulated points becomes a center of light spreading in all directions, which the magnet draws together to form the magnetic curve passing through this point. Each of the points corresponds to such a luminous magnetic curve. If the platinum wire is not insulated, light emanates from each of its points and we get a growing magnetic surface, as a geometric point of magnetic current passing through the individual points of the platinum wire.

The English chemist and physicist **W. Crookes** (1832-1919) observed that, if one placed an object in front of the cathode rays, they cast a shadow. In 1879 he delivered a lecture "Radiant Matter" to the British Association for the advancement of science. For him these rays were streams of particles. He called them "radiant matter".





ON

RADIANT MATTER.

A LECTURE DELIVERED

TO THE

BRITISH ASSOCIATION FOR THE ADVANCEMENT
OF SCIENCE,

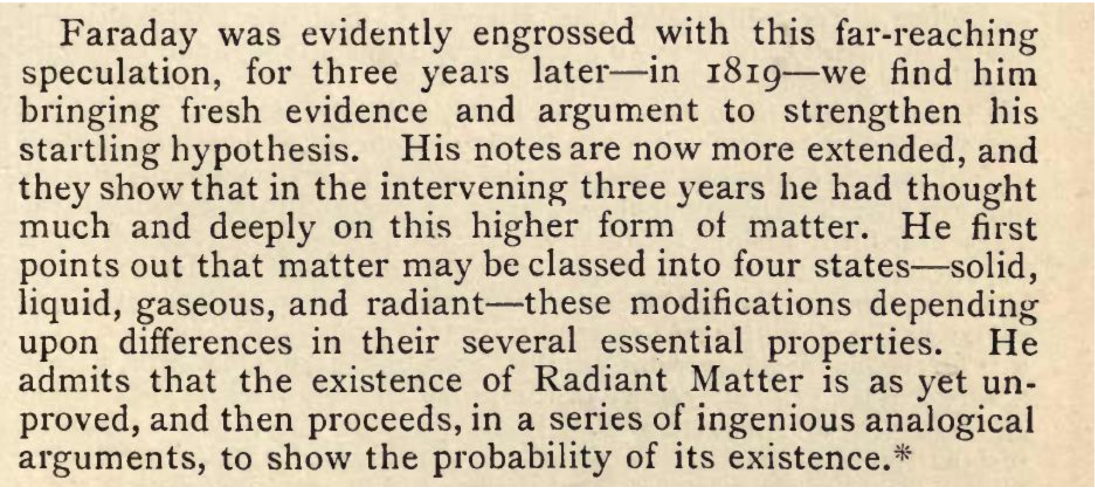
AT SHEFFIELD,

FRIDAY, AUGUST 22, 1879.

BY

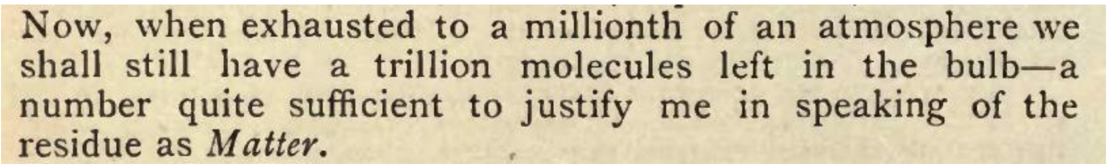
WILLIAM CROOKES, F.R.S.

Thus, cathode rays which propagate in straight lines cause fluorescence (absorption and luminescence) and great heat on substances when impact. He believed that this matter is the fourth state of matter. In fact, the name and the belief turn back to the Faraday's work on this subject.



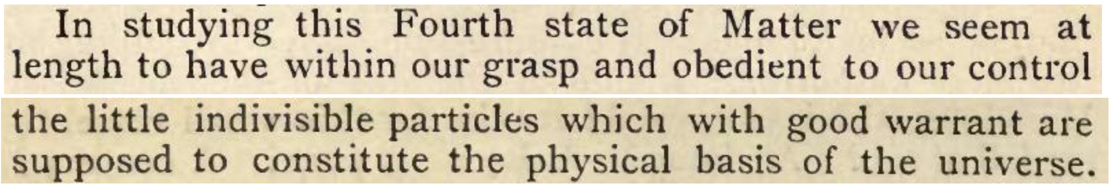
> Faraday was evidently engrossed with this far-reaching speculation, for three years later—in 1819—we find him bringing fresh evidence and argument to strengthen his startling hypothesis. His notes are now more extended, and they show that in the intervening three years he had thought much and deeply on this higher form of matter. He first points out that matter may be classed into four states—solid, liquid, gaseous, and radiant—these modifications depending upon differences in their several essential properties. He admits that the existence of Radiant Matter is as yet unproved, and then proceeds, in a series of ingenious analogical arguments, to show the probability of its existence.*

Using different kind of vacuumed glass tube with different shapes he showed that,



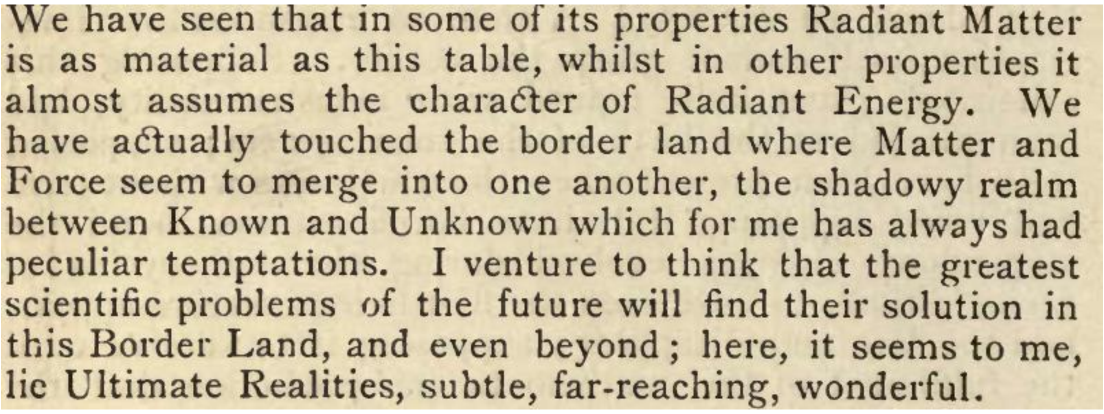
> Now, when exhausted to a millionth of an atmosphere we shall still have a trillion molecules left in the bulb—a number quite sufficient to justify me in speaking of the residue as *Matter*.

He also states that,



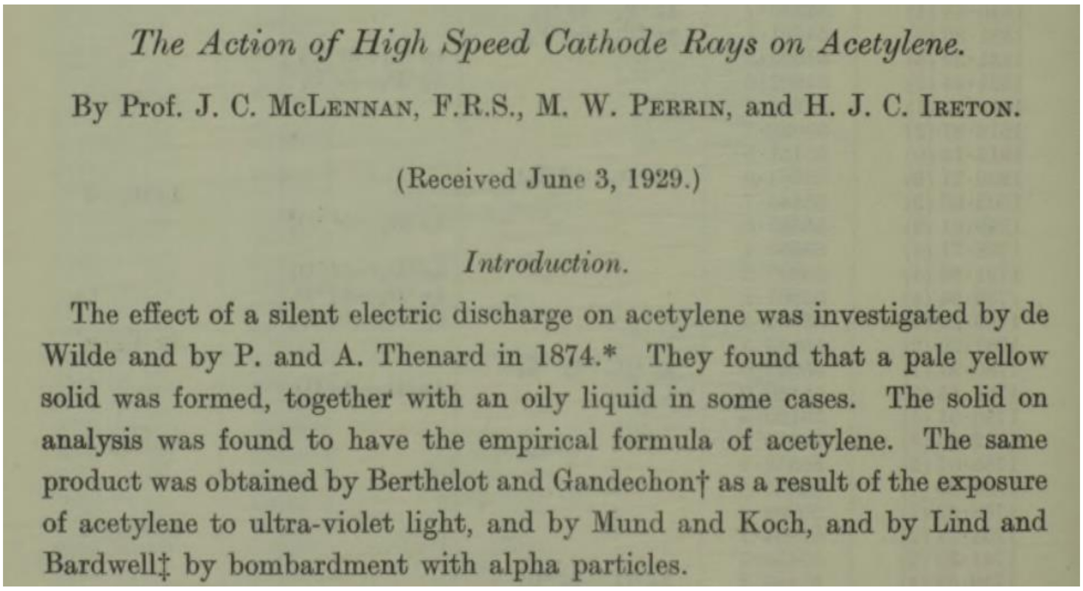
> In studying this Fourth state of Matter we seem at length to have within our grasp and obedient to our control the little indivisible particles which with good warrant are supposed to constitute the physical basis of the universe.

He even put on forward and says



> We have seen that in some of its properties Radiant Matter is as material as this table, whilst in other properties it almost assumes the character of Radiant Energy. We have actually touched the border land where Matter and Force seem to merge into one another, the shadowy realm between Known and Unknown which for me has always had peculiar temptations. I venture to think that the greatest scientific problems of the future will find their solution in this Border Land, and even beyond; here, it seems to me, lie Ultimate Realities, subtle, far-reaching, wonderful.

In 1895 the author of "Les Atoms" **Jean Perrin** (1870-1942) found that the cathode rays transfer negative charges. They were deflected in a magnetic field as well. This was showing that, "the cathode rays should be a stream of negatively charged high-speed particles". In 1929 working with **J. C. McLennan** (1868 1935) and **H. J. C. Ireton** (1827-1973) Perrin published an article "The Action of High-Speed Cathode Rays on Acetylene". Here they considered the impact of cathode rays on Acetylene as a colorless gas with chemical compound of C_2H_2. They start as,



The Action of High Speed Cathode Rays on Acetylene.

By Prof. J. C. McLennan, F.R.S., M. W. Perrin, and H. J. C. Ireton.

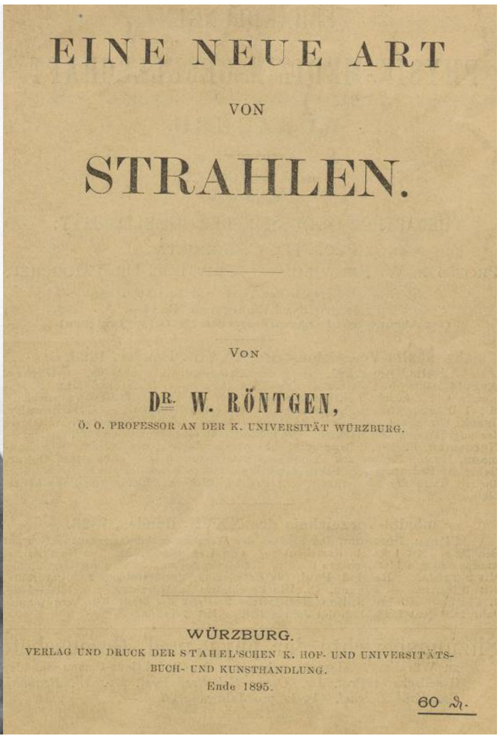
(Received June 3, 1929.)

Introduction.

The effect of a silent electric discharge on acetylene was investigated by de Wilde and by P. and A. Thenard in 1874.* They found that a pale yellow solid was formed, together with an oily liquid in some cases. The solid on analysis was found to have the empirical formula of acetylene. The same product was obtained by Berthelot and Gandechon† as a result of the exposure of acetylene to ultra-violet light, and by Mund and Koch, and by Lind and Bardwell‡ by bombardment with alpha particles.

Using a "high voltage source for the operation of the cathode ray tube" and a "reaction cell" their experiment concluded that, the number of reacting acetylene molecules and the number of ions produced by each passing electron "through the window" are in the same order. Thus "the reaction takes place following the ionization of the acetylene by the cathode rays.

However, in year 1895 another remarkable discovery happened. It was the discovery of X-rays by William Röntgen (1845-1923). He was a German mechanical engineer and physicist who produced and detected electromagnetic radiation in a wavelength range of these rays. Such rays are also called known Röntgen rays after him. His work on this topic was published in three "Mittheilung" communications in 1896 as *Ueber eine neue Art von Strahlen* "on new kind of rays".



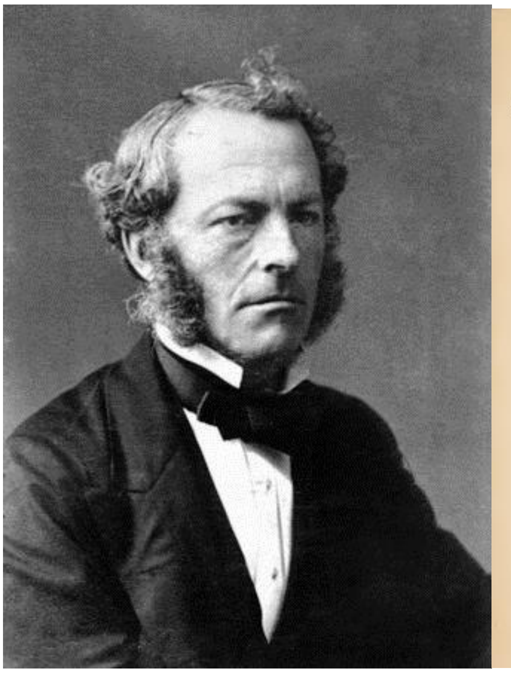
EINE NEUE ART
VON
STRAHLEN.

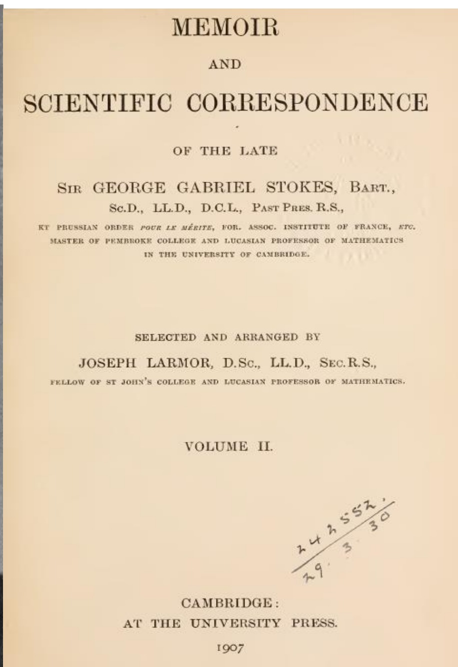
VON
DR. W. RÖNTGEN,
Ö. O. PROFESSOR AN DER K. UNIVERSITÄT WÜRZBURG.

WÜRZBURG.
VERLAG UND DRUCK DER STAHEL'SCHEN K. HOF- UND UNIVERSITÄTS-BUCH- UND KUNSTHANDLUNG.
Ende 1895.

60 Pf.

The contributor of "Navier–Stokes equations" in fluid mechanics and "Stokes theorem" in vector calculus **Sir George Gabriel Stokes** (1819-1903) is another British physicist and mathematician who has important works in our discussion. His papers appeared on aberration of light, electromagnetic spectrum, diffraction, polarization and colors during 1845 to 1851 on several journals such as *Transactions of the Cambridge Philosophical Society* journal and *Royal Society Magazine*.





MEMOIR
AND
SCIENTIFIC CORRESPONDENCE
OF THE LATE
SIR GEORGE GABRIEL STOKES, BART.,
Sc.D., LL.D., D.C.L., PAST PRES. R.S.,
KT PRUSSIAN ORDER *POUR LE MÉRITE*, FOR. ASSOC. INSTITUTE OF FRANCE, *ETC.*
MASTER OF PEMBROKE COLLEGE AND LUCASIAN PROFESSOR OF MATHEMATICS
IN THE UNIVERSITY OF CAMBRIDGE.

SELECTED AND ARRANGED BY
JOSEPH LARMOR, D.Sc., LL.D., SEC.R.S.,
FELLOW OF ST JOHN'S COLLEGE AND LUCASIAN PROFESSOR OF MATHEMATICS.

VOLUME II.

242552
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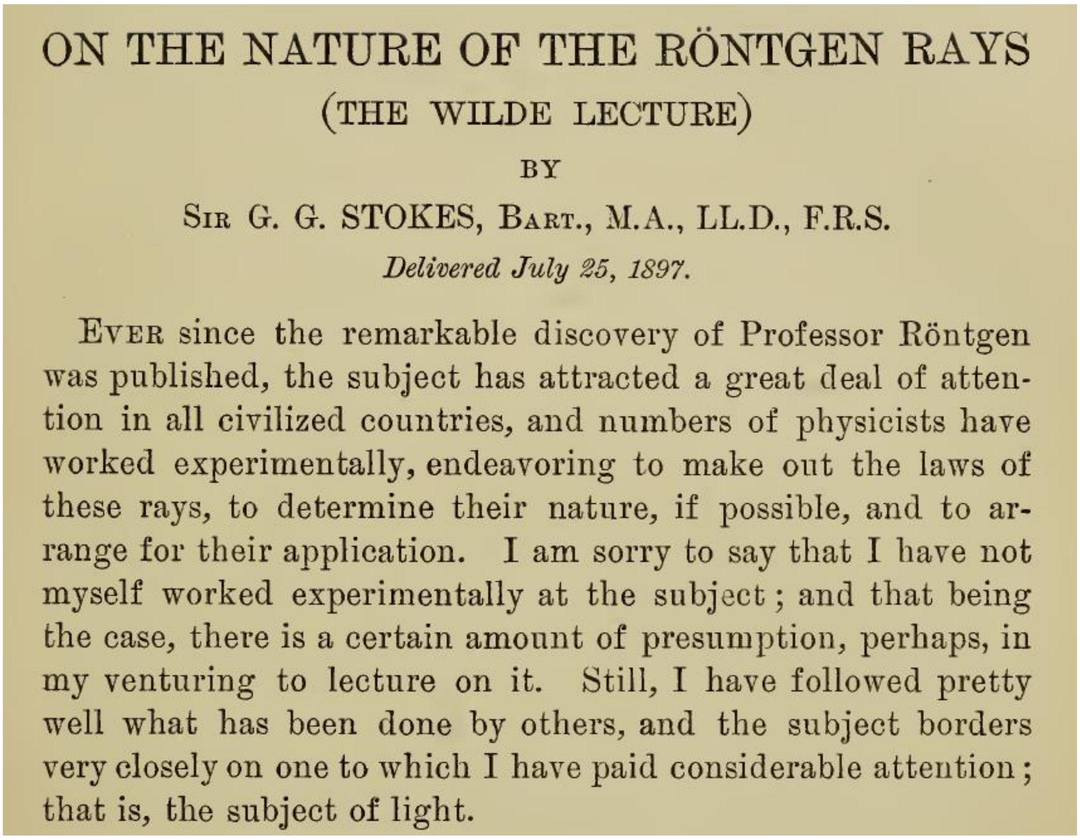
CAMBRIDGE:
AT THE UNIVERSITY PRESS.
1907

After his death his works published in collection of five volumes titled as " mathematical and physical papers " and " memoir and scientific correspondence". His paper on cathode rays and Röntgen rays appeared as below.



ON THE NATURE OF THE RÖNTGEN RAYS

(THE WILDE LECTURE)

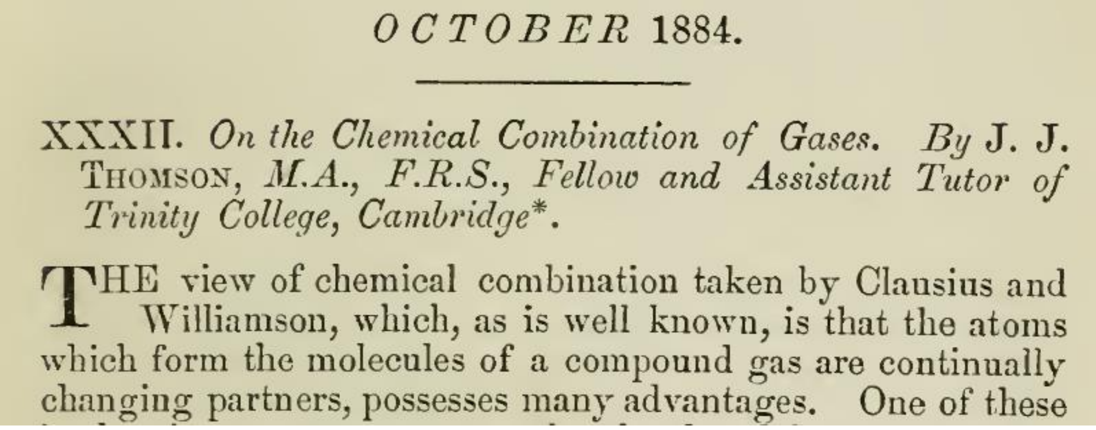
BY

SIR G. G. STOKES, BART., M.A., LL.D., F.R.S.

Delivered July 25, 1897.

EVER since the remarkable discovery of Professor Röntgen was published, the subject has attracted a great deal of attention in all civilized countries, and numbers of physicists have worked experimentally, endeavoring to make out the laws of these rays, to determine their nature, if possible, and to arrange for their application. I am sorry to say that I have not myself worked experimentally at the subject; and that being the case, there is a certain amount of presumption, perhaps, in my venturing to lecture on it. Still, I have followed pretty well what has been done by others, and the subject borders very closely on one to which I have paid considerable attention; that is, the subject of light.

Brinish physicist and chemist appeared to put an end to the whole process of finding "electron" as an elementary particle is **Joseph John Thomson** (1856-1940). One of his first papers published in philosophical magazine in 1884 was investigated the mathematical properties of gases "results of the chemical combination".

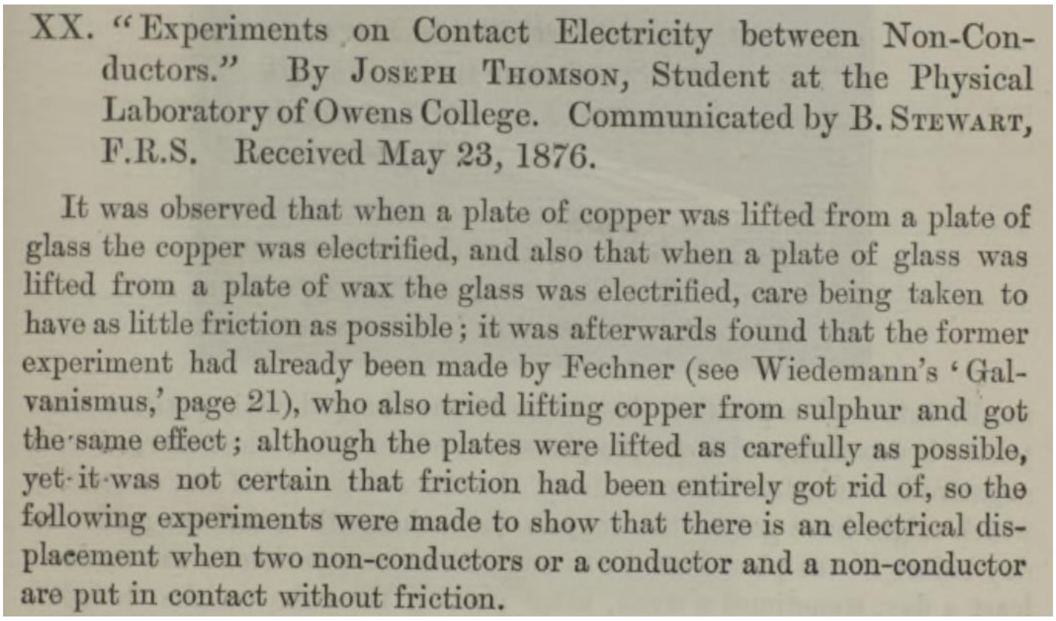


OCTOBER 1884.

XXXII. *On the Chemical Combination of Gases. By* J. J. THOMSON, *M.A., F.R.S., Fellow and Assistant Tutor of Trinity College, Cambridge*.*

THE view of chemical combination taken by Clausius and Williamson, which, as is well known, is that the atoms which form the molecules of a compound gas are continually changing partners, possesses many advantages. One of these

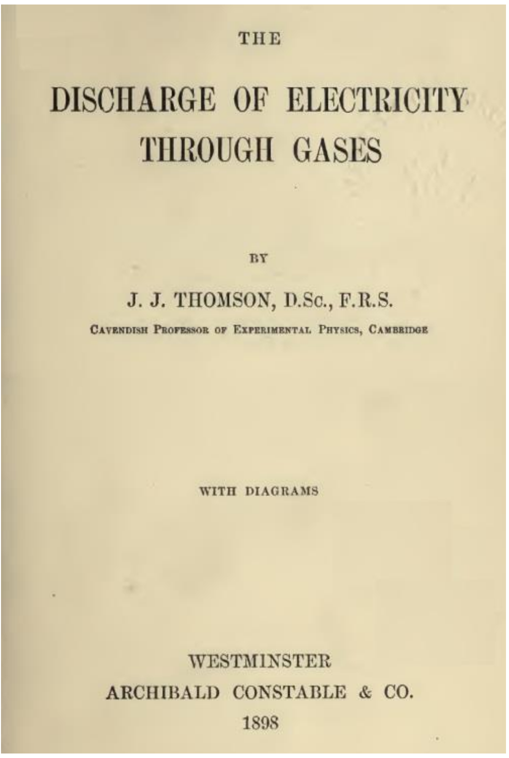
However, the earlier paper on electricity between insulators published in 1876 when he was a student at Owens College.



XX. "Experiments on Contact Electricity between Non-Conductors." By Joseph Thomson, Student at the Physical Laboratory of Owens College. Communicated by B. Stewart, F.R.S. Received May 23, 1876.

It was observed that when a plate of copper was lifted from a plate of glass the copper was electrified, and also that when a plate of glass was lifted from a plate of wax the glass was electrified, care being taken to have as little friction as possible; it was afterwards found that the former experiment had already been made by Fechner (see Wiedemann's 'Galvanismus,' page 21), who also tried lifting copper from sulphur and got the same effect; although the plates were lifted as carefully as possible, yet it was not certain that friction had been entirely got rid of, so the following experiments were made to show that there is an electrical displacement when two non-conductors or a conductor and a non-conductor are put in contact without friction.

In 1897 Thomson showed that, cathode rays were composed negatively charged particles. " The cathode-rays moved in the electric field of a capacitor with a superimposed magnetic field". He delivered his findings in four lectures to University of Princeton as "on the discharge of electricity through gases". It was published in 1898.



THE

DISCHARGE OF ELECTRICITY
THROUGH GASES

BY

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WITH DIAGRAMS

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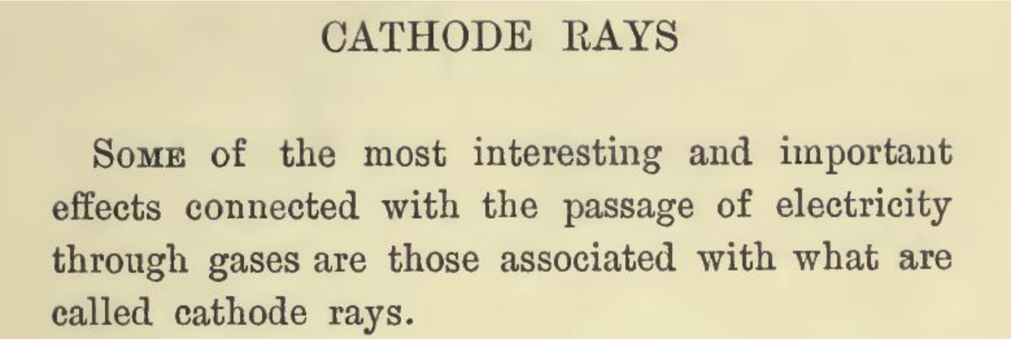
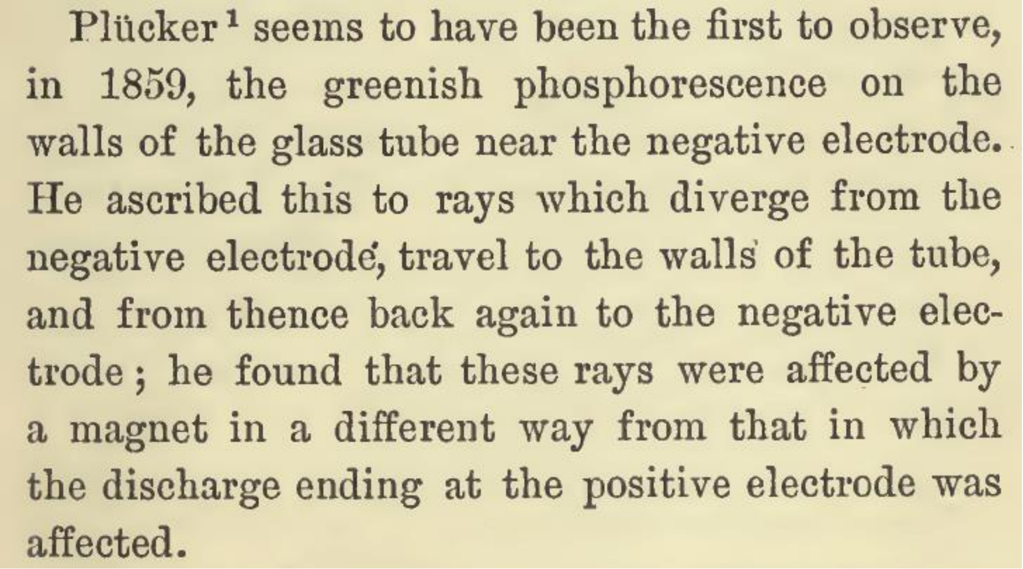
In this book he considered the discharge of electricity through gases, photo-electric effects and cathode rays as well. In the part about the cathode rays he writes.



CATHODE RAYS

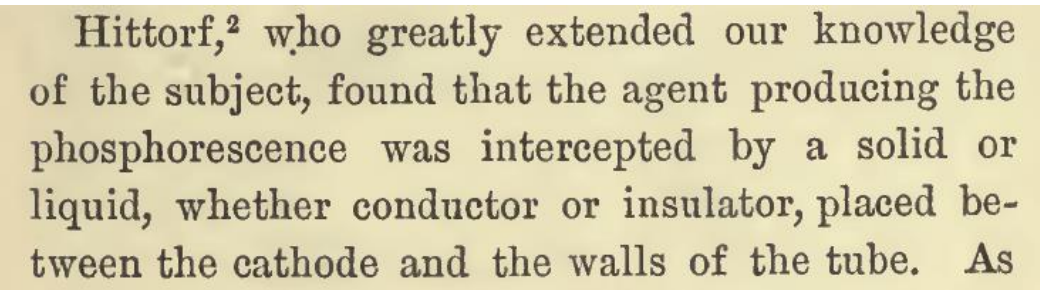
Some of the most interesting and important effects connected with the passage of electricity through gases are those associated with what are called cathode rays.

Through associating the deflection of a beam of cathode rays to electric and magnetic fields he gained more vigorous measurements of the mass-to-charge ratio. This confirmed his previous estimation. However, before his contribution he talks about previous endeavors.

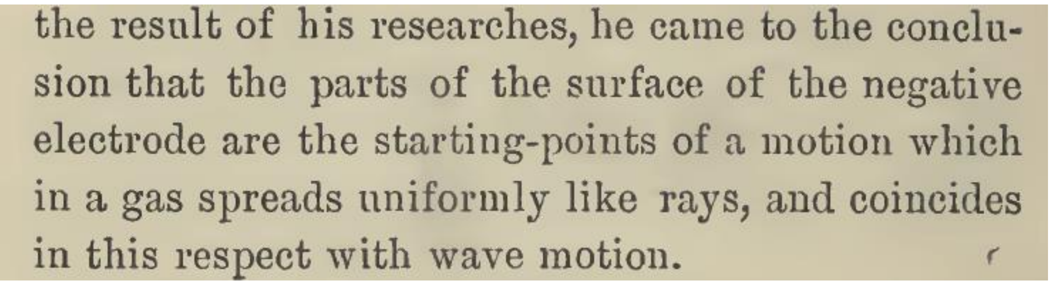


Plücker[1] seems to have been the first to observe, in 1859, the greenish phosphorescence on the walls of the glass tube near the negative electrode. He ascribed this to rays which diverge from the negative electrode, travel to the walls of the tube, and from thence back again to the negative electrode; he found that these rays were affected by a magnet in a different way from that in which the discharge ending at the positive electrode was affected.

Then

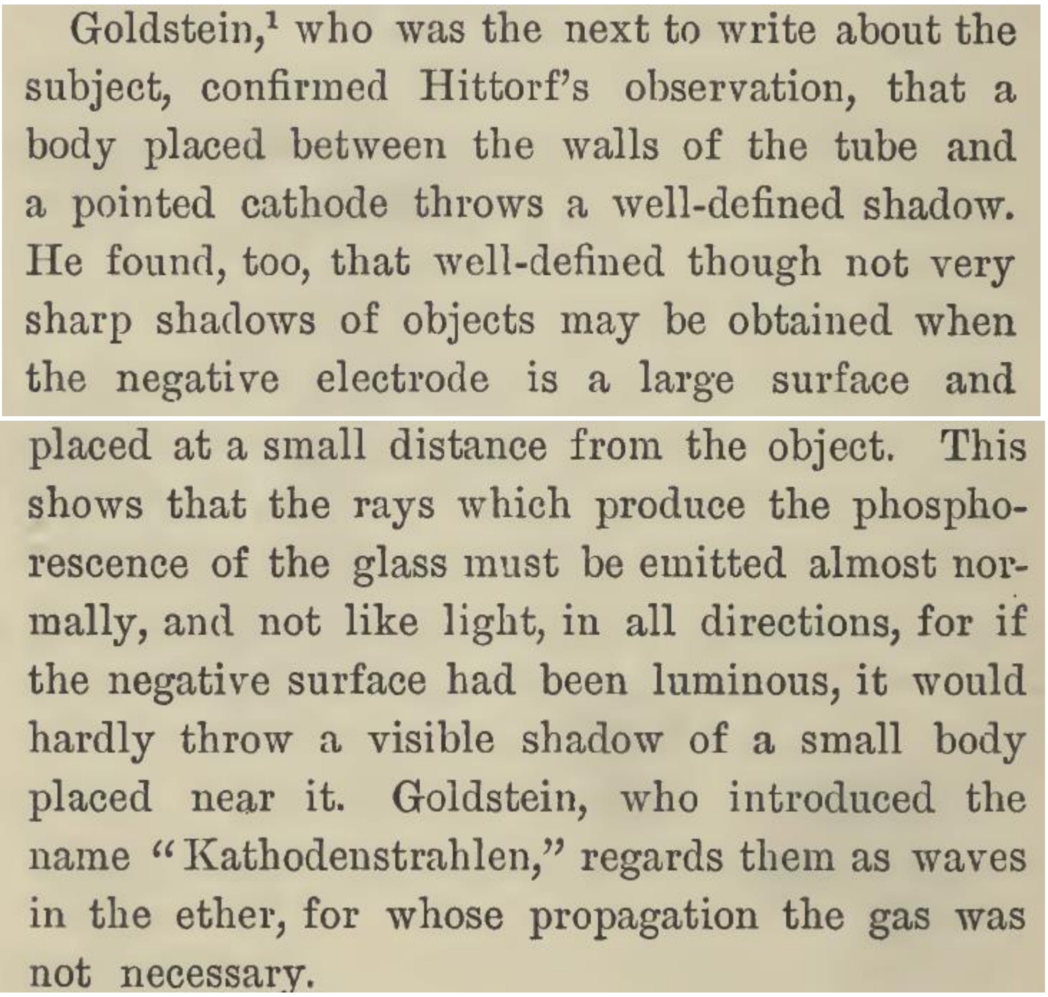


Hittorf,[2] who greatly extended our knowledge of the subject, found that the agent producing the phosphorescence was intercepted by a solid or liquid, whether conductor or insulator, placed between the cathode and the walls of the tube. As



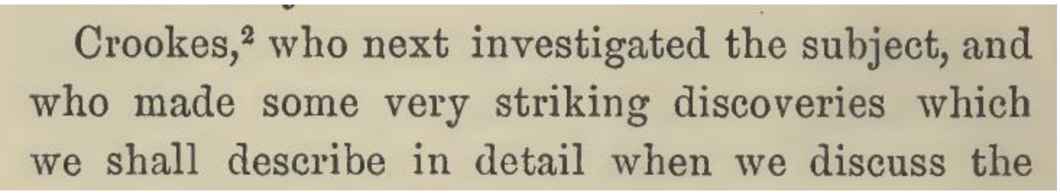
the result of his researches, he came to the conclusion that the parts of the surface of the negative electrode are the starting-points of a motion which in a gas spreads uniformly like rays, and coincides in this respect with wave motion.

Then he continues with Goldstein as,

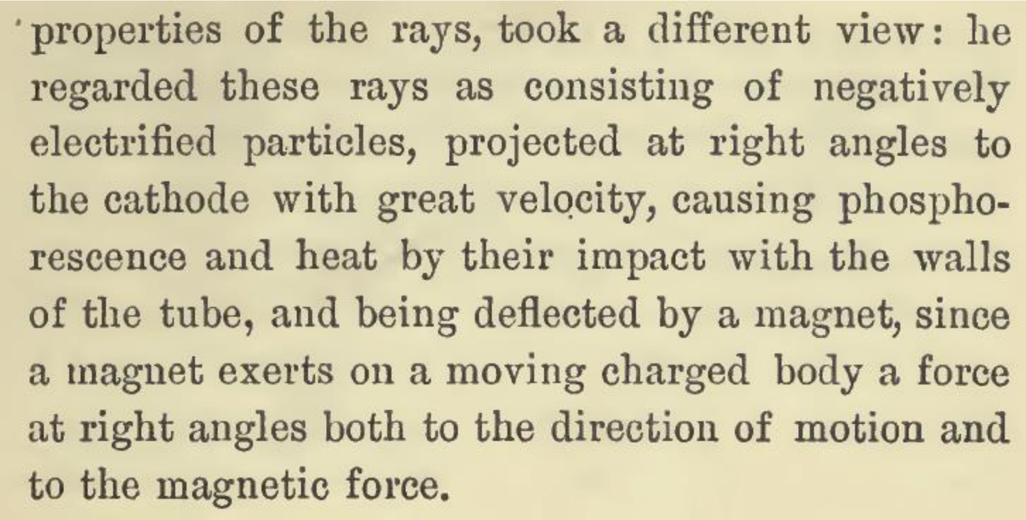


Goldstein,[1] who was the next to write about the subject, confirmed Hittorf's observation, that a body placed between the walls of the tube and a pointed cathode throws a well-defined shadow. He found, too, that well-defined though not very sharp shadows of objects may be obtained when the negative electrode is a large surface and placed at a small distance from the object. This shows that the rays which produce the phosphorescence of the glass must be emitted almost normally, and not like light, in all directions, for if the negative surface had been luminous, it would hardly throw a visible shadow of a small body placed near it. Goldstein, who introduced the name "Kathodenstrahlen," regards them as waves in the ether, for whose propagation the gas was not necessary.

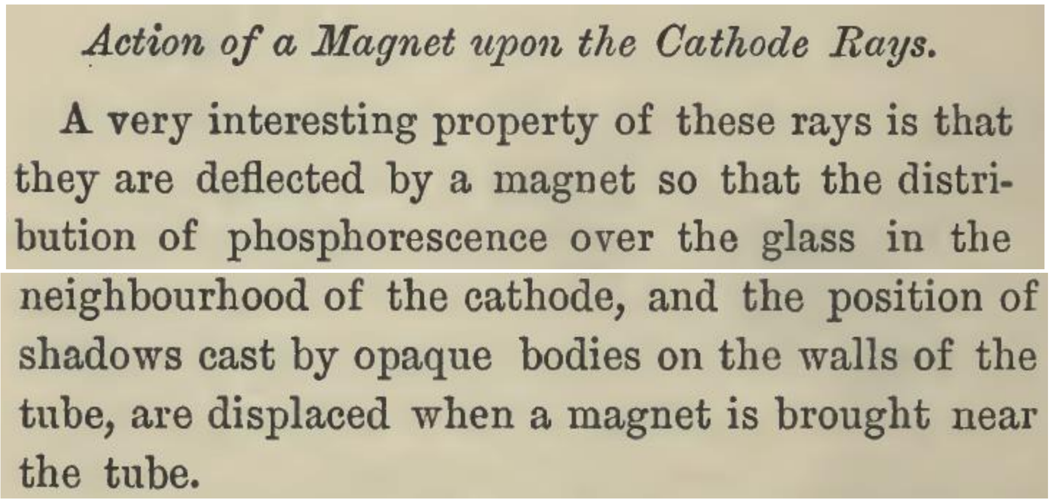
About Sir Crooks he writes,



Crookes,[2] who next investigated the subject, and who made some very striking discoveries which we shall describe in detail when we discuss the


properties of the rays, took a different view: he regarded these rays as consisting of negatively electrified particles, projected at right angles to the cathode with great velocity, causing phosphorescence and heat by their impact with the walls of the tube, and being deflected by a magnet, since a magnet exerts on a moving charged body a force at right angles both to the direction of motion and to the magnetic force.

After considering properties of cathode rays, their produced thermal and mechanical effects, he introduces the effects of a magnet on these rays.


Action of a Magnet upon the Cathode Rays.

A very interesting property of these rays is that they are deflected by a magnet so that the distribution of phosphorescence over the glass in the neighbourhood of the cathode, and the position of shadows cast by opaque bodies on the walls of the tube, are displaced when a magnet is brought near the tube.

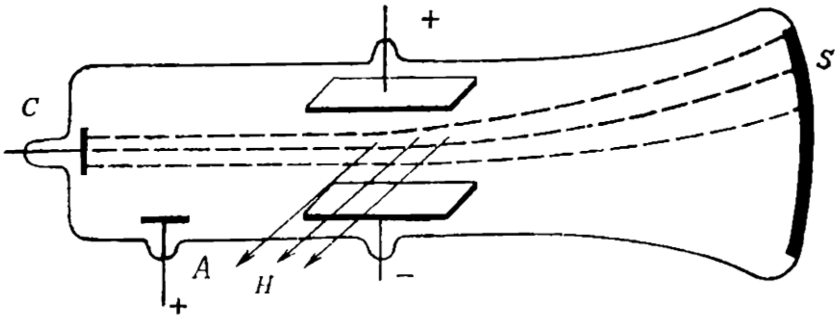
He showed that, in an electric field of strength E the presumed particles are deflected over an angle.

$$\Theta = \frac{e}{m} E \frac{l}{v^2}$$

Here e is the charge of the particle, m is its mass, l, the length of the capacitor and v, the (unknown) velocity of the particle. Note the ratio of e/m. It is the charge to mass ratio which about these particles were very large.

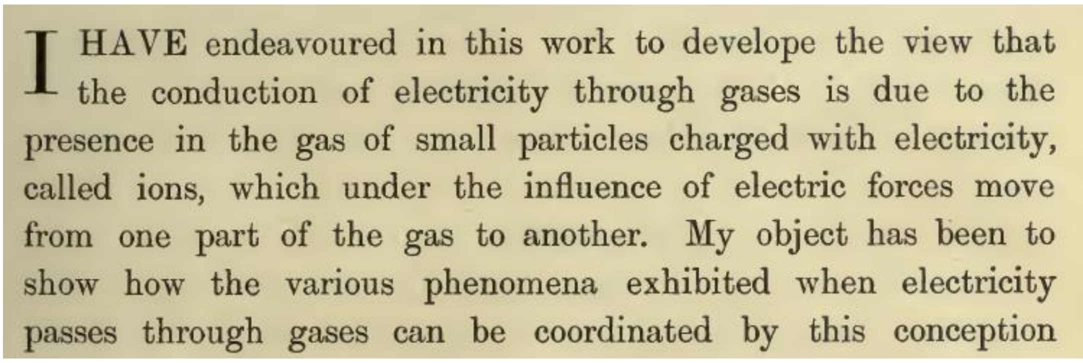
For magnetic field B, the deflection angles a may be shown to be

$$\alpha = \frac{e}{m} B \frac{l}{v}$$



In the case $\theta \approx \alpha$, using the value of v one can calculate the ratio of specific charge of electron i.e. e/m. The value of this ratio is constant in different types of gases. In cathode rays this ratio is about 10^3 times that of the lightest possible hydrogen ion H^+. Thus, Thomson concluded that "the mass of the cathode-ray particles was 10^3 times smaller than that of H+". This was the first to formulation for a new elementary particle of matter namely "the electron".

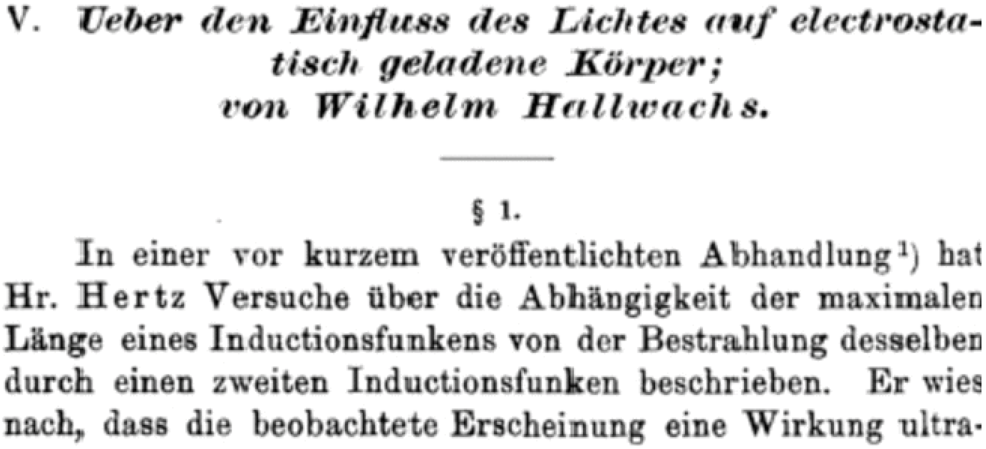
Thomson was also awarded Nobel Prize in 1906 for his "theoretical and experimental investigations on the conduction of electricity by gases". In the preface he says;



I HAVE endeavoured in this work to develope the view that the conduction of electricity through gases is due to the presence in the gas of small particles charged with electricity, called ions, which under the influence of electric forces move from one part of the gas to another. My object has been to show how the various phenomena exhibited when electricity passes through gases can be coordinated by this conception

The discovery of the first elementary particle paved the way for other experiments of physics in this respect.

Another evidence for existing of electron Photo electric effect was discovered by **Heinrich Hertz** in early 1887. During his experiments to prove Maxwell’s electromagnetic theory of light he observed that placing the spark gap in different situation of light exposing from a dark box to a glass box increase the length of the spark. **Wilhelm Hallwachs** (1859-1922) also confirmed these results in 1888. He published it as *Ueber den Einfluss des Lichtes auf electrostatisch geladene Körper* " On the influence of light on electrostatically charged bodies " in *Annalen der physik*.



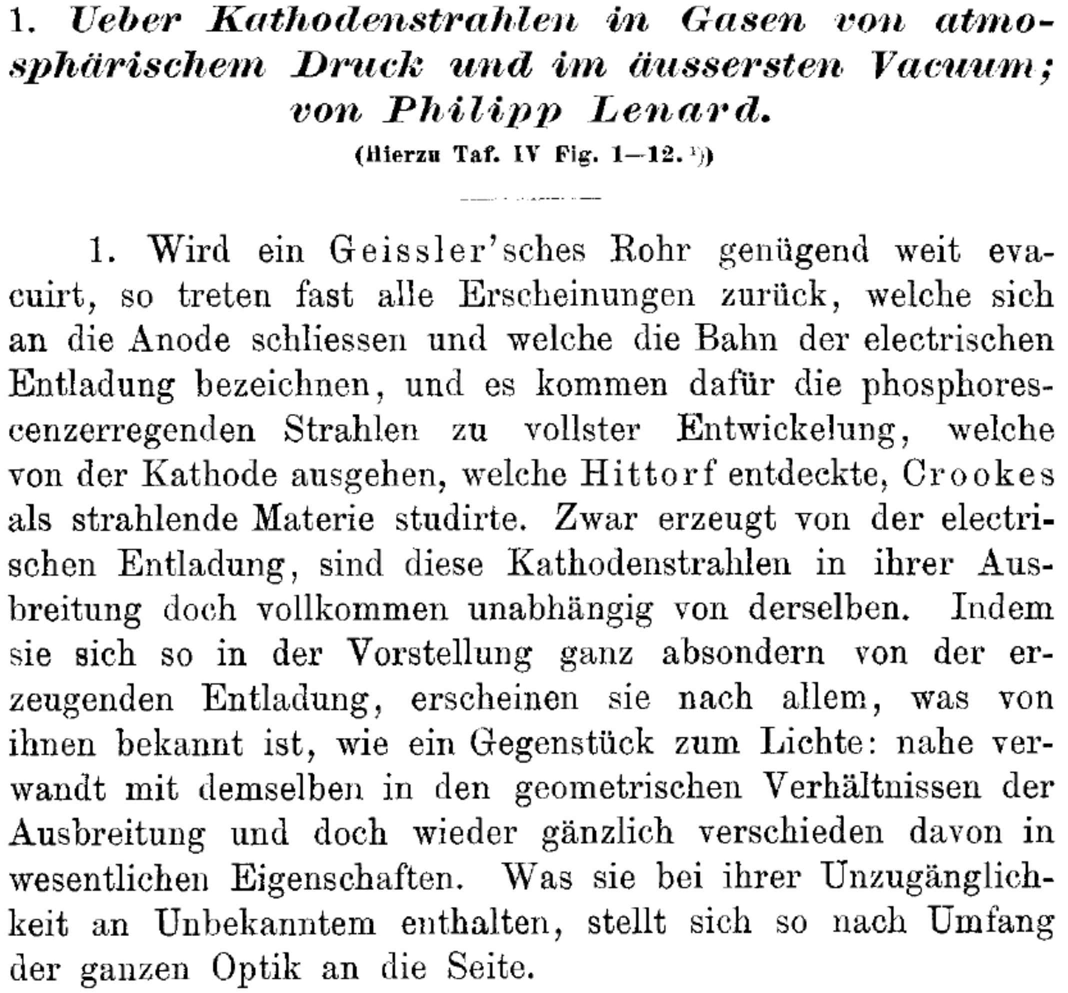
V. *Ueber den Einfluss des Lichtes auf electrostatisch geladene Körper;*
von Wilhelm Hallwachs.

§ 1.

In einer vor kurzem veröffentlichten Abhandlung[1]) hat Hr. Hertz Versuche über die Abhängigkeit der maximalen Länge eines Inductionsfunkens von der Bestrahlung desselben durch einen zweiten Inductionsfunken beschrieben. Er wies nach, dass die beobachtete Erscheinung eine Wirkung ultra-

He showed that UV light on a Zinc plate connected to a battery generated a current (because of electron emission). In this phenomenon electrons ejects from the surface of a metal when light is incident on it. These ejected electrons are called "photoelectrons". In fact, Thompson found that the current amount varied with respect to radiation intensity and frequency.

Based on Thomson's experiment **Philipp Lenard** (1862– 1947) conducted "many experiments with aluminum foil of various thickness". He found in 1899 that, "the particles produced by the photo electric effect had the same specific charge as the electron did". He published his results in *Ueber Kathodenstrahlen in Gasen von atmosphärischem Druck und im äussersten Vacuum* " On cathode rays in gases at atmospheric pressure and in extreme vacuum" in *Annalen der physik.* This work brought him Nobel prize in 1905.



1. *Ueber Kathodenstrahlen in Gasen von atmosphärischem Druck und im äussersten Vacuum; von Philipp Lenard.*

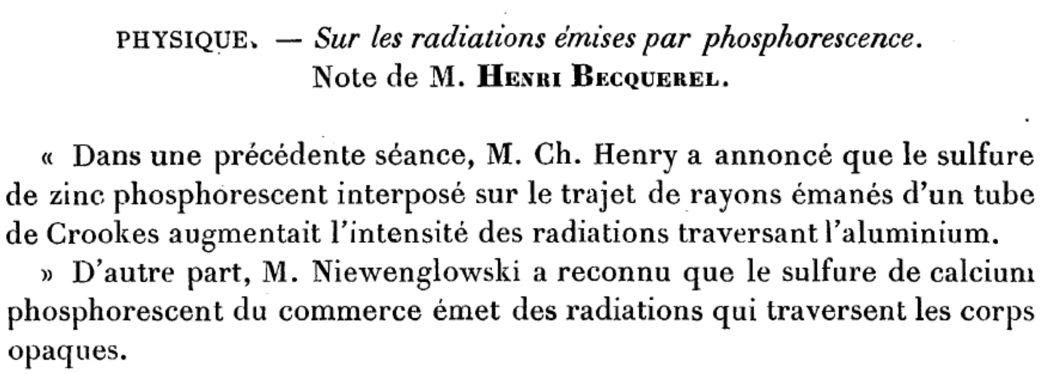
(Hierzu Taf. IV Fig. 1—12.[1])

1. Wird ein Geissler'sches Rohr genügend weit evacuirt, so treten fast alle Erscheinungen zurück, welche sich an die Anode schliessen und welche die Bahn der electrischen Entladung bezeichnen, und es kommen dafür die phosphorescenzerregenden Strahlen zu vollster Entwickelung, welche von der Kathode ausgehen, welche Hittorf entdeckte, Crookes als strahlende Materie studirte. Zwar erzeugt von der electrischen Entladung, sind diese Kathodenstrahlen in ihrer Ausbreitung doch vollkommen unabhängig von derselben. Indem sie sich so in der Vorstellung ganz absondern von der erzeugenden Entladung, erscheinen sie nach allem, was von ihnen bekannt ist, wie ein Gegenstück zum Lichte: nahe verwandt mit demselben in den geometrischen Verhältnissen der Ausbreitung und doch wieder gänzlich verschieden davon in wesentlichen Eigenschaften. Was sie bei ihrer Unzugänglichkeit an Unbekanntem enthalten, stellt sich so nach Umfang der ganzen Optik an die Seite.

If a Geissler tube is evacuated far enough, almost all the phenomena which are connected to the anode and which indicate the course of the electrical discharge recede, and the phosphorescent rays which emanate from the cathode, come into full development Hittorf discovered Crookes studied as radiant matter. It is true that these cathode rays are produced by the electric discharge, but in their propagation, they are completely independent of it. By completely separating themselves in the imagination from the producing discharge, they appear, according to all that is known of them, as a counterpart to the light: Closely related to it in the geometrical relationships of expansion and yet again completely different from it in essential properties. What they contain of the unknown, given their inaccessibility, is placed alongside the whole optics in scope.

In his long running paper (44 pages) he showed that, it is just the emission of electrons that is the essence of the photoelectric effect.

Another evidence for the existence of the electron as an independent particle was the phenomenon of "radioactivity". It was discovered by French physicist **A. H. Becquerel** (1852-1908). Enthused by Röntgen's discovery of x-rays he started his own investigations. He published his findings in six papers in *Comptes rendus des séances de l'Académie des Sciences*, Vol. 122. On February 26, 1896 as *Sur les radiations émises par phosphorescence* "On the radiations emitted by phosphorescent bodies" and Sur *les radiations invisibles émises par les corps phosphorescents* "On the invisible radiations emitted by phosphorescent bodies".



PHYSIQUE. — *Sur les radiations émises par phosphorescence.* Note de M. **Henri Becquerel.**

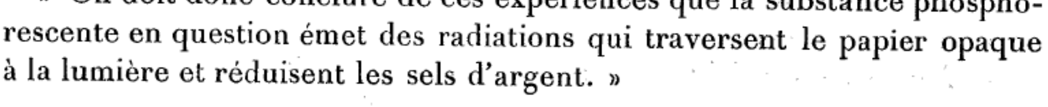
« Dans une précédente séance, M. Ch. Henry a annoncé que le sulfure de zinc phosphorescent interposé sur le trajet de rayons émanés d'un tube de Crookes augmentait l'intensité des radiations traversant l'aluminium.

» D'autre part, M. Niewenglowski a reconnu que le sulfure de calcium phosphorescent du commerce émet des radiations qui traversent les corps opaques.

In a previous session, Mr. Ch. Henry announced that the phosphorescent sulphide of zinc interposed on the path of rays emanating from a Crookes tube increased the intensity of the radiations passing through the aluminum.

On the other hand, M. Niewenglowski has recognized that commercially available phosphorescent sulphide of calcium emits radiations which pass through opaque bodies.

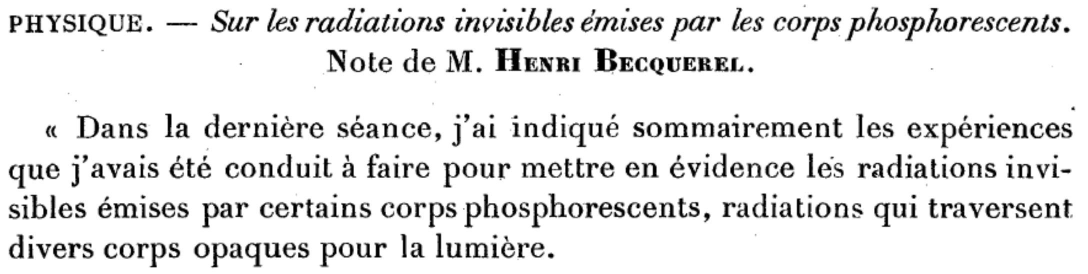
Then he set some experiments and conclude that,



» On doit donc conclure de ces expériences que la substance phosphorescente en question émet des radiations qui traversent le papier opaque à la lumière et réduisent les sels d'argent. »

We must therefore conclude from these experiments that the phosphorescent substance in question emits radiations which pass through the paper opaque to the light and reduce the silver salts.

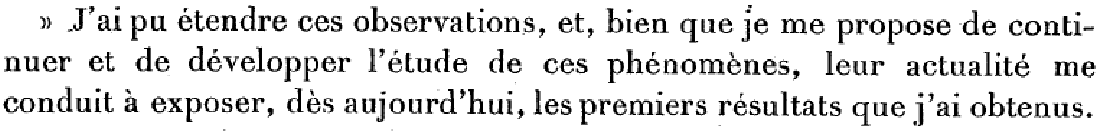
In continuation of his research he published,



PHYSIQUE. — *Sur les radiations invisibles émises par les corps phosphorescents.* Note de M. **Henri Becquerel.**

« Dans la dernière séance, j'ai indiqué sommairement les expériences que j'avais été conduit à faire pour mettre en évidence les radiations invisibles émises par certains corps phosphorescents, radiations qui traversent divers corps opaques pour la lumière.

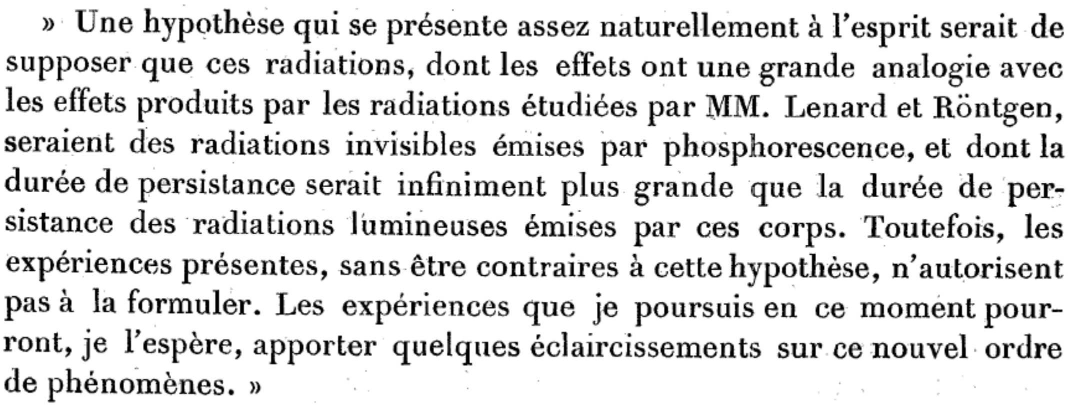
In the last session, I indicated summarily the experiments which I had been led to make to bring to light the invisible radiations emitted by certain phosphorescent bodies, radiations which cross various opaque bodies for light.



» J'ai pu étendre ces observations, et, bien que je me propose de continuer et de développer l'étude de ces phénomènes, leur actualité me conduit à exposer, dès aujourd'hui, les premiers résultats que j'ai obtenus.

I was able to extend these observations, and, although I propose to continue and develop the study of these phenomena, their topicality leads me to expose, as of today, the first results which I obtained.

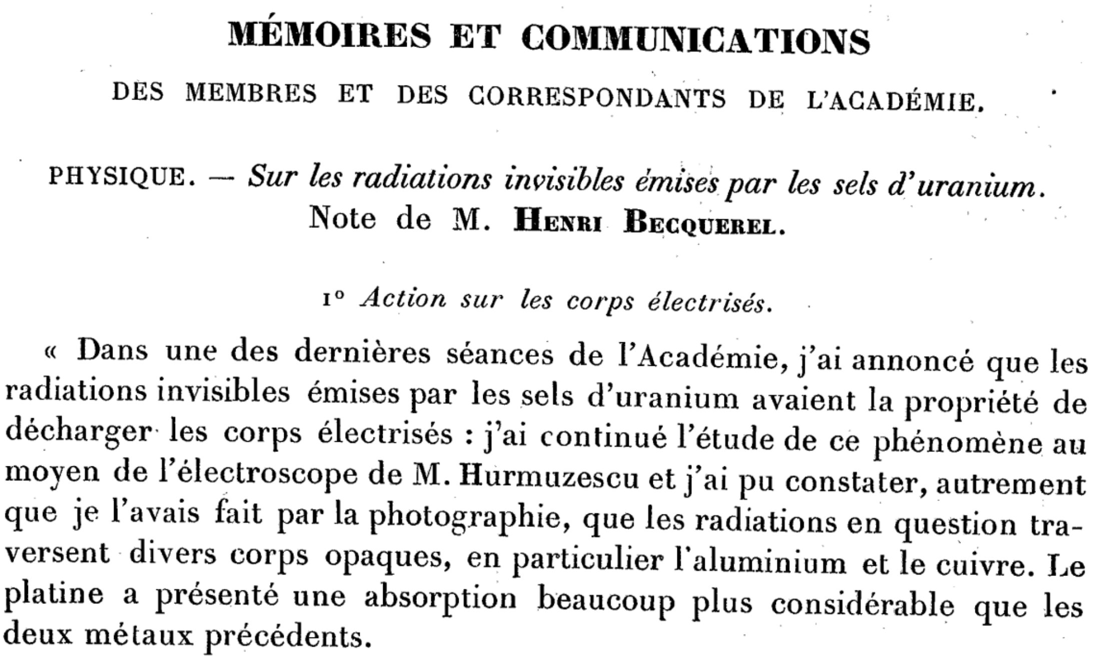
He concludes that,



» Une hypothèse qui se présente assez naturellement à l'esprit serait de supposer que ces radiations, dont les effets ont une grande analogie avec les effets produits par les radiations étudiées par MM. Lenard et Röntgen, seraient des radiations invisibles émises par phosphorescence, et dont la durée de persistance serait infiniment plus grande que la durée de persistance des radiations lumineuses émises par ces corps. Toutefois, les expériences présentes, sans être contraires à cette hypothèse, n'autorisent pas à la formuler. Les expériences que je poursuis en ce moment pourront, je l'espère, apporter quelques éclaircissements sur ce nouvel ordre de phénomènes. »

A hypothesis which presents itself quite naturally to the mind would be to suppose that these radiations, the effects of which have a great analogy with the effects produced by the radiations studied by MM. Lenard and Rontgen, would be invisible radiations emitted by phosphorescence, and whose duration of persistence would be infinitely greater than the duration of persistence of the luminous radiations emitted by these bodies. However, the present experiences, without being contrary to this hypothesis, do not allow us to formulate it. The experiments which I am pursuing at the moment may, I hope, shed some light on this new order of phenomena.

He continued his research and published *Sur quelquespropriétés nouvelles des radiations invisibles émises par divers corps phosphorescents* "on some new properties of the invisible radiations emitted by various phosphorescent bodies", and *Sur les radiations invisibles émises par les sels d'uranium* " On the invisible radiations emitted by uranium salts".



MÉMOIRES ET COMMUNICATIONS

DES MEMBRES ET DES CORRESPONDANTS DE L'ACADÉMIE.

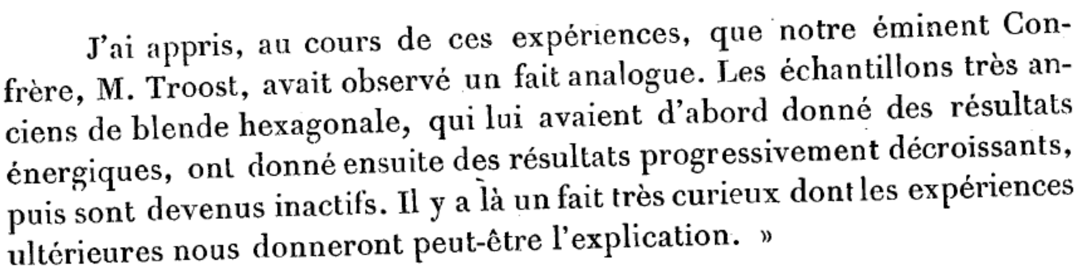
PHYSIQUE. — *Sur les radiations invisibles émises par les sels d'uranium.*
Note de M. **Henri Becquerel.**

1° *Action sur les corps électrisés.*

« Dans une des dernières séances de l'Académie, j'ai annoncé que les radiations invisibles émises par les sels d'uranium avaient la propriété de décharger les corps électrisés : j'ai continué l'étude de ce phénomène au moyen de l'électroscope de M. Hurmuzescu et j'ai pu constater, autrement que je l'avais fait par la photographie, que les radiations en question traversent divers corps opaques, en particulier l'aluminium et le cuivre. Le platine a présenté une absorption beaucoup plus considérable que les deux métaux précédents.

In one of the last sessions of the Academy, I announced that the invisible radiations emitted by uranium salts had the property of discharging electrified bodies. I continued the study of this phenomenon by means of the electroscope of M. Hurmuzescu and I was able to ascertain, otherwise than I had done by photography, that the radiations in question pass through various opaque bodies, in particular aluminum and copper. Platinum exhibited a much greater absorption than the two previous metals.

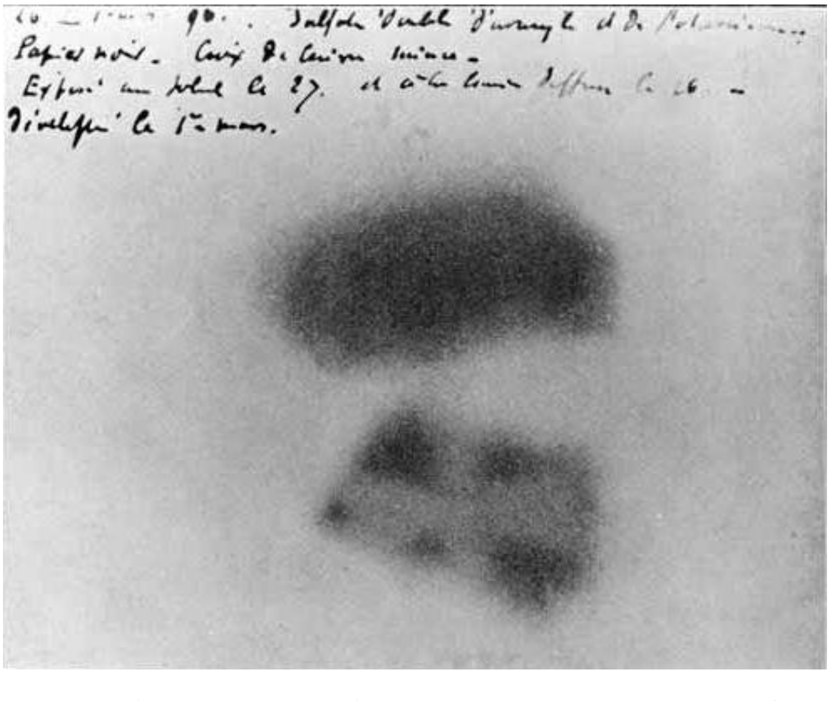
He goes on with persistence and excitement in radioactive emission of various uranium salts. Then considers absorption by various substances, refraction and deviations exhibited by various substances. However, sulphides rendered highly phosphorescent with the discharge of the battery, but it didn't show any manifest activity through the black paper. In the end he declares that,



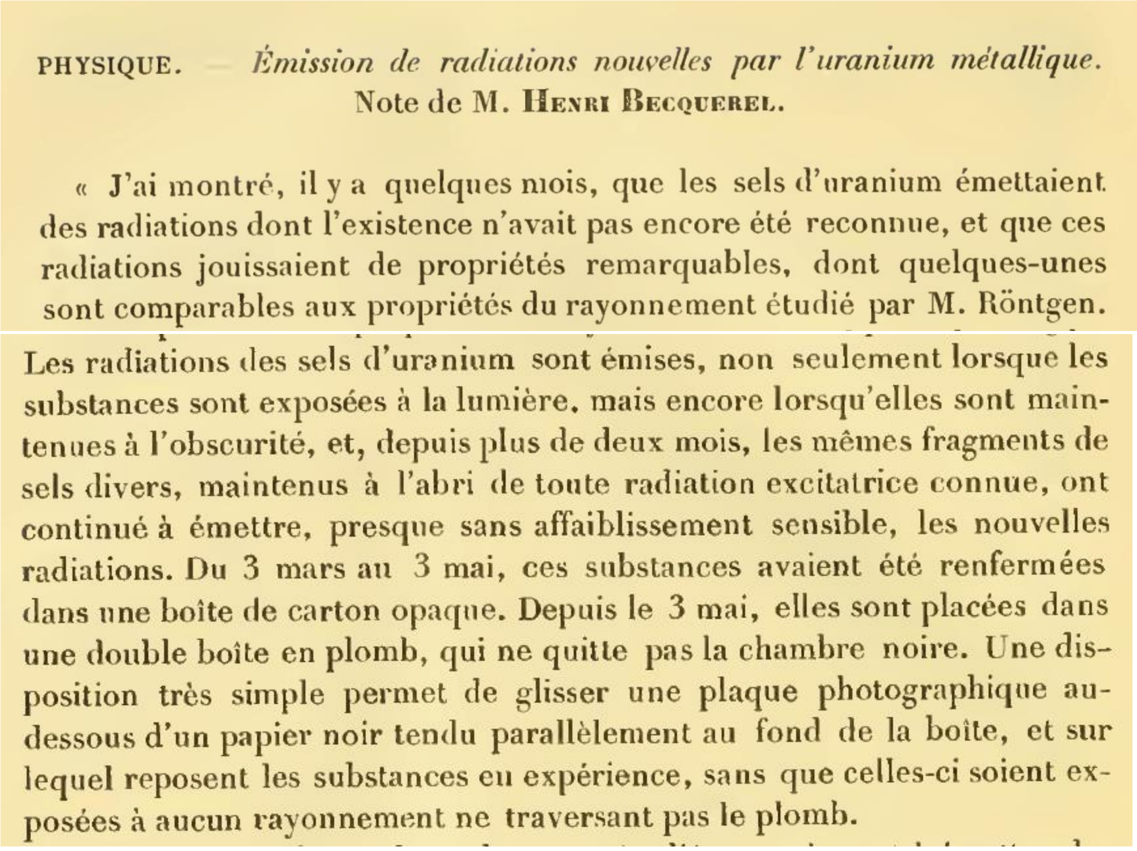
J'ai appris, au cours de ces expériences, que notre éminent Confrère, M. Troost, avait observé un fait analogue. Les échantillons très anciens de blende hexagonale, qui lui avaient d'abord donné des résultats énergiques, ont donné ensuite des résultats progressivement décroissants, puis sont devenus inactifs. Il y a là un fait très curieux dont les expériences ultérieures nous donneront peut-être l'explication. »

I learned, during these experiments, that our eminent colleague, Mr. Troost, had observed a similar fact. The very old samples of hexagonal blende, which had at first given him energetic results, then gave progressively decreasing results, then became inactive. There is a very curious fact here which later experiments will perhaps give us the explanation.

Using a photographic plate made by him shows the effects of exposure to radioactivity. "A metal Maltese cross, placed between the plate and radioactive uranium salt, left a clearly visible shadow on the plate". This discovery brought him Noble prize in 1900 with Marie Skłodowska and Pierre Curie for their discovery of radioactivity. This plate is showed below.



Then perusing his researches on uranium and with some responses to Charles Henry and published *Sur les propriétés différentes des radiations invisibles emises par les sels d'uranium, et du rayonnement de la paroi anticathodiqued'un* *tube de Crookes* "On the different properties of the invisible radiations emitted by uranium salts, and of the radiation from the anticathode wall of a Crookes tube". Then he went on with some collaboration with Henri Moissan (1852-1907) working on uranium. In his experiments he noticed that "uranium salts emitted some invisible rays that caused blackening of a photographic plate". He ends with a paper on metallic uranium.



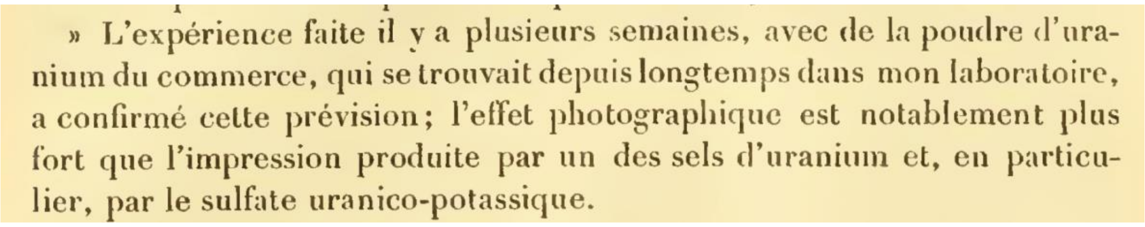
PHYSIQUE. — *Émission de radiations nouvelles par l'uranium métallique.* Note de M. Henri Becquerel.

« J'ai montré, il y a quelques mois, que les sels d'uranium émettaient des radiations dont l'existence n'avait pas encore été reconnue, et que ces radiations jouissaient de propriétés remarquables, dont quelques-unes sont comparables aux propriétés du rayonnement étudié par M. Röntgen.

Les radiations des sels d'uranium sont émises, non seulement lorsque les substances sont exposées à la lumière, mais encore lorsqu'elles sont maintenues à l'obscurité, et, depuis plus de deux mois, les mêmes fragments de sels divers, maintenus à l'abri de toute radiation excitatrice connue, ont continué à émettre, presque sans affaiblissement sensible, les nouvelles radiations. Du 3 mars au 3 mai, ces substances avaient été renfermées dans une boîte de carton opaque. Depuis le 3 mai, elles sont placées dans une double boîte en plomb, qui ne quitte pas la chambre noire. Une disposition très simple permet de glisser une plaque photographique au-dessous d'un papier noir tendu parallèlement au fond de la boîte, et sur lequel reposent les substances en expérience, sans que celles-ci soient exposées à aucun rayonnement ne traversant pas le plomb.

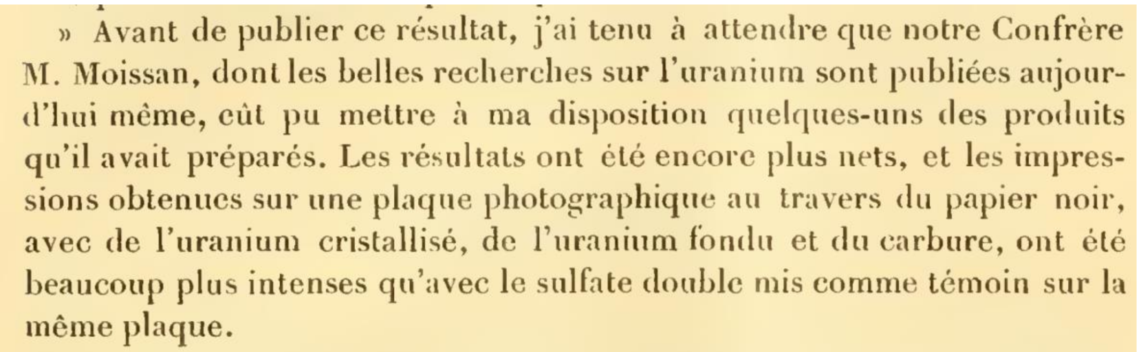
I showed, a few months ago, that uranium salts emit radiations whose existence had not yet been recognized, and that these radiations enjoy remarkable properties, some of which are comparable to the properties of radiation studied by M. Rontgen. The radiations of the uranium salts are emitted, not only when the substances are exposed to light, but also when they are kept in the dark, and, for more than two months, the same fragments of various salts, kept in the dark, shielded from all known excitatory radiation, continued to emit, almost without noticeable attenuation, the new radiations. From March 3 to May 3, these substances had been enclosed in an opaque cardboard box. Since May 3, they have been placed in a double lead box, which does not leave the darkroom. A very simple arrangement allows a photographic plate to be slid under a black paper stretched parallel to the bottom of the box, and on which the substances in experiment rest, without these being exposed to any radiation that does not pass through the lead.

Then with some descriptions of previous experiments he goes on with Moissan's contributions.



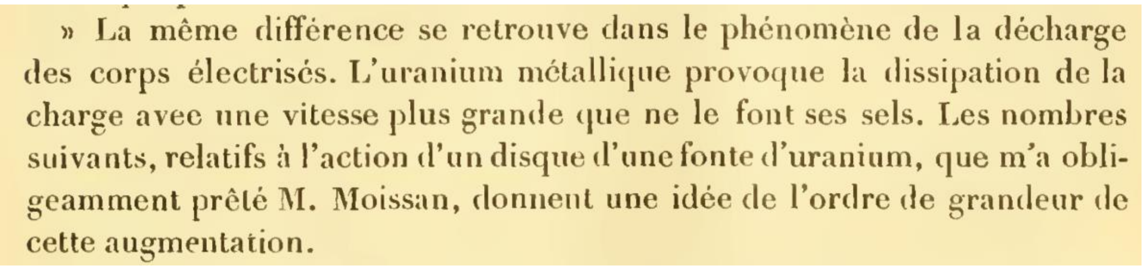
» L'expérience faite il y a plusieurs semaines, avec de la poudre d'uranium du commerce, qui se trouvait depuis longtemps dans mon laboratoire, a confirmé cette prévision; l'effet photographique est notablement plus fort que l'impression produite par un des sels d'uranium et, en particulier, par le sulfate uranico-potassique.

The experiment made several weeks ago with commercial uranium powder, which had been in my laboratory for a long time, confirmed this prediction; the photographic effect is notably stronger than the impression produced by one of the uranium salts and, in particular, by the uranico-potassic sulphate.



» Avant de publier ce résultat, j'ai tenu à attendre que notre Confrère M. Moissan, dont les belles recherches sur l'uranium sont publiées aujourd'hui même, eût pu mettre à ma disposition quelques-uns des produits qu'il avait préparés. Les résultats ont été encore plus nets, et les impressions obtenues sur une plaque photographique au travers du papier noir, avec de l'uranium cristallisé, de l'uranium fondu et du carbure, ont été beaucoup plus intenses qu'avec le sulfate double mis comme témoin sur la même plaque.

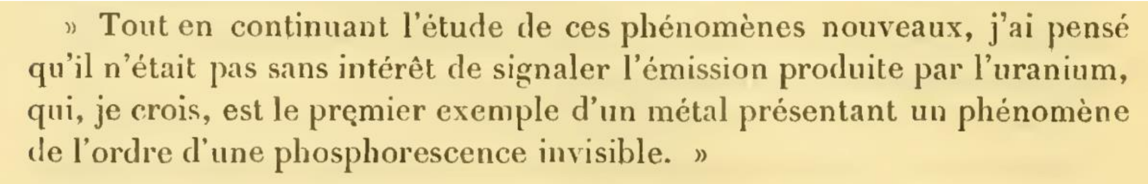
Moissan, whose fine researches on uranium are published today, could have made some of the products he had prepared available to me. The results were even clearer, and the impressions obtained on a photographic plate through black paper, with crystallized uranium, molten uranium and carbide, were much more intense than with the double sulphate put as witness on the same plate.



» La même différence se retrouve dans le phénomène de la décharge des corps électrisés. L'uranium métallique provoque la dissipation de la charge avec une vitesse plus grande que ne le font ses sels. Les nombres suivants, relatifs à l'action d'un disque d'une fonte d'uranium, que m'a obligeamment prêté M. Moissan, donnent une idée de l'ordre de grandeur de cette augmentation.

The same difference is found in the phenomenon of the discharge of electrified bodies. Uranium metal causes charge to dissipate with a greater rate than do its salts. The following numbers, relating to the action of a disk of uranium melt, which M. Moissan kindly lent me, give an idea of the order of magnitude of this increase.

Then he goes on with some details about his experiment and ends that,

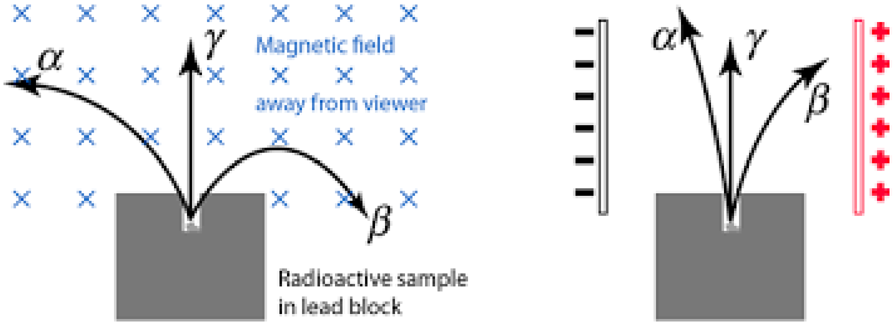


» Tout en continuant l’étude de ces phénomènes nouveaux, j’ai pensé qu’il n’était pas sans intérêt de signaler l’émission produite par l’uranium, qui, je crois, est le premier exemple d’un métal présentant un phénomène de l’ordre d’une phosphorescence invisible. »

While continuing the study of these new phenomena, I thought that it was not without interest to announce the emission produced by uranium, which, I believe, is the first example of a metal presenting a phenomenon of the order of an invisible phosphorescence.

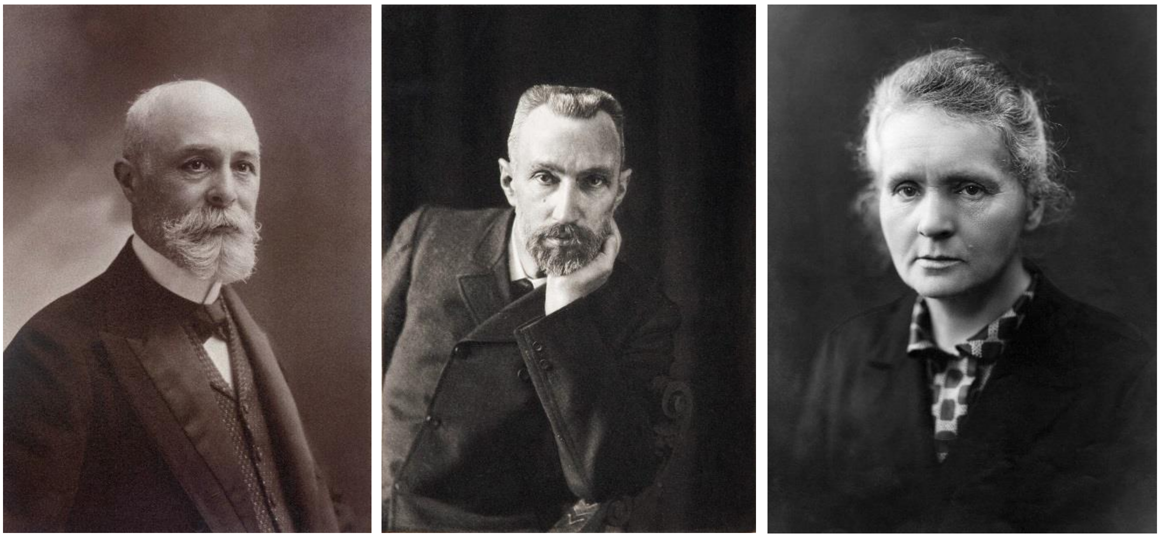
Subsequent researches showed that in a magnetic field these enigmatic rays were composed of three components. They were called alpha-, beta and gamma-rays. The alpha particle has the shortest range and thus strong interaction with matter. While the gamma ray is the next with extreme penetrating property. Beta ray is in the third place with a strong radioactivity interacts with matter.

More investigations revealed that, when Gamma-rays is passed through the magnetic field they will be undisturbed. While for alpha- and Beta-rays, there is a "judging from the way they were deflected, behaved like positive and negative charges, respectively". This is showed in figure below (www. Hyperphysics.com)



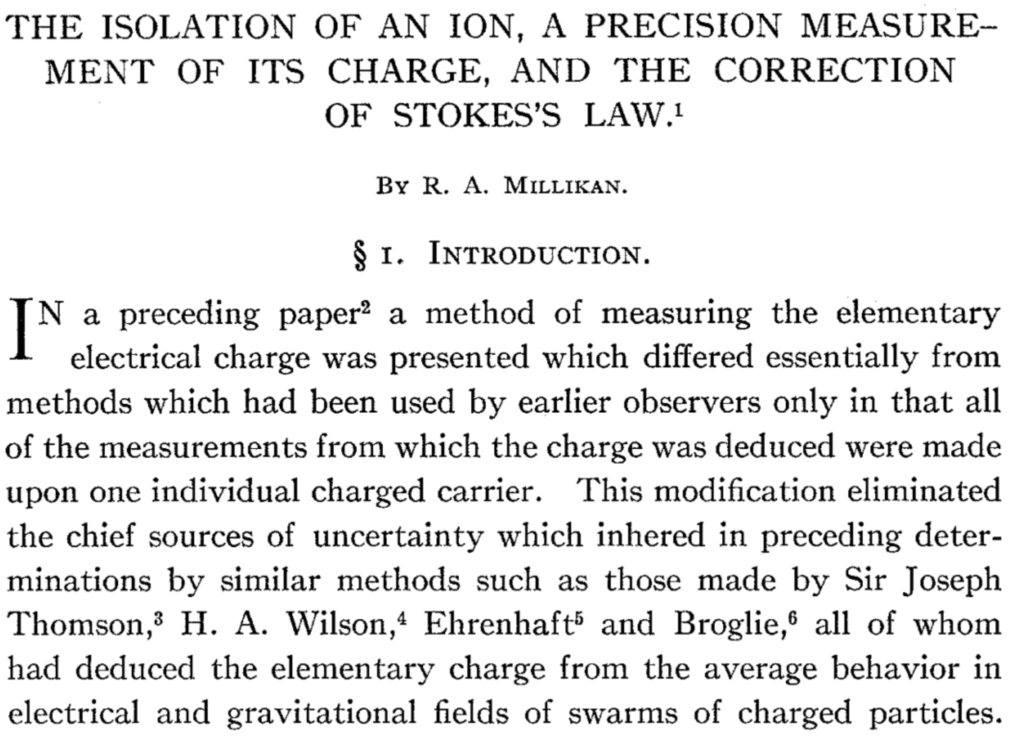
In 1900, by measuring the specific charge of beta-particles, Becquerel found that beta-rays were in fact a stream of electrons. There was no doubt now that the electron was indeed an independent particle and, even more, a building block of the Universe.

Two years later of Becquerel's discovery of the phenomena of radioactivity in matter, in 1989, Marie Salomea Skłodowska (1867-1934) and Pierre Curie (1859 –1906) discovered that there exists other matter with similar radiation of uranium. They were called polonium and radium. These matters emit stronger radiation than uranium. After them this phenomenon is coined as "radioactivity".



Considering electrolysis data and cathode-rays Faraday and others evaluated averaged charge of the electron. But what is amount of the electron’s charge and mass distinctly?

American experimental physicist **Robert Andrews Millikan** (1868-1953) was the first to directly measured the electron charge. In his experiment performed in 1909 he produced oil droplets in the space between two horizontal capacitor plates. Millikan published the results as "The Isolation of an Ion, a Precision Measurement of Its Charge, and the Correction off Stokes's Law" in Physical Review in the year 1911.



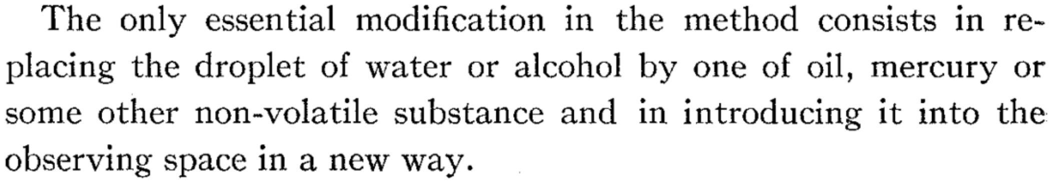
THE ISOLATION OF AN ION, A PRECISION MEASUREMENT OF ITS CHARGE, AND THE CORRECTION OF STOKES'S LAW.[1]

By R. A. Millikan.

§ 1. Introduction.

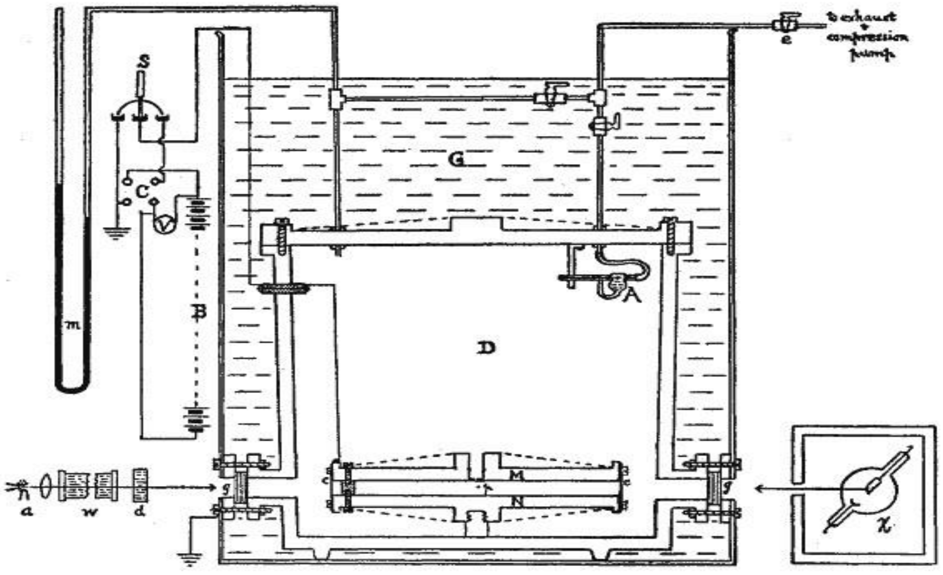
In a preceding paper[2] a method of measuring the elementary electrical charge was presented which differed essentially from methods which had been used by earlier observers only in that all of the measurements from which the charge was deduced were made upon one individual charged carrier. This modification eliminated the chief sources of uncertainty which inhered in preceding determinations by similar methods such as those made by Sir Joseph Thomson,[3] H. A. Wilson,[4] Ehrenhaft[5] and Broglie,[6] all of whom had deduced the elementary charge from the average behavior in electrical and gravitational fields of swarms of charged particles.

He continues with a brief introduction to previous method of such experiments which used water or alcohol droplets in stead of oil he used.



The only essential modification in the method consists in replacing the droplet of water or alcohol by one of oil, mercury or some other non-volatile substance and in introducing it into the observing space in a new way.

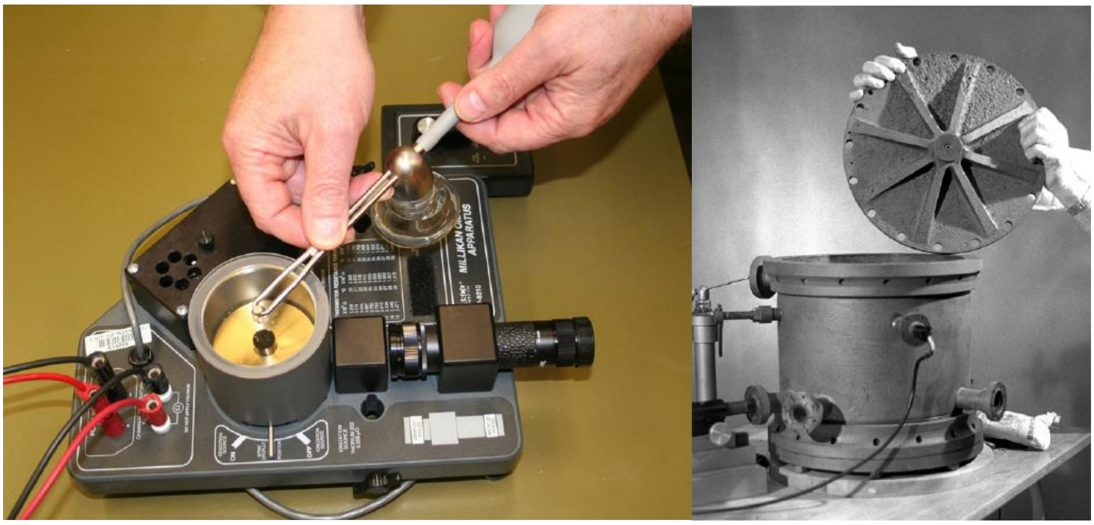
The apparatus he used for this experiment was as below.



Millikan starts to explain this device as below

> By means of a commercial "atomizer"[1] A a cloud of fine droplets of oil is blown with the aid of dust-free air into the dust-free chamber C. One or more of the droplets of this cloud is allowed to fall through a pin-hole p into the space between the plates M, N of a horizontal air condenser and the pin-hole is then closed by means of an electromagnetically operated cover not shown in the diagram.

Here he means the cover in the picture below, the modern and classical ones!



> A narrow parallel beam of light from an arc lamp enters the condenser through the first window and emerges through the last. The other window serves for observing, with the aid of a short focus telescope placed about 2 feet distant, the illuminated oil droplet as it floats in the air between the plates. The appearance of this drop is that of a brilliant star on a black background.

The drop falls under gravity but before reaching the lower plate it encounters with 3000 to 8000 volts of electrical field strength produced by battery B. the note is that under receiving a "frictional charges of the proper sign and strength" the drop acts against gravity toward the upper plate. Thus, a switch S is placed to prevent the strike with the upper plat and reverse it to fall under gravity. Using this switch, we have a "short-circuited" plate. He describes this process as below,

> Before it strikes it the plates are short-circuited by means of the switch S and the time required by the drop to fall under gravity the distance corresponding to the space between the cross hairs of the observing telescope is accurately determined. Then the rate at which the droplet moves up under the influence of the field is measured by timing it through the same distance when the field is on.

He repeated this process and indefinitely checked the speeds until the drop catches an ion by existing air or any usual way such as X-ray. This event and its exact distance are revealed for the observer by the "charge in the speed of the droplet under the influence of the field".

> From the sign and magnitude of this change in speed, taken in connection with the constant speed under gravity, the sign and the exact value of the charge carried by the captured ion are determined. The error in a single observation need not exceed one third of one per cent. It is from the values of the speeds observed that all of the conclusions above mentioned are directly and simply deduced.

The interesting point was that, when the drop carries one elementary charge it can be completely neutralized by capturing an ion with the opposite sign. In this case its speed does not affect by the field. Therefore, the amount of charge of the drop is same of the captured ion.

Then he deduces these relative values of changes for any given droplet. He starts with relation below.

$$\frac{v_1}{v_2} = \frac{mg}{Ie_n - mg} \quad or\ e_n = \frac{mg}{I}\left(\frac{v_1 + v_2}{v_1}\right)$$

Here m is the apparent mass of any droplet and e_n stands for the charges they carry. While, v_1 and v_2 are the speeds under gravity and electrical field with strength I. However, if we consider the viscosity coefficient of the medium (μ), the acting force (F) on a spherical droplet with radius (r) and the velocity of the droplet (v), then we have **Stokes' law**, for the frictional force.

$$F = 6\pi\mu r v$$

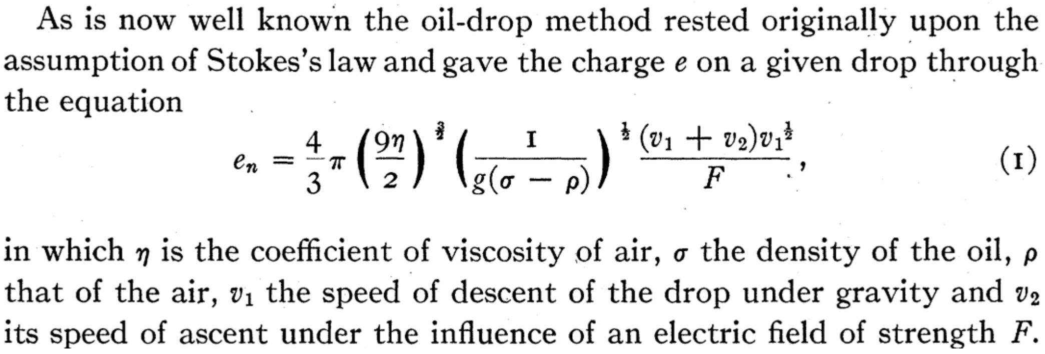
Now with substituting droplet density (σ) and medium density (ρ) we have Stokes relation for the rate of fall of a droplet under gravity.

$$v_1 = \frac{2}{9}\frac{gr^2}{\mu}(\sigma - \rho)$$

Finally, with eliminating of m from the first relation with the help of above relation and mass relation

$$m = \frac{4}{3}\pi r^3(\sigma - \rho)$$

We get the new relation for charge e_n. In the strat of his next paper on subject Milikan write this as below.

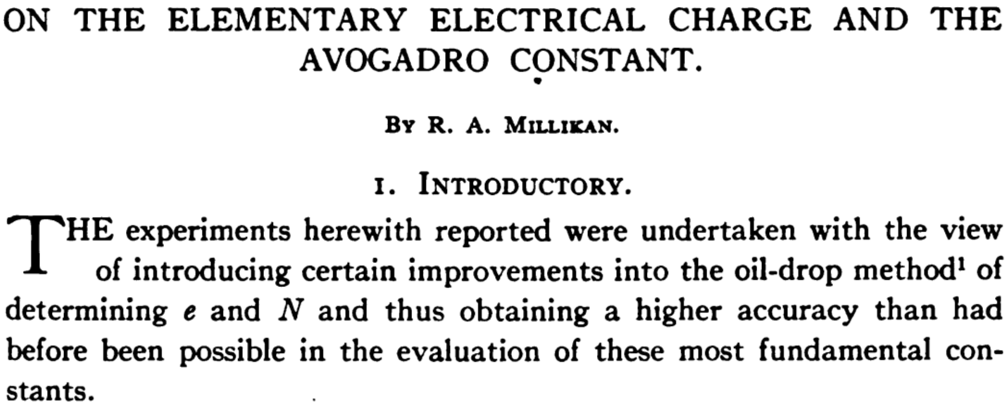


As is now well known the oil-drop method rested originally upon the assumption of Stokes's law and gave the charge e on a given drop through the equation

$$e_n = \frac{4}{3}\pi\left(\frac{9\eta}{2}\right)^{\frac{3}{2}}\left(\frac{1}{g(\sigma - \rho)}\right)^{\frac{1}{2}}\frac{(v_1 + v_2)v_1^{\frac{1}{2}}}{F}, \qquad (1)$$

in which η is the coefficient of viscosity of air, σ the density of the oil, ρ that of the air, v_1 the speed of descent of the drop under gravity and v_2 its speed of ascent under the influence of an electric field of strength F.

He published this paper in 1913 as "on the elementary electrical charge and the Avogadro constant" in "The Physical Review".



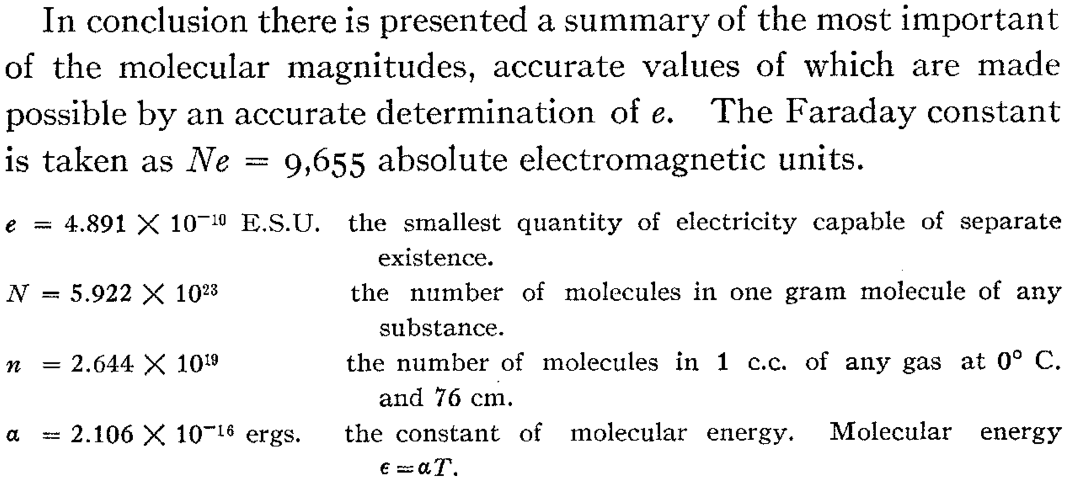
ON THE ELEMENTARY ELECTRICAL CHARGE AND THE AVOGADRO CONSTANT.

By R. A. Millikan.

1. Introductory.

THE experiments herewith reported were undertaken with the view of introducing certain improvements into the oil-drop method[1] of determining e and N and thus obtaining a higher accuracy than had before been possible in the evaluation of these most fundamental constants.

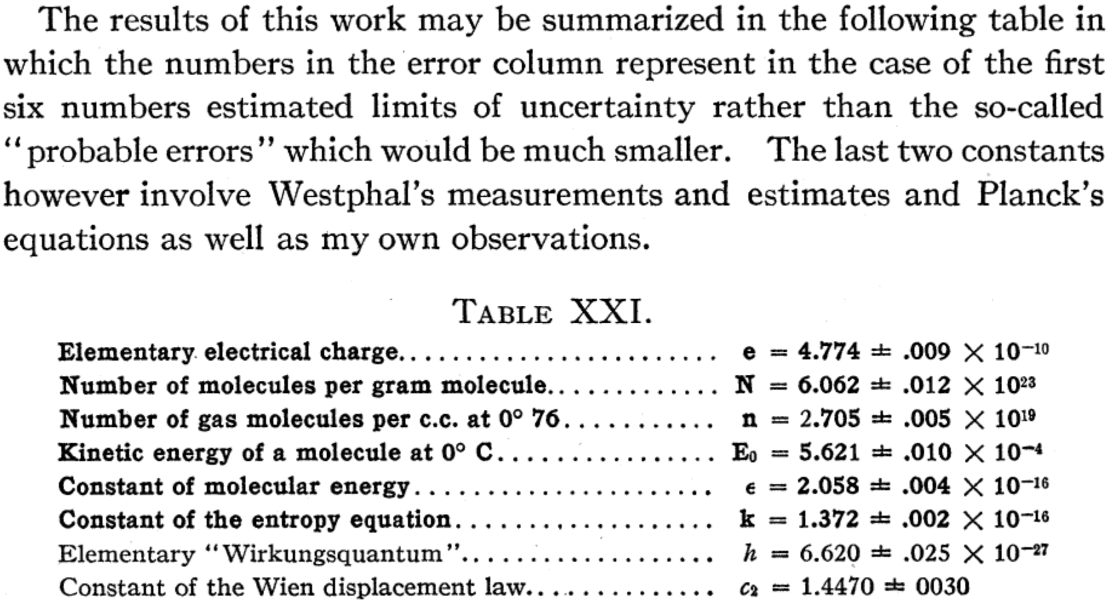
However, in the conclusion of his first paper "The Isolation of an Ion, a Precision Measurement of Its Charge, and the Correction off Stokes's Law" he finishes with some important constants as below.



In conclusion there is presented a summary of the most important of the molecular magnitudes, accurate values of which are made possible by an accurate determination of e. The Faraday constant is taken as $Ne = 9{,}655$ absolute electromagnetic units.

$e = 4.891 \times 10^{-10}$ E.S.U.	the smallest quantity of electricity capable of separate existence.
$N = 5.922 \times 10^{23}$	the number of molecules in one gram molecule of any substance.
$n = 2.644 \times 10^{19}$	the number of molecules in 1 c.c. of any gas at 0° C. and 76 cm.
$\alpha = 2.106 \times 10^{-16}$ ergs.	the constant of molecular energy. Molecular energy $\epsilon = \alpha T$.

But in this second one he ended with some corrections as



The results of this work may be summarized in the following table in which the numbers in the error column represent in the case of the first six numbers estimated limits of uncertainty rather than the so-called "probable errors" which would be much smaller. The last two constants however involve Westphal's measurements and estimates and Planck's equations as well as my own observations.

TABLE XXI.

Elementary electrical charge	$\mathbf{e} = 4.774 \pm .009 \times 10^{-10}$
Number of molecules per gram molecule	$\mathbf{N} = 6.062 \pm .012 \times 10^{23}$
Number of gas molecules per c.c. at 0° 76	$\mathbf{n} = 2.705 \pm .005 \times 10^{19}$
Kinetic energy of a molecule at 0° C	$\mathbf{E}_0 = 5.621 \pm .010 \times 10^{-4}$
Constant of molecular energy	$\epsilon = 2.058 \pm .004 \times 10^{-16}$
Constant of the entropy equation	$\mathbf{k} = 1.372 \pm .002 \times 10^{-16}$
Elementary "Wirkungsquantum"	$h = 6.620 \pm .025 \times 10^{-27}$
Constant of the Wien displacement law	$c_2 = 1.4470 \pm 0030$

In this paper he mentions that having a more correct value of radius r (or as he notes "a") of the droplet requires a precise value of e and define it as below.

$$r = \sqrt[3]{\frac{3Fe}{4\pi g(\sigma - \rho)} \frac{v_1}{(v_1 + v_2)}}$$

In the first paper, after introducing the formula for e_n he goes on with "primary observations upon the catching of ions by oil-drops". These observations were done in 4.5 hours under the influence of the field *F* and Gravity *G* taken upon oil-drops one by one to determine e_n. He tabulated these primary results for negative drops. We bring the first part of his table here.

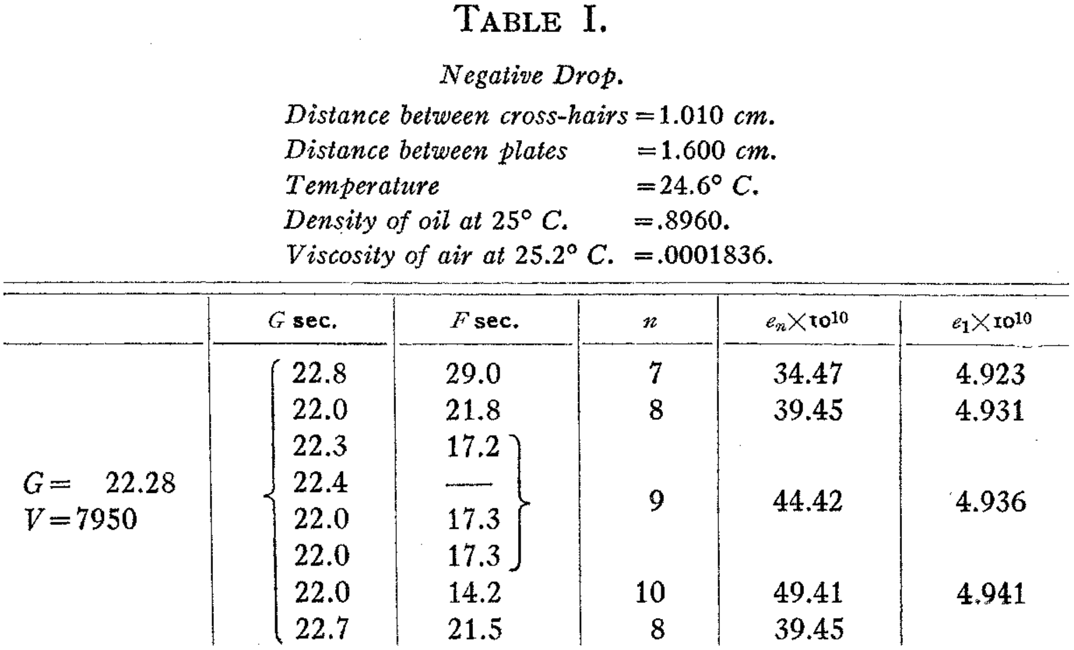


TABLE I.

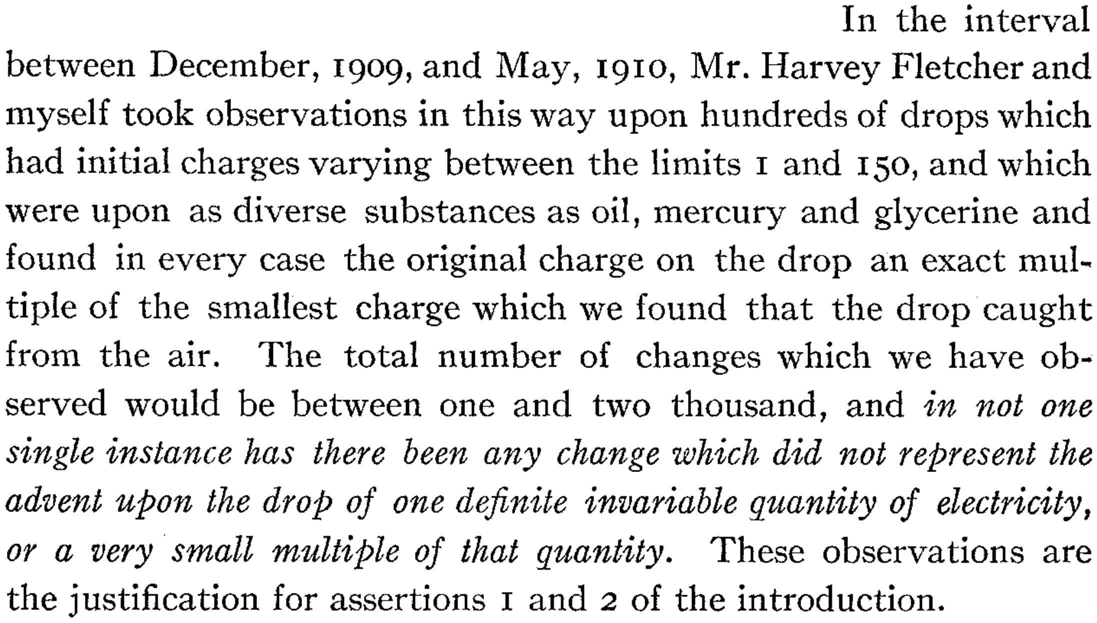
Negative Drop.

Distance between cross-hairs = 1.010 *cm.*
Distance between plates = 1.600 *cm.*
Temperature = 24.6° *C.*
Density of oil at 25° *C.* = .8960.
Viscosity of air at 25.2° *C.* = .0001836.

	G sec.	*F* sec.	*n*	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
G = 22.28 *V* = 7950	22.8	29.0	7	34.47	4.923
	22.0	21.8	8	39.45	4.931
	22.3	17.2	9	44.42	4.936
	22.4	——			
	22.0	17.3			
	22.0	17.3			
	22.0	14.2	10	49.41	4.941
	22.7	21.5	8	39.45	

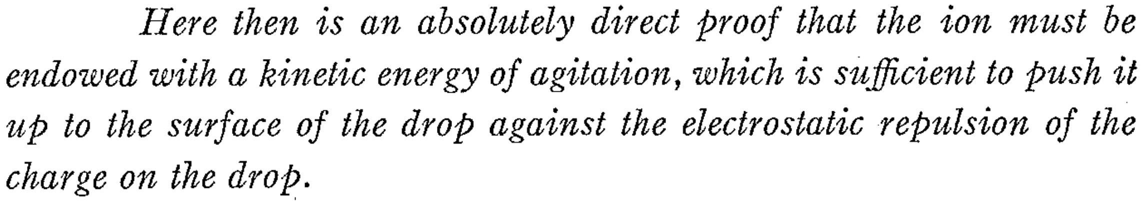
Column *G* stands for successive times in seconds taken by droplet to fall under gravity. While for *F* this is the time of ascent under the field. The column *n* stand for the numbers by them the values of proceeding column must be divided to obtain the numbers in the last column for e_1. The column e_n stands for average values of all observations of the table based on the values of column *n*. He ends this table with an average value of $e_{1s} = 4.917$.

Now the exact value of the charge carries by any droplets is gained in terms of mg from the difference values of e_n. The value of m is calculated approximately by Stokes's law. Then the approximate value of charge on the captured ion is given by the difference between the values of e_n. The mean value of all these differences would be 4.93×10^{-10} electrical units. He describes the at most accuracy of these experiments as below.



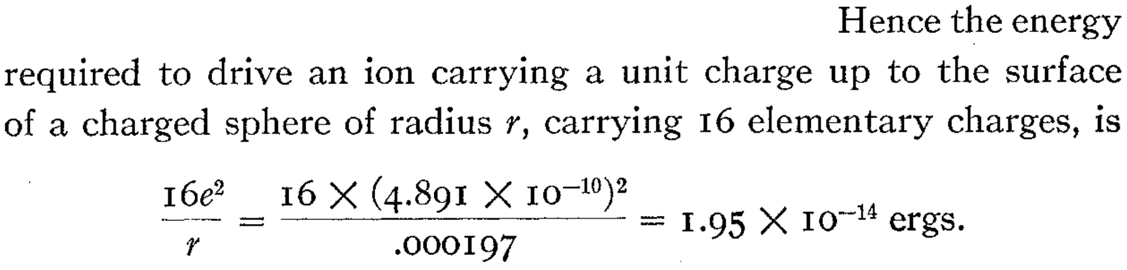
> In the interval between December, 1909, and May, 1910, Mr. Harvey Fletcher and myself took observations in this way upon hundreds of drops which had initial charges varying between the limits 1 and 150, and which were upon as diverse substances as oil, mercury and glycerine and found in every case the original charge on the drop an exact multiple of the smallest charge which we found that the drop caught from the air. The total number of changes which we have observed would be between one and two thousand, and *in not one single instance has there been any change which did not represent the advent upon the drop of one definite invariable quantity of electricity, or a very small multiple of that quantity.* These observations are the justification for assertions 1 and 2 of the introduction.

In his observations on the energy of agitation of a molecule he gets to the point that,



> *Here then is an absolutely direct proof that the ion must be endowed with a kinetic energy of agitation, which is sufficient to push it up to the surface of the drop against the electrostatic repulsion of the charge on the drop.*

He continues as: since the radius of the droplet in this case is 0.000197 and the mean value of elementary charge based on above observations is 4.891×10^{-10} then,



> Hence the energy required to drive an ion carrying a unit charge up to the surface of a charged sphere of radius r, carrying 16 elementary charges, is
>
> $$\frac{16e^2}{r} = \frac{16 \times (4.891 \times 10^{-10})^2}{.000197} = 1.95 \times 10^{-14} \text{ ergs.}$$

However, his paper is very interesting to go on more but since we do not have enough space to consider his way of calculation on the whole thus, we consider the process in more concise way as the following.

We mentioned that, along with the spraying the droplets they attained charge due to friction. Of course, if "there was a voltage applied across the plates". In the absent the voltage the drop with weight p will fall with the velocity $v_1 = fp$ which f is a proportionality factor. While in the presence of electric field E the droplet carries the charge q and the velocity of falling v_2 will be $v_2 = f(qE - p)$. These are under the assumption that, "gravity and the electric field act in opposite directions". Therefore, we get,

$$q = p/Ev_1(v_1 + v_2)$$

It must be noted that, to measure the charge of a droplet, Millikan needed to ionize the air in the medium around the droplet. To this aim he employed the X-rays ("invisible rays emitted by solids bombarded by a stream of fast electrons"). Acquiring these ions of the air by the droplets the sign of their charges alters from q to q' with the velocity of v_2 to v_2' this simply defined as

$$\Delta q = q' - q \quad or\ \Delta q = p/Ev_1(v_2' - v_2)$$

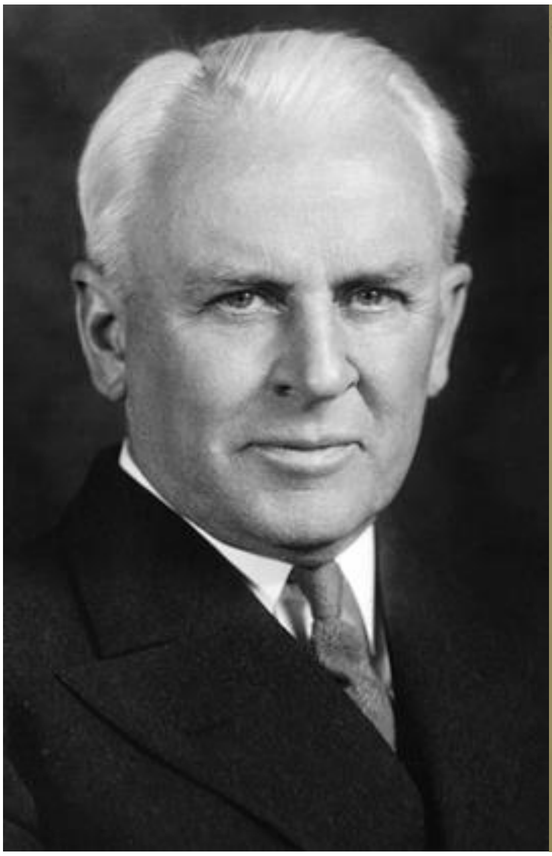
Now we say that, the value of the quantity p/Ev_1 for a given droplet is constant.

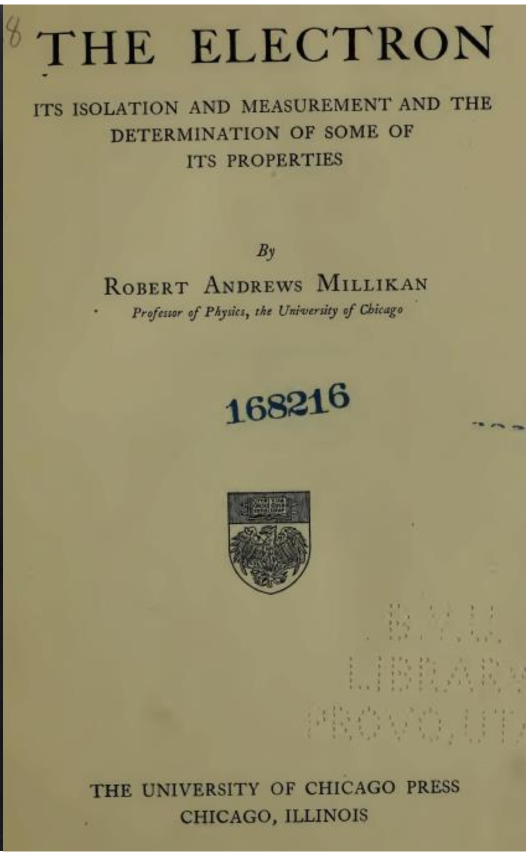
As mentioned in Millikan's first paper to find the charge of a droplet, two quantities are required. Its accurate traveling distance and its corresponding time. His experiment demonstrated that, apart from the size of the droplet,

$$\Delta q = ne$$

Such that, Δq is an "integral" multiple (n) of a "minimum" charge e. This shows the atomic nature of electricity and the reality of the minimum portion of electricity was proved. For these experiments, Millikan was rewarded the Nobel prize in 1923. He also published several course books in physics such as; *A Laboratory course in physics for secondary schools* in 1906 and *Practical physics* in 1922. However, his most famous boo is called *The Electron: Its Isolation and Measurements and the*

Determination of Some of its Properties which was published *The University of Chicago Press* in 1917.





8

THE ELECTRON

ITS ISOLATION AND MEASUREMENT AND THE
DETERMINATION OF SOME OF
ITS PROPERTIES

By

ROBERT ANDREWS MILLIKAN
Professor of Physics, the University of Chicago

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The modern definition for an electron says this subatomic elementary particle carries negative charge belongs to the first generation of the lepton particle family. It is denoted by e or $-e$. They represent the electric charge carried by a single proton or the magnitude of the negative electric charge carried by a single electron respectively. n the SI system of units, the value of the elementary charge is exactly defined as

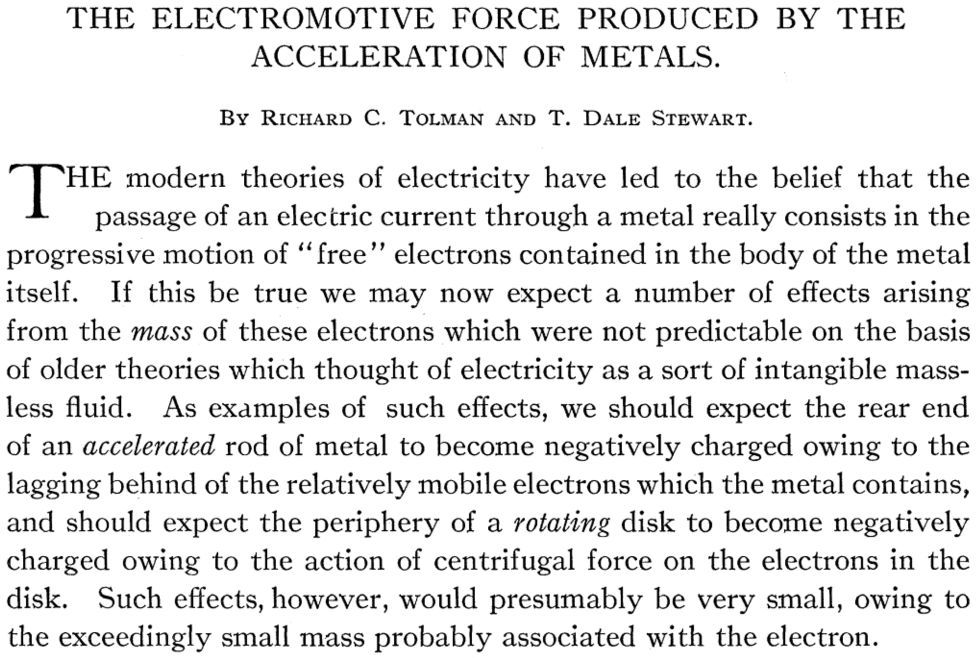
$$e = 1.602176634 \times 10^{-19} \text{coulombs}$$

We consider the question; how much does an electron weigh? Using the value of e and Thomson's result for the specific charge of the electron, e/m, we are now in a position to calculate the electron's mass m_e. We Obtain

$$m_e = 9.1093837015 \times 10^{-31} - 31 \text{ kg}$$

It turns out that the electron is 1830 times lighter than a hydrogen atom. In fact, it is the lightest stable subatomic particle.

In 1916 two American experimental chemist and physicists provided a direct proof for the existence of mass in an electron. They were **Thomas Dale Stewart** (1890–1958) from University of Chicago and **Richard Chace Tolman** (1881–1948) at Caltech. They published their works in physical review as "the electromotive force produce by the acceleration of metals."



THE ELECTROMOTIVE FORCE PRODUCED BY THE ACCELERATION OF METALS.

By Richard C. Tolman and T. Dale Stewart.

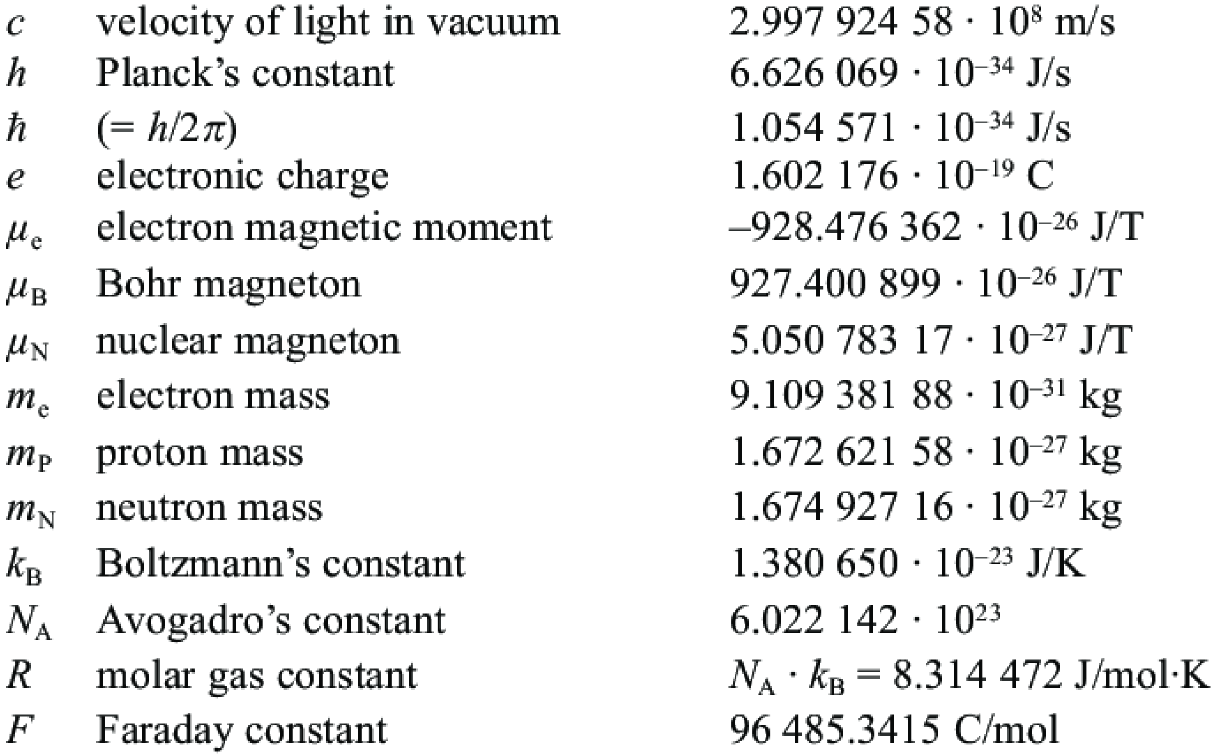
THE modern theories of electricity have led to the belief that the passage of an electric current through a metal really consists in the progressive motion of "free" electrons contained in the body of the metal itself. If this be true we may now expect a number of effects arising from the *mass* of these electrons which were not predictable on the basis of older theories which thought of electricity as a sort of intangible massless fluid. As examples of such effects, we should expect the rear end of an *accelerated* rod of metal to become negatively charged owing to the lagging behind of the relatively mobile electrons which the metal contains, and should expect the periphery of a *rotating* disk to become negatively charged owing to the action of centrifugal force on the electrons in the disk. Such effects, however, would presumably be very small, owing to the exceedingly small mass probably associated with the electron.

Using an ordinary coil of conducting wire they set their experiment. Firstly, the coil first set into rapid rotation. Then it was brusquely stopped. This was done since they assumed that the electron has mass and thus the inertia. If this is true after sudden stop of the coil the electric current should flow. That was the case. Thus, it was proved that the electron has mass.

We leave our curiosity about their work here since their results are strongly related to the constant c namely the speed of light which we will investigate its story in book two. Now let's sum up or pretty long way of electron.

The discovery of the electron was done using a large number of experiments and mathematical formulation. It was Thomson who displayed it existence. Millikan’s experiments, showed the existence of the minimum portion of charge it carries. While Stewart and Tolman investigation measured its mass. This discovery paved the way for investigating the quantum nature of electric charge. Properties such as half-integer value of electron's "spin" (intrinsic angular momentum).

Do you remember what of which of these constants considered in book one?



c	velocity of light in vacuum	$2.997\,924\,58 \cdot 10^{8}$ m/s
h	Planck’s constant	$6.626\,069 \cdot 10^{-34}$ J/s
$\hbar$	$(= h/2\pi)$	$1.054\,571 \cdot 10^{-34}$ J/s
e	electronic charge	$1.602\,176 \cdot 10^{-19}$ C
μ_e	electron magnetic moment	$-928.476\,362 \cdot 10^{-26}$ J/T
μ_B	Bohr magneton	$927.400\,899 \cdot 10^{-26}$ J/T
μ_N	nuclear magneton	$5.050\,783\,17 \cdot 10^{-27}$ J/T
m_e	electron mass	$9.109\,381\,88 \cdot 10^{-31}$ kg
m_P	proton mass	$1.672\,621\,58 \cdot 10^{-27}$ kg
m_N	neutron mass	$1.674\,927\,16 \cdot 10^{-27}$ kg
k_B	Boltzmann’s constant	$1.380\,650 \cdot 10^{-23}$ J/K
N_A	Avogadro’s constant	$6.022\,142 \cdot 10^{23}$
R	molar gas constant	$N_A \cdot k_B = 8.314\,472$ J/mol·K
F	Faraday constant	96 485.3415 C/mol

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